Exercise Problems - Week 6

Statistical Mechanics and Thermodynamics

SoSe 2018

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JGU Mainz - Institut für Physik

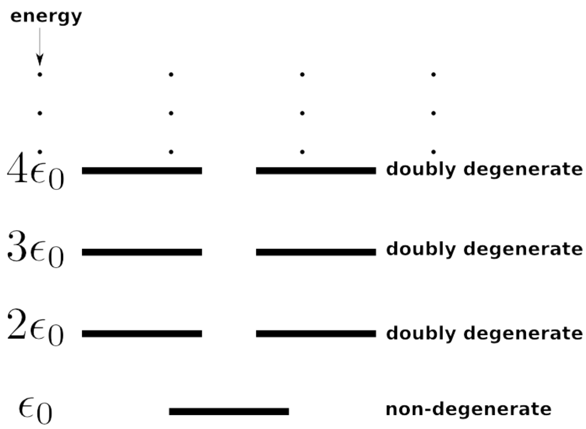
Choose 3 of the 5 problems below. Each problem is worth 12 points.

Problem 1

1. Show that the Hamiltonian of an LC circuit is that of a harmonic oscillator. Identify the resonance frequency, and obtain the discrete energy levels of this system if it is quantized.
2. Calculate the mean energy of this system <u>if</u> it is at thermodynamic equilibrium with its surroundings which are at an absolute temperature T.

Problem 2

1. Shown in the figure below is a set of available single-particle states and their energies. By properly filling these single-particle states, find the energies and degeneracies for a system of four identical non-interacting particles in the systems lowest energy level and in its first excited energy level, assuming that these particles are:
   - identical spinless bosons,
   - identical spinless fermions.
2. At a finite temperature T, calculate the ratio $r = P_1/P_0$, again for the two cases defined above. P_1 is the probability for finding the four-particle system in the first excited energy level, and P_0 is the probability for finding the same system in the lowest energy level.



Problem 3

Consider a one-dimensional (non-harmonic) oscillator with energy given by

$$E = \frac{p^2}{2m} + bx^4$$

where p is the momentum and b is some constant. Suppose this oscillator is in thermal equilibrium with a heat bath at a sufficiently high temperature T so that classical mechanics is valid.

1. Compute its mean kinetic energy as a fraction of kT.
2. Compute its mean potential energy as a fraction of kT.
3. Consider a collection of such non-interacting oscillators all at thermal equilib- rium in one-dimension. What is the specific heat (per particle) of this system?

[Hint: You might use

$$\int_0^\infty x^{n-1}e^{-x}dx = \Gamma(n), \quad n \neq -1, -2, \ldots$$

or an integration by parts in solving this problem.]

Problem 4

A molecule has energy $E = \frac{1}{2}mv^2 + \lambda j$, where v is the velocity, λ is a constant with the dimension of energy, and j is an internal quantum number which can take all odd integer values (no degeneracy). It is in a cubic box of volume V with its walls maintained at the absolute temperature T.

1. Calculate the average energy of the molecule as a function of T assuming that it is in thermodynamic equilibrium with the walls of the box.
2. Calculate the free energy of the molecule as a function of T .
3. Calculate the entropy S of the molecule as a function of T .

Problem 5

The system consists of two noninteractiong quantum particles of the same nature each of which can be in the state $\psi_1(\mathbf{r}) = \psi_1(-\mathbf{r})$ or $\psi_2(\mathbf{r}) = -\psi_2(-\mathbf{r})$ where $\psi_1(\mathbf{r})$ and $\psi_2(\mathbf{r})$ are known functions.

1. Construct the wave function of the system assuming that the particles are a) spinless bosons; b) fermions with spin 1/2 and the total spin of the system S=1 (both particles are in the same spin sate).
2. Calculate the probability to find both particles in the half-space $z > 0$ for both cases (a and b). Which probability (for the case a or case b) is larger? Copare both probabilities with the case of the classical distinguishable particles.