# Spin-Orbit Twisted Spin Waves in 2 Dimensional Electron Liquids


## Florent Perez

*Institut des NanoSciences de Paris, Université Pierre et Marie Curie, CNRS-UMR7588*


*October 25th, 2016*


**Theory**

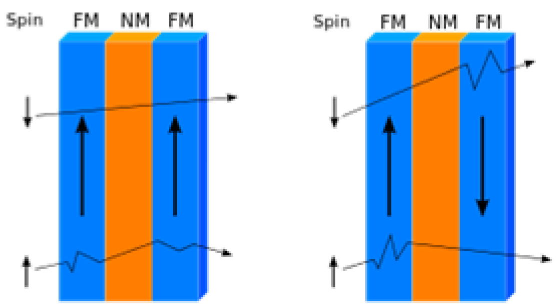
**C. A. Ullrich, G. Vignale**
University of Missouri, USA

**I. D'Amico**
University of York, UK

# Spintronic thematics evolution

Spintronics started in the mid 90’s, with Giant magneto resistance (2007 Nobel Prize)

## SPIN-POLARIZED CURRENT



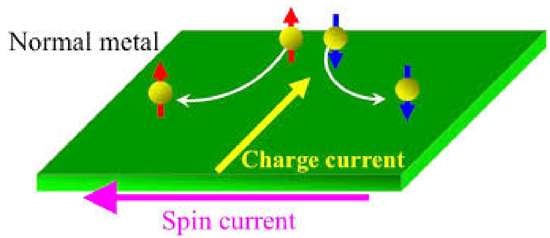

***SPIN-VALVE GMR 90’s***

$J_{charge} \neq 0$

$J^{z}_{spin} \neq 0$

**STORAGE**

## PURE SPIN CURRENT



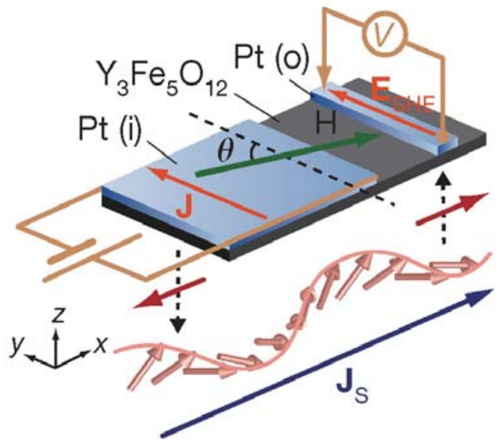

***SPIN-HALL EFFECT 2004***

$J_{charge} = 0$

$J^{z}_{spin} \neq 0$

**LESS DISSIPATIVE TREATMENT**

## MAGNONICS




***SPIN-WAVE SPINTRONICS 2006-***

$J_{charge} = 0, J^{z}_{spin} = 0$

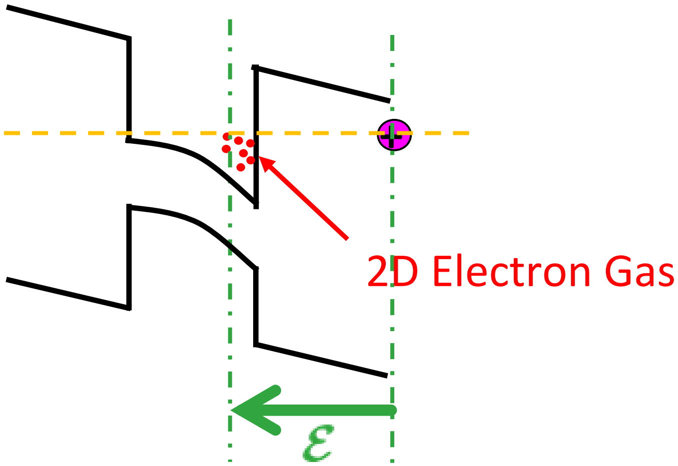
$J^{+}_{spin} \neq 0$

**EVEN LESS DISSIPATIVE TREATMENT**

# SCOPE

- Spin-wave control by spin-orbit interaction: important facts
- Twisted spin waves (our work)
- Comparison with chiral spin waves in conducting ferromagnet
- Perspectives

# (1) SPIN-ORBIT in the CONDUCTION BAND of a 2D Galilean invariant



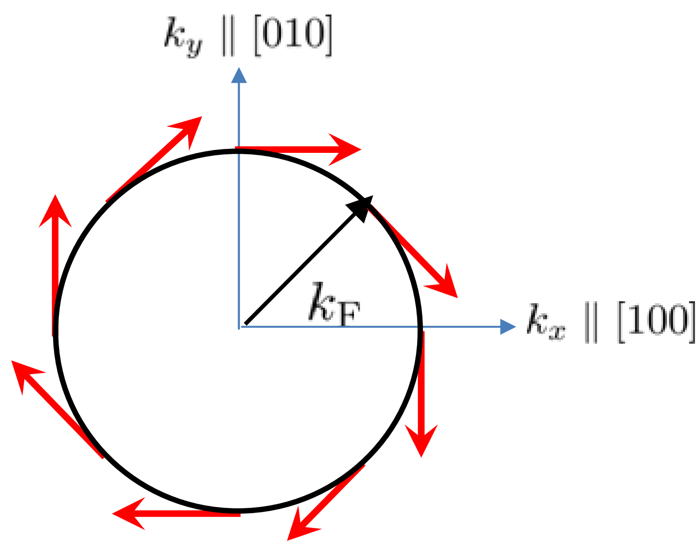

Structural Inversion Asymetry

**Rashba term**

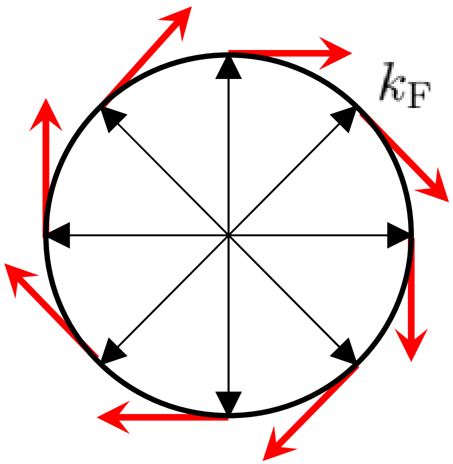
$$\mathbf{B}_{\mathrm{SO}} = -\frac{1}{c^2}\mathbf{v} \times \mathcal{E}$$

$$\hat{H}_{SO} = \mathbf{B}_{SO}(\mathbf{k}) \cdot \frac{\widehat{\sigma}}{2}$$

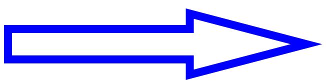
$$\mathbf{B}_{\mathrm{SO}}(\mathbf{k}) = 2\,\overbrace{\alpha \begin{pmatrix} k_y \\ -k_x \end{pmatrix}}$$



# (2) SPIN-ORBIT TORQUE in CONDUCTION BAND : Inverse Spin-Galvanic

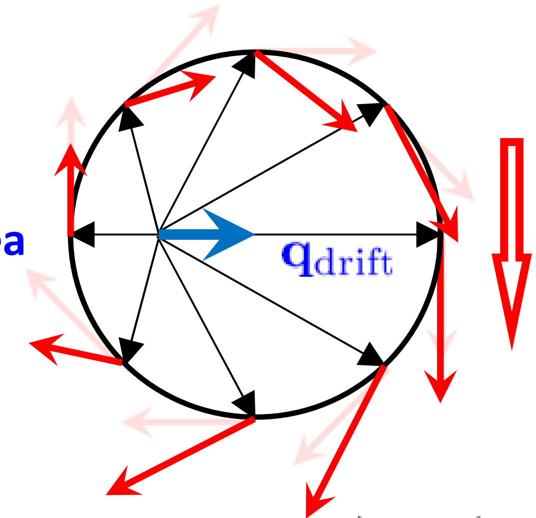



$\langle \mathbf{S}(\mathbf{k}) \rangle_{\mathbf{k}} = 0$



**Electric field**

**Drifting Fermi sea**

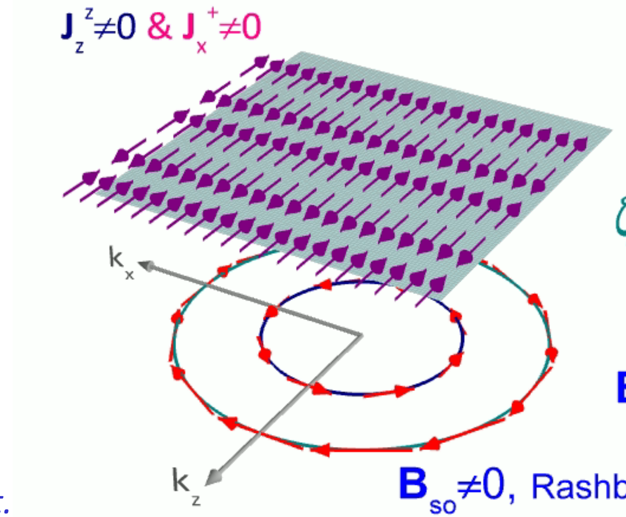



$\langle \mathbf{S}(\mathbf{k}) \rangle_{\mathbf{k}} \parallel \mathbf{B}_{\mathrm{SO}}(\mathbf{q}_{\mathrm{drift}})$

**Steady state spin-polarization**

$$\hbar \frac{d\hat{\sigma}}{dt} = \mathbf{B}_{SO}\left(\mathbf{k}+d\mathbf{k}\right) \times \hat{\sigma}$$

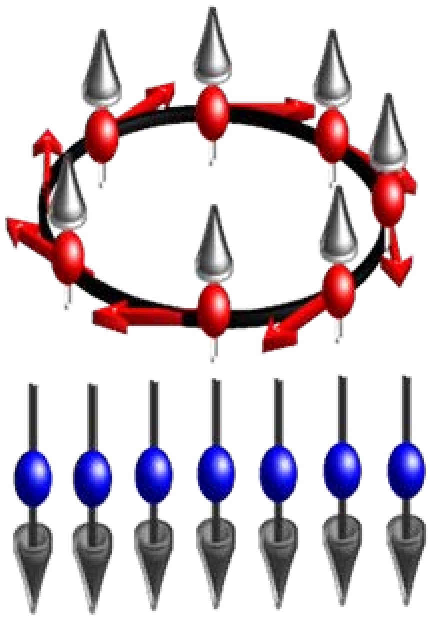
*V. M. Edelstein, Solid State Comm. 1990*
*J. Sinova et al., Phys. Rev. Lett. 2004*
*S. Ganichev et al., Nature 2002, J.Mag Mag Mat.*



# (3) SPIN-ORBIT TORQUE in CB : 2 SPIN SYSTEMS

$$\langle \mathbf{S}(\mathbf{k}) \rangle_{\mathbf{k}} = \mathbf{S}_{z0}$$

**Electric field**



$$\left\langle \hat{\mathbf{M}} \right\rangle = \hat{\mathbf{M}}_{z0}$$

**Edelstein effect**

$$\langle \mathbf{S}(\mathbf{k}) \rangle_{\mathbf{k}} = \mathbf{S}_{z0} + \delta\mathbf{S}_y$$

$$\hat{H}_{\mathrm{DMS}} = -J_{sd}\, \hat{\mathbf{S}} \cdot \hat{\mathbf{M}}$$

$$\frac{d\hat{\mathbf{M}}}{dt} = \underbrace{-J_{sd}\, \delta\mathbf{S}_y \times \hat{\mathbf{M}}}_{\mathbf{T}}$$

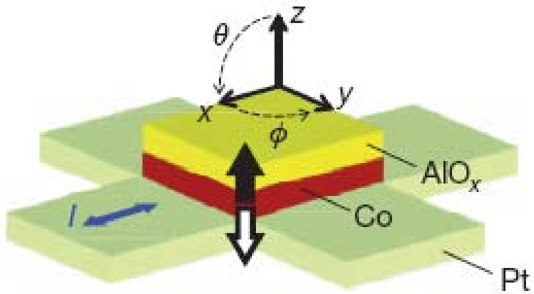
*A. Manchon et al., Phys. Rev. B 2009*

# (4) SPIN-ORBITRONIC : HYBRID STRUCTURES : Rashba/FM

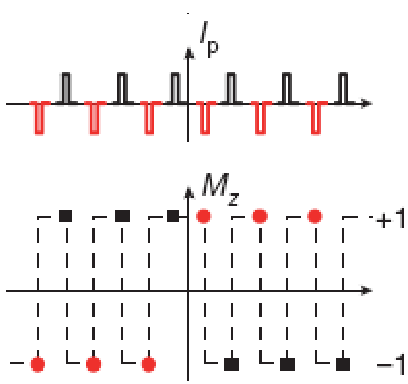
## Magnetization switching

$$\left\langle \hat{\mathbf{M}} \right\rangle = \hat{\mathbf{M}}_{z0}$$

**Electric field**

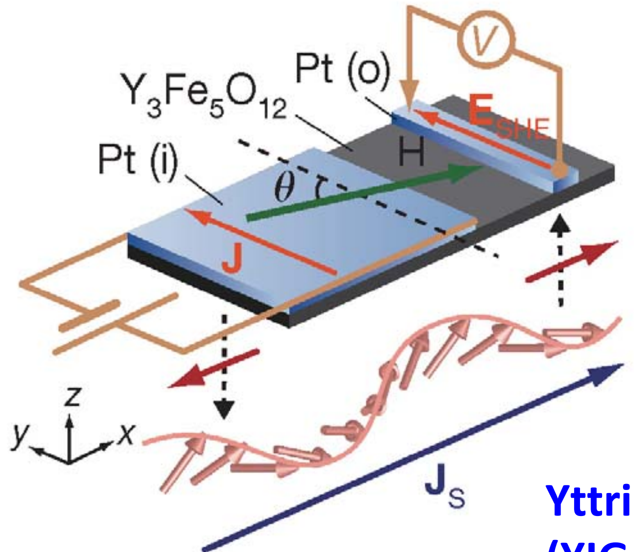



$$\langle \mathbf{S}(\mathbf{k}) \rangle_{\mathbf{k}} = \mathbf{S}_{z0} + \delta \mathbf{S}_y$$




*I. Miron et al., Nat. Letter 2011*

## Spin wave transistor




**Yttrium-Iron-Garnet (YIG, insulator)**

*Kajiwara et al., Nat. Lett. 2010*

# (5) STATE OF The ART for Controlling SW with SO

**Natural Route :**

**Hybrid heterostructure with Structural Asymetry => Rashba type SO**

**=**

**Ferromagnet Insulator (Magnons, localized spins, no SO, long lived)**

**+**

**Adjacent conducting material with strong SO**

**Less natural Route :**

**Itinerant magnet (Spin Waves of conducting spins, short lived)**

**+**

**Conduction with SO**

**(Spin-orbit and Spin-Spin interactions are supported by the same medium)**

**+**

**Interplay of SO and Coulomb-exchange ?**

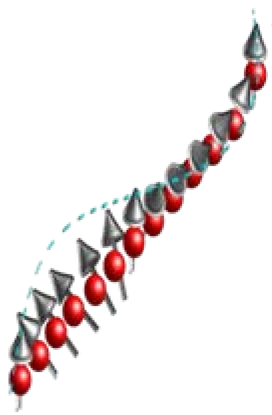
# SCOPE

- Spin-wave control by spin-orbit interaction
- Twisted spin waves (our work)
  - Model system : SP2DEG, Spin waves by Raman
  - First and 2nd interpretations
  - Final : Twisted Spin Waves
  - Group velocity control
- Comparison with chiral spin waves in conducting ferromagnet

# Spin-Orbit and Spin Waves from first principles in a model system

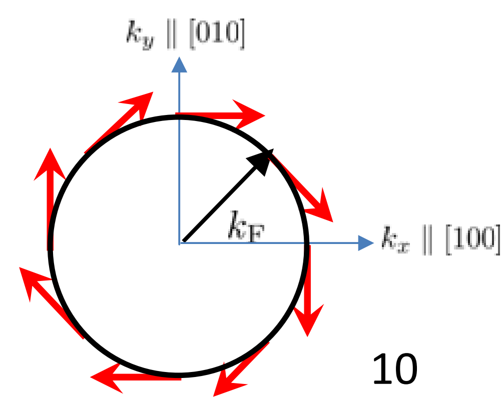
**Model system : Spin-polarized two-dimensional electron gas**

$$\hat{H}_{2DEG} = \underbrace{\underbrace{\sum_i \frac{\hat{\mathbf{p}}_i^2}{2m^*}}_{\hat{H}_K} + Z\underbrace{\sum_i \frac{1}{2}\hat{\sigma}_{z,i}}_{\hat{H}_Z} + \hat{H}_{\text{Coulomb}}}_{\text{Spin Waves}} + \underbrace{\sum_i \mathbf{B}_{SO}(\mathbf{k}_i)\cdot\frac{\widehat{\sigma_i}}{2}}_{\hat{H}_{SO}}$$



**HSP2DEG**

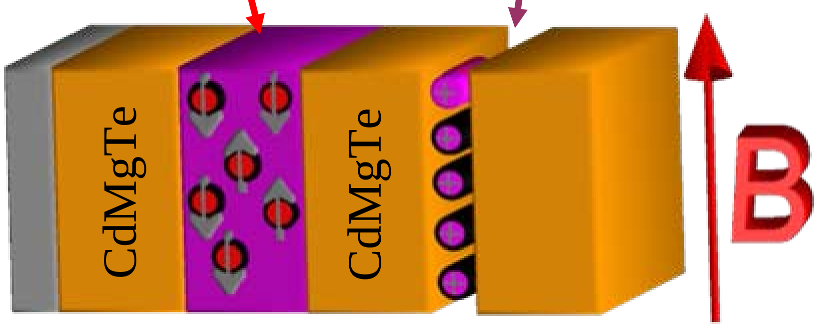
**HSO**



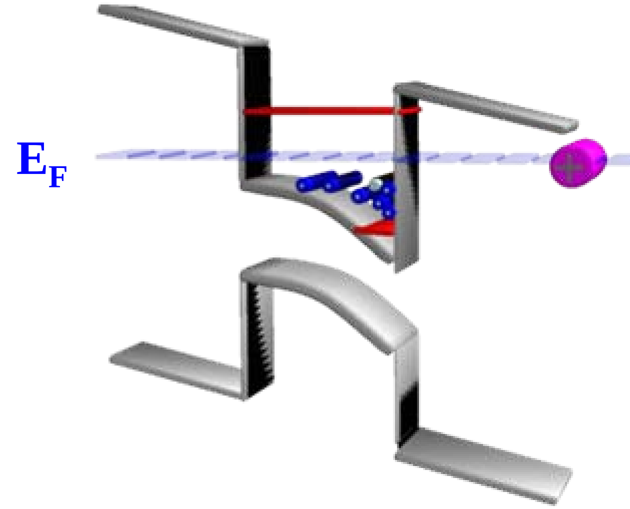

F. Baboux, F. Perez *et al.*, Phys. Rev. Lett. **109**, 166401 (2012) ;
F. Baboux, F. Perez *et al.*, PRB Rapid Comm. **87**, 121303 (2013)
Theory : C. Ullrich et al., PRB 2002 & 2003 ; I. d'Amico

# Model system : CdMnTe doped quantum well

Mn ($x$<1%) Iodine ($n_{2D}$ ~ 1.5-4 ×$10^{11}$cm$^{-2}$)

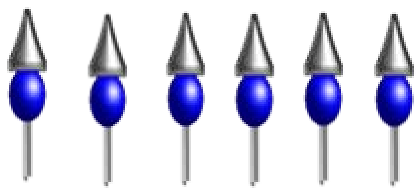



$Cd_{1-x}Mn_xTe$

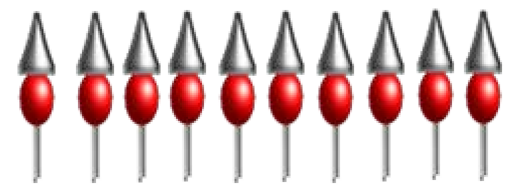



**Two interacting spin sub-systems :**

**e$^-$ 1/2-spin system**



**Mn 5/2-spin system**

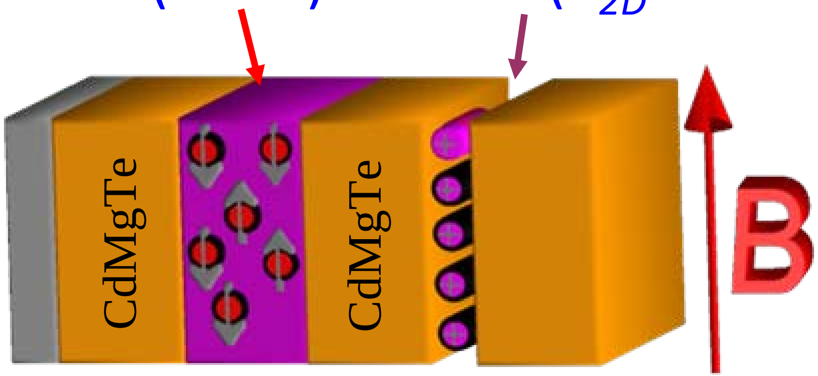


$$\hat{H}_{s-d} = -\alpha \sum_{i,j} \chi^2\left(y_j\right)\hat{\mathbf{s}}_i \cdot \hat{\mathbf{I}}_j$$

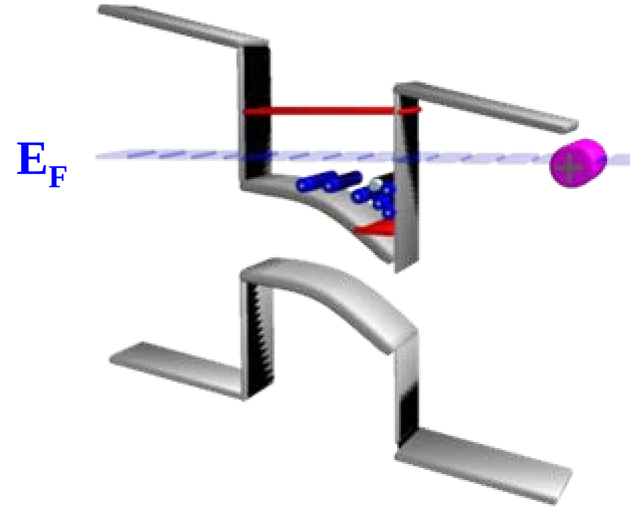
$$= \Delta \times \hat{S}_{z,q=0} + K \times \hat{M}_{z,q=0} + \hat{H}_{\mathrm{Corr}}$$

# Model system : CdMnTe doped quantum well

Mn ($x$<1%)   Iodine ($n_{2D}$ ~ 1.5-4 ×$10^{11}$cm$^{-2}$)



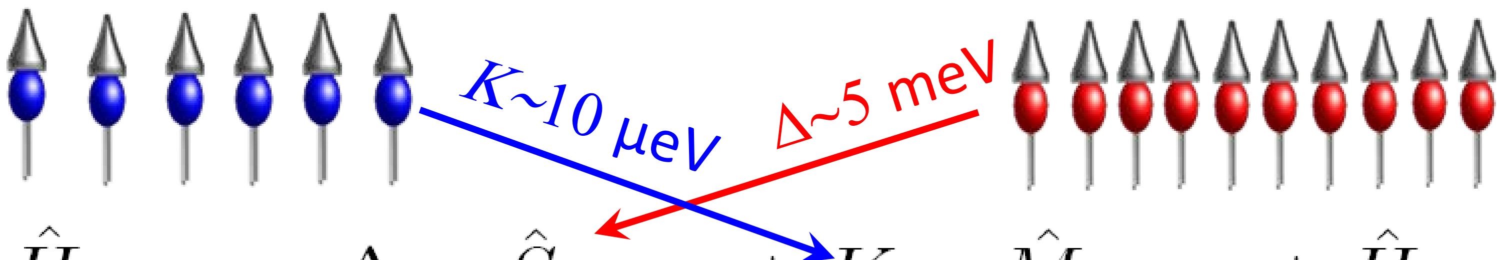

$Cd_{1-x}Mn_xTe$




**Two interacting spin sub-systems :**

**$e^-$ 1/2-spin system**

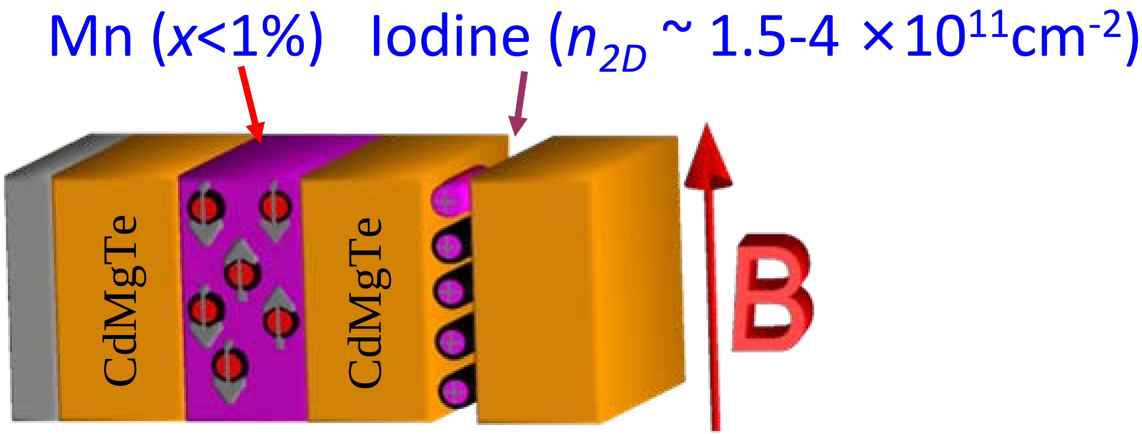
**Mn 5/2-spin system**



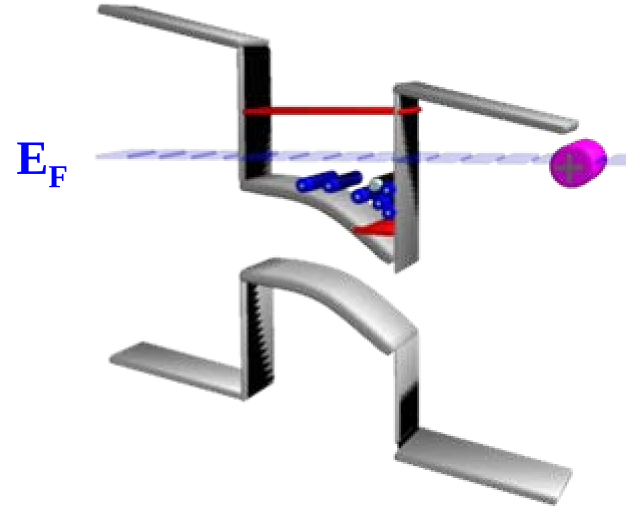

$$\hat{H}_{s-d} = \Delta \times \hat{S}_{z,q=0} + K \times \hat{M}_{z,q=0} + \hat{H}_{\mathrm{Corr}}$$

$$d_{Mn-Mn} \ll \lambda_F \Leftrightarrow K \ll \Delta$$

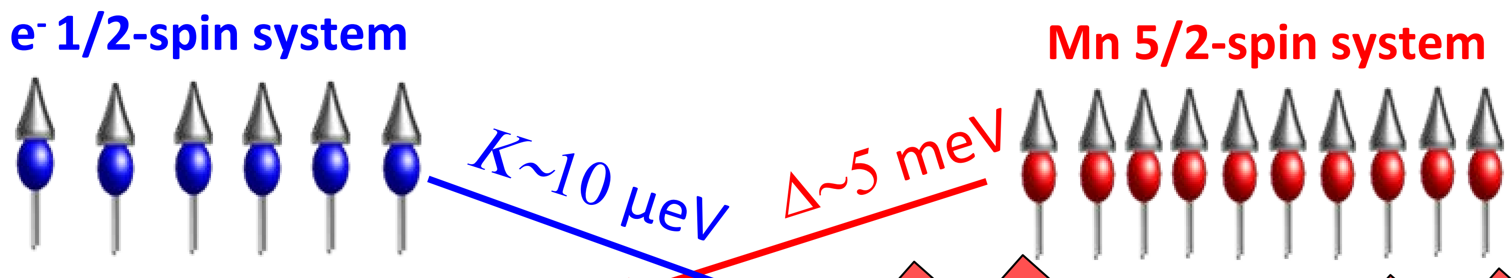
# Model system : CdMnTe doped quantum well




