Exercise Problems - Week 3

Statistical Mechanics and Thermodynamics

SoSe 2018

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Choose 3 of the 5 problems below. Each problem is worth 12 points.

Problem 1

In an isolated system of N identical particles (N large) each particle can be in two energy states: $\epsilon_1 = 0$ and $\epsilon_2 = \epsilon > 0$. The total energy of the system is E. Find, as a function of E,

1. The entropy of the system.
2. The temperature of the system.

Note that $\log n! = n \log n - n$

Problem 2

There are approximately $(NM)^n/(n!)^2$ ways of removing n atoms from a lattice with N sites and distribute them over M interstitial sites to obtain n Frenkel defects. The energy of an atom at an interstitial site is $+\epsilon$ relative to the energy at the lattice site, taken as zero.

1. Obtain an expression for the (Boltzmann) entropy $S_B(E)$ of the system in the microcanonical ensemble.
2. Calculate the temperature T as a function of E, and find the most probable excitation energy E and defect number n as a function of T.
[**Hint**: You may need to use: $ln\ N! \approx (N\ ln\ N) - N$.]

Problem 3

Consider a large number N of identical non-interacting particles, each of which can be in only one of two states, with energies ϵ_0 and ϵ_1. Denote by n_0 and n_1 the occupation numbers of each of these states.

1. Find the total energy E as a function of n_0.
2. Find the number of states N available to the system, as a function of n_0
3. Find S, the entropy, as a function of E.
4. Find the temperature T as a function of E.
5. Using $E_0 = N\epsilon_0$ and $\Delta\epsilon = (\epsilon_1 - \epsilon_0)$, show that

$$E = E_0 + \frac{N\Delta\epsilon}{1 + \exp\left[\Delta\epsilon/k_B T\right]}$$

[**Hint:** $\log n! \approx n \log n - n$, for large n.]

Problem 4

Show that the expression for the entropy derived in Sec. 1.4

$$S(N, V, E) = N k_B \ln\left[\frac{V}{h^3}\left(\frac{4\pi m E}{3N}\right)^{3/2}\right] + \frac{3}{2} N k_B$$

is not extensive and leads to the wrong result when considering the mixing of two gases. Show how it needs to be fixed and that it leads to the expression

$$S(N, V, E) = N k_B \ln\left[\frac{V}{N h^3}\left(\frac{4\pi m E}{3N}\right)^{3/2}\right] + \frac{5}{2} N k_B$$

Problem 5

Study the statistical mechanics of an extreme relativistic gas (along the lines of Sec. 1.4 in the main book) for which the single-particle energy of states are

$$\epsilon(n_x, n_y, n_z) = \frac{hc}{2L}(n_x^2 + n_y^2 + n_z^2)^{1/2}$$

Obtain its entropy and show that the ratio $C_P/C_V = 4/3$.