**Exercise Problems - Assignment 6**

# Statistical Mechanics and Thermodynamics

WS 2018

Due: 10th Dec 2018

JGU Mainz - Institut für Physik

## Choose 3 of the 5 problems below. Each problem is worth 12 points.

### Problem 1

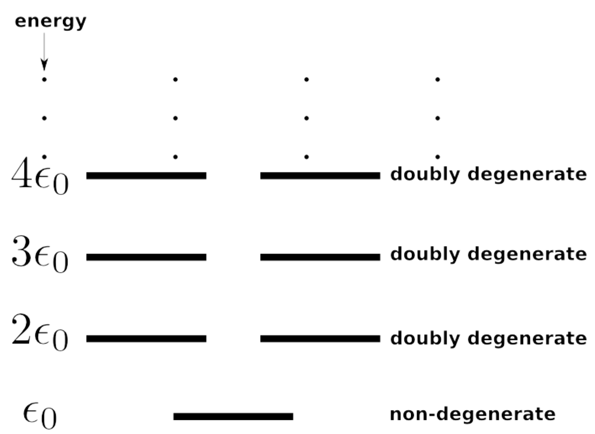
1. Show that the Hamiltonian of an $LC$ circuit is that of a harmonic oscillator. Identify the resonance frequency, and obtain the discrete energy levels of this system if it is quantized.

2. Calculate the mean energy of this system <u>if</u> it is at thermodynamic equilibrium with its surroundings which are at an absolute temperature $T$.

### Problem 2

1. Shown in the figure below is a set of available single-particle states and their energies. By properly filling these single-particle states, find the energies and degeneracies for a system of four identical non-interacting particles in the systems lowest energy level and in its first excited energy level, assuming that these particles are:
   - identical spinless bosons,
   - identical spinless fermions.

2. At a finite temperature T, calculate the ratio $r = P_1/P_0$, again for the two cases defined above. $P_1$ is the probability for finding the four-particle system in the first excited energy level, and $P_0$ is the probability for finding the same system in the lowest energy level.




### Problem 3

Consider a one-dimensional (non-harmonic) oscillator with energy given by

$$E = \frac{p^2}{2m} + bx^4$$

where p is the momentum and b is some constant. Suppose this oscillator is in thermal equilibrium with a heat bath at a sufficiently high temperature T so that classical mechanics is valid.

1. Compute its mean kinetic energy as a fraction of $kT$.

2. Compute its mean potential energy as a fraction of $kT$.

3. Consider a collection of such non-interacting oscillators all at thermal equilib- rium in one-dimension. What is the specific heat (per particle) of this system?

[Hint: You might use

$$\int_0^\infty x^{n-1} e^{-x} dx = \Gamma(n), \quad n \neq -1, -2, \ldots$$

or an integration by parts in solving this problem.]

## Problem 4

A molecule has energy $E = \frac{1}{2}mv^2 + \lambda j$, where v is the velocity, $\lambda$ is a constant with the dimension of energy, and $j$ is an internal quantum number which can take all odd integer values (no degeneracy). It is in a cubic box of volume V with its walls maintained at the absolute temperature T.

1. Calculate the average energy of the molecule as a function of T assuming that it is in thermodynamic equilibrium with the walls of the box.
2. Calculate the free energy of the molecule as a function of T .
3. Calculate the entropy S of the molecule as a function of T .

## Problem 5

The system consists of two noninteractiong quantum particles of the same nature each of which can be in the state $\psi_1(\mathbf{r}) = \psi_1(-\mathbf{r})$ or $\psi_2(\mathbf{r}) = -\psi_2(-\mathbf{r})$ where $\psi_1(\mathbf{r})$ and $\psi_2(\mathbf{r})$ are known functions.

1. Construct the wave function of the system assuming that the particles are a) spinless bosons; b) fermions with spin 1/2 and the total spin of the system S=1 (both particles are in the same spin sate).
2. Calculate the probability to find both particles in the half-space $z > 0$ for both cases (a and b). Which probability (for the case a or case b) is larger? Copare both probabilities with the case of the classical distinguishable particles.