$Cd_{1-x}Mn_xTe$



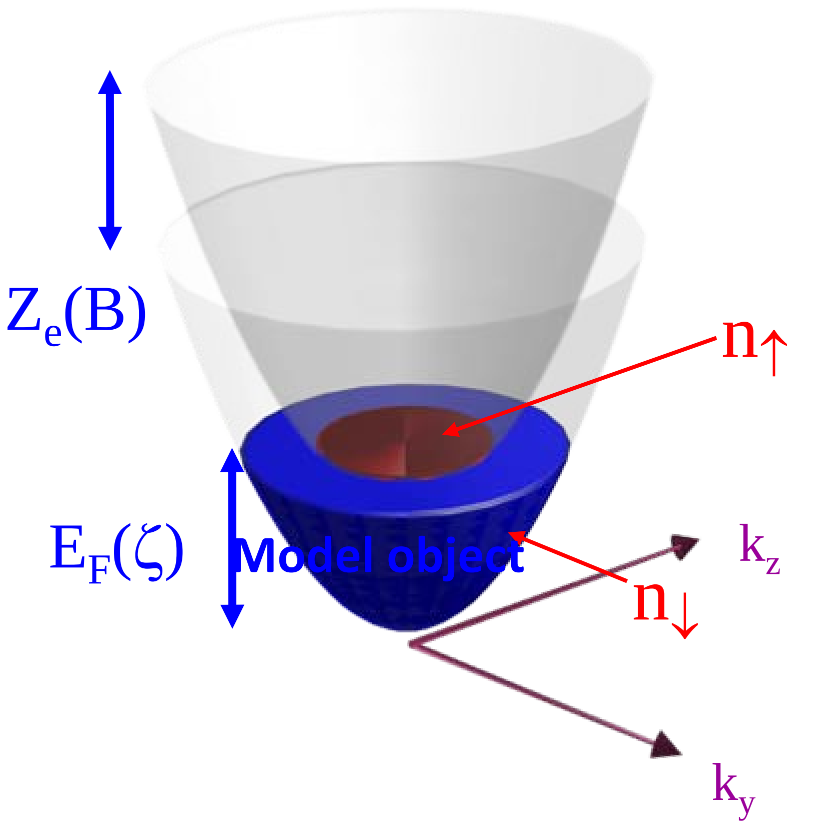

**Two interacting spin sub-systems :**




$$\hat{H}_{s-d} = \Delta \times \hat{S}_{z,q=0} + K \times \hat{M}_{z,q=0} + \hat{H}_{\mathrm{Corr}}$$

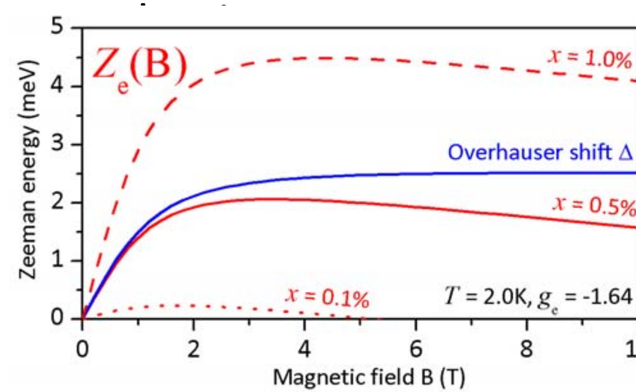
$$Z_e(B) = \underbrace{xN_0\alpha \langle I_z(B,T)\rangle}_{\Delta} - |g_e|\,\mu_B B$$

# SP2DEG



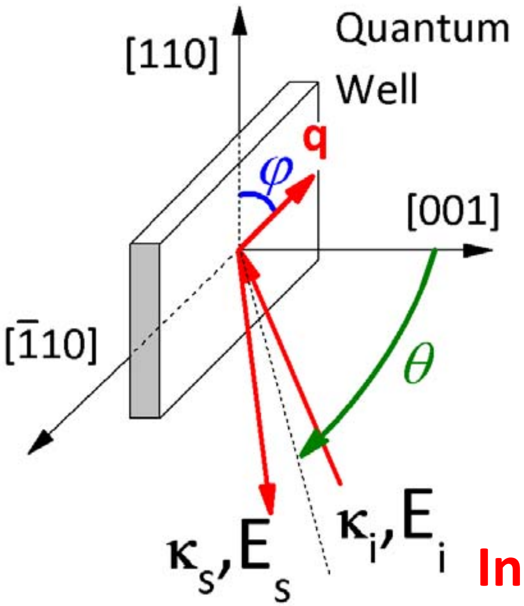

No Landau quantization

- Spin quantization dominates over orbital quantization (opp. GaAs)
- Mobilities are up to $10^5$ $cm^2/Vs$
- High spin polarization degree (up to 100%)
- Tunable, paramagnetic ($T_c$<1K)




*B. Jusserand, F. Perez et al. Phys. Rev. Lett. (2003)*
*F. Perez, et al. Phys. Rev. Lett. (2007)*

# Raman setup




**Incoming photon**

**Scattered photon**

## Raman : Inelastic scattering

### Conservation laws :

energy :

$$\hbar\omega = E_i - E_s$$

wavevector :

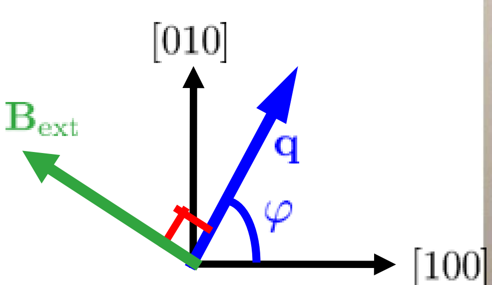
$$\mathbf{q} = (\boldsymbol{\kappa}_i - \boldsymbol{\kappa}_s)_{\|}$$

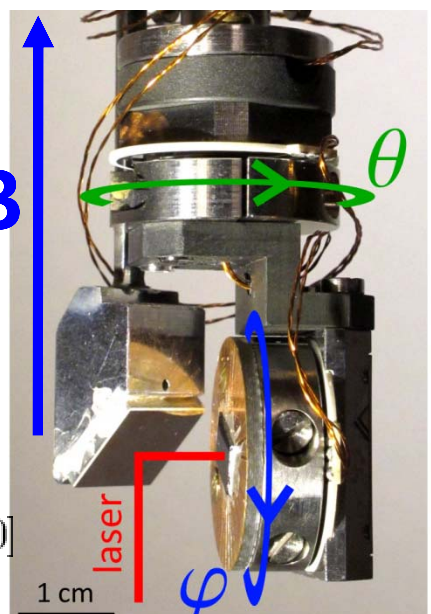
Dispersion $\hbar\omega(\mathbf{q})$

$$q = \frac{4\pi}{\lambda}\sin\theta$$

$$\lambda \simeq 770 \text{ nm}$$

$$0 < q < 15\ \mu\text{m}^{-1} \sim 0.1\, k_F$$



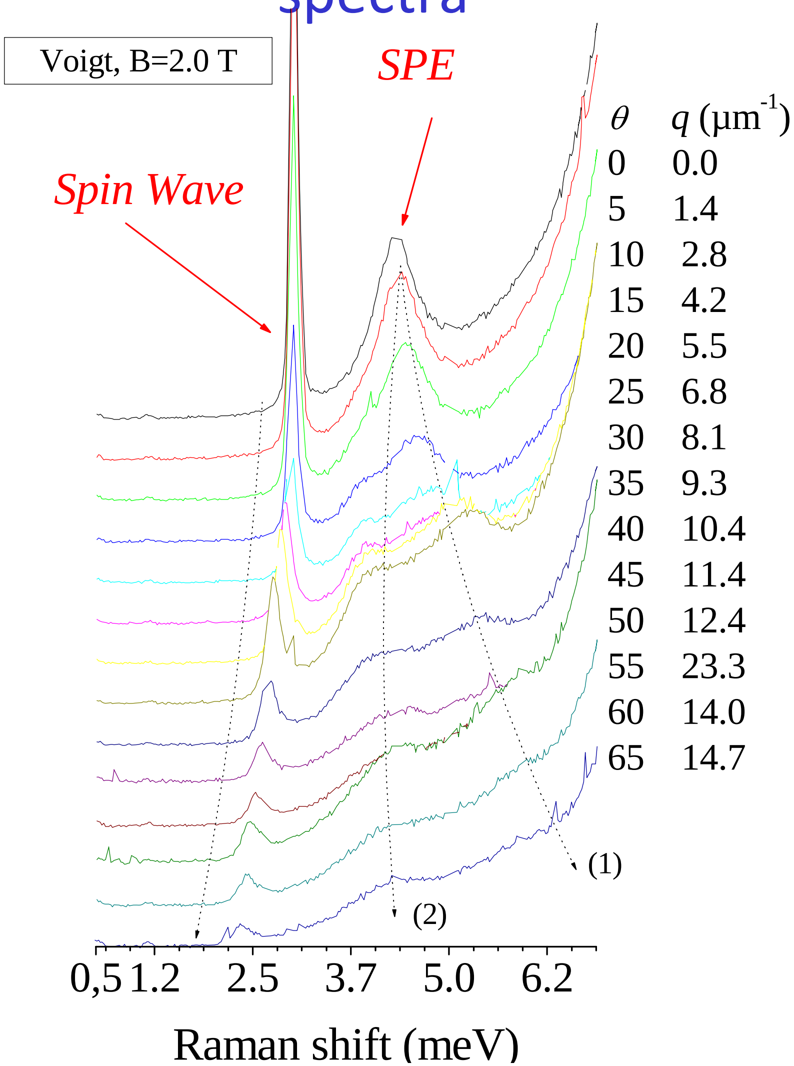




### Selection rules :

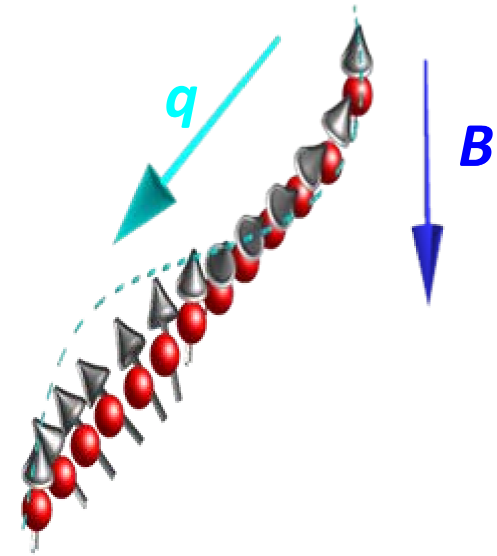
**Crossed polarizations ⇔ transverse spin excitations**

# Spin wave dispersion : *q*>0

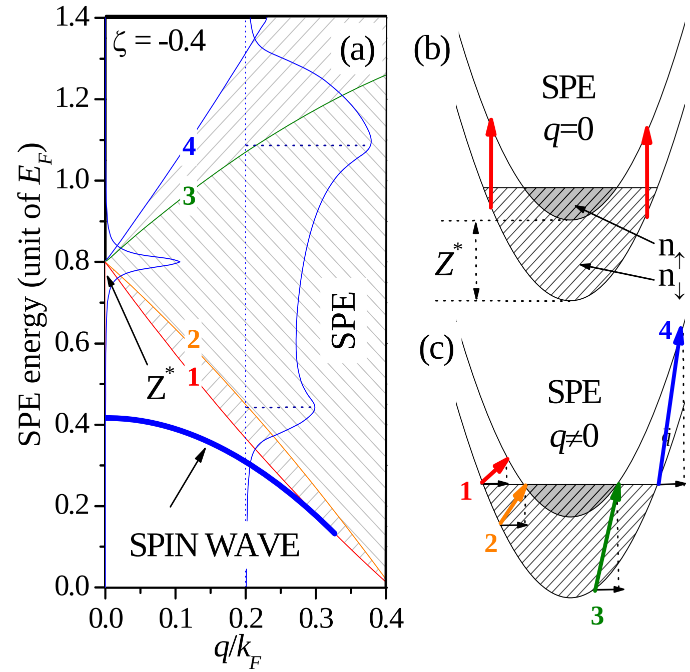
## Crossed polarized Raman spectra




## Spin Wave

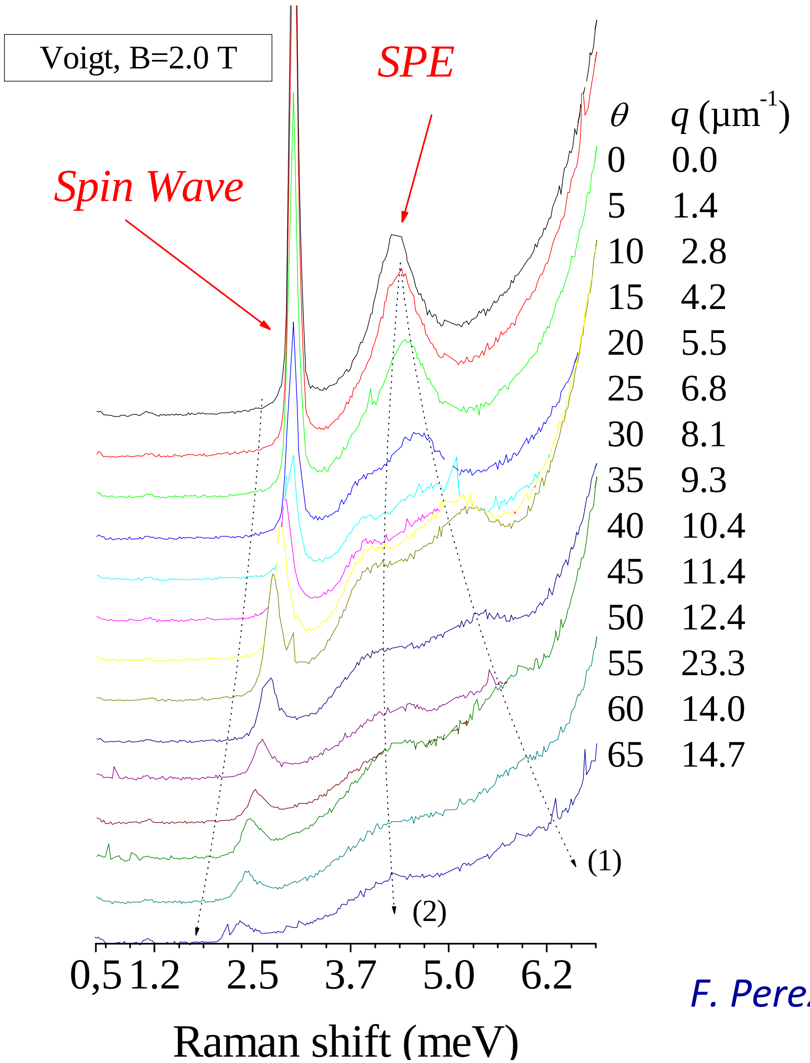


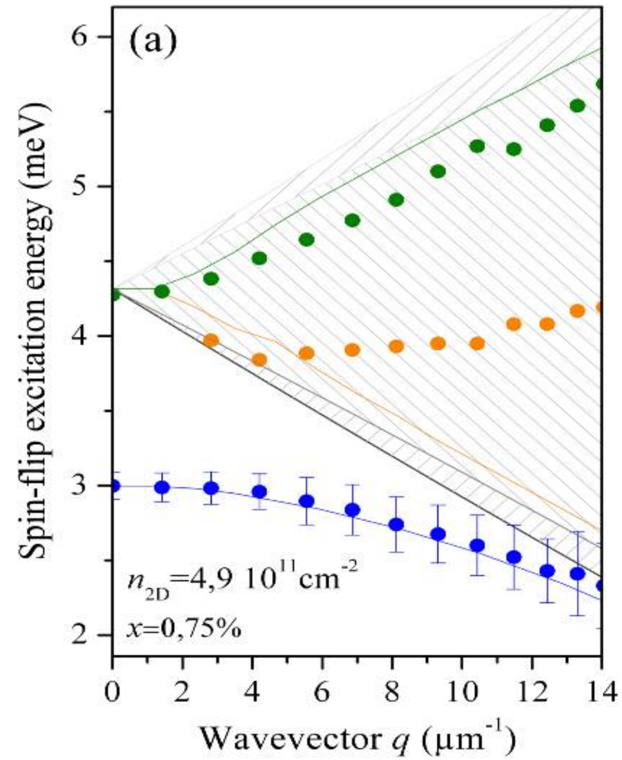

## SPE



# Spin wave dispersion : *q*>0

## Crossed polarized spectra

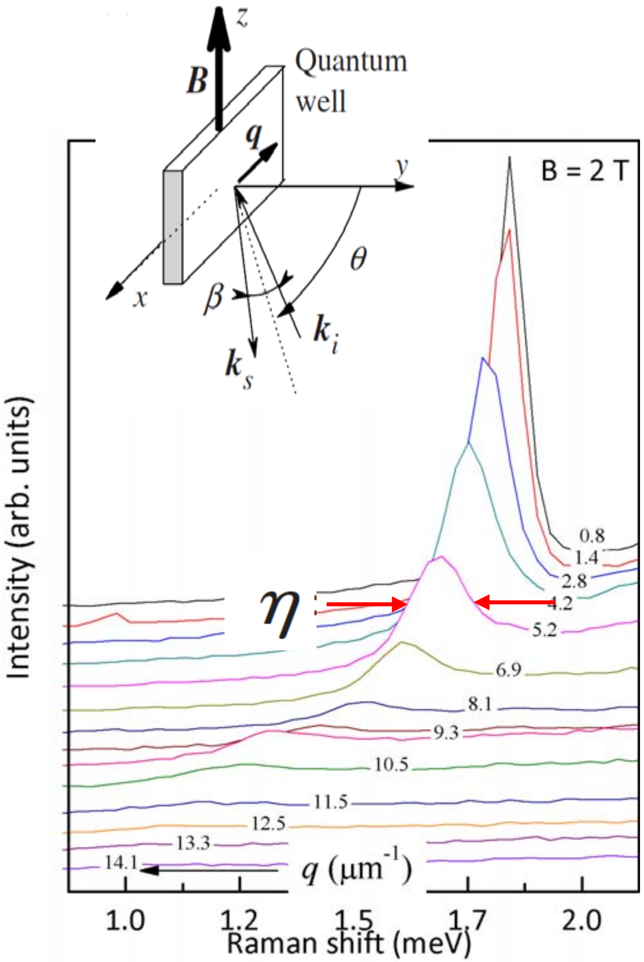


