Exercise Problems - Week 1

Statistical Mechanics and Thermodynamics

SoSe 2018

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JGU Mainz - Institut für Physik

Choose 4 of the 6 problems below. Each problem is worth 12 points.

Problem 1

An engine with one mole of a monoatomic gas as the working fluid undergoes the following cycle:

(a) An adiabatic expansion from pressure P_0, volume V_0 to a volume $2V_0$;

(b) An isothermal compression from $2V_0$ to V_0;

(c) At constant volume to the original state.

1. Find the efficiency of the engine
2. What is the change in the entropy of the gas during the second leg of the cycle?
3. What is the change in internal energy of the gas in the first leg of the cycle?

Problem 2

An engine with one mole of a monoatomic ideal gas as a working substance undergoes the following process:
$1 \to 2$ isobaric expansion $V_0 \to 2V_0$
$2 \to 3$ isothermal expansion $P_0 \to P_0/2$
$3 \to 4$ isobaric compression at $P_0/2$
$4 \to 1$ isothermal compression at T_0

1. Find the net work done in the process in terms of P_0, V_0
2. Find the net heat absorbed in the total process in terms of P_0, V_0
3. Find the entropy change in going from 1 to 2 in terms of R
4. Find the entropy change in going from 2 to 3 in terms of R
5. What is the net entropy change in the total process?

Problem 3

A cylinder closed at both ends equipped with insulating (adiabatic) walls, and is divided into two parts with a frictionless, insulating, movable piston. The gases on both sides are initially at equilibrium with identical pressure, volume, and temperature (P_0, V_0, T_0). The gas is ideal with $C_V = 3R/2$ and $C_P/C_V = 5/3$. By means of a heating coil in the gas on the left hand side, heat is slowly supplied to the gas on the left until the pressure reaches $32P_0$. In terms of P_0, V_0 and T_0

1. What is the final right hand volume?
2. What is the final right hand temperature?
3. What is the final left hand temperature?
4. How much heat must be supplied to the gas on the left?
5. How much work is done on the gas on the right?
6. What is entropy change of the gas on the right?
7. Compute the entropy change of the gas on the left.

Problem 4

This problem refers to a classical monoatomic gas.

1. On a $T - S$ digram, draw an isothermal expansion, an isobaric expansion, and a constant volume process.
2. Draw a Carnot cycle on a $T - S$ diagram. Assume that the Carnot cycle operates between T_1 and T_2, with $T_1 > T_2$.
3. Derive the expression for the efficiency of a Carnot cycle using this $T - S$ diagram.

Problem 5

1. A body of finite, temperature independent, heat capacity is originally at temperature T_1 which is higher than the temperature of a heat reservoir T_2. Suppose that a heat engine operates in infinitesimal cycles between the body and the reservoir, until it lowers the temperature of the body from T1 to T2 extracting Q (heat) and does work W during the process. Prove that the maximum amount of work obtainable is

$$W_{\max} = Q - T_2 \left(S_1 - S_2 \right),$$

   where $S_1 - S_2$ is the entropy decrease of the body.

2. Two identical bodies, each of constant heat capacity C are at the same initial temperature T_i. A refrigerator operates between these two bodies until one of them is cooled to T_2. Calculate using the same principle as the one in the previous question the final temperature of the high temperature body in terms of T_i and T_2. Show that the minimum amount of work needed is

$$W = C \left(\frac{T_i^2}{T_2} + T_2 - 2T_i \right).$$

Problem 6

1. A membrane separates two chambers of equal volume. Only one side contains gas, the other is a vacuum. The system as a whole is thermally isolated from the world, and its walls are rigid. If the membrane is burst (neglect the energy associated with the membrane) which of the following quantities will remain unchanged after the system adjusts to the doubled volume?
   (a) total entropy S
   (b) total particle number N
   (c) total energy E
   (d) temperature T
   (e) pressure P
   (f) chemical potential μ
2. During which periods of time can one describe the entropy of the system as a unique function of energy, volume, and particle number? Comment on your answers for each part.
   (a) Initially.
   (b) During the expansion.
   (c) After the adjustment to the doubled volume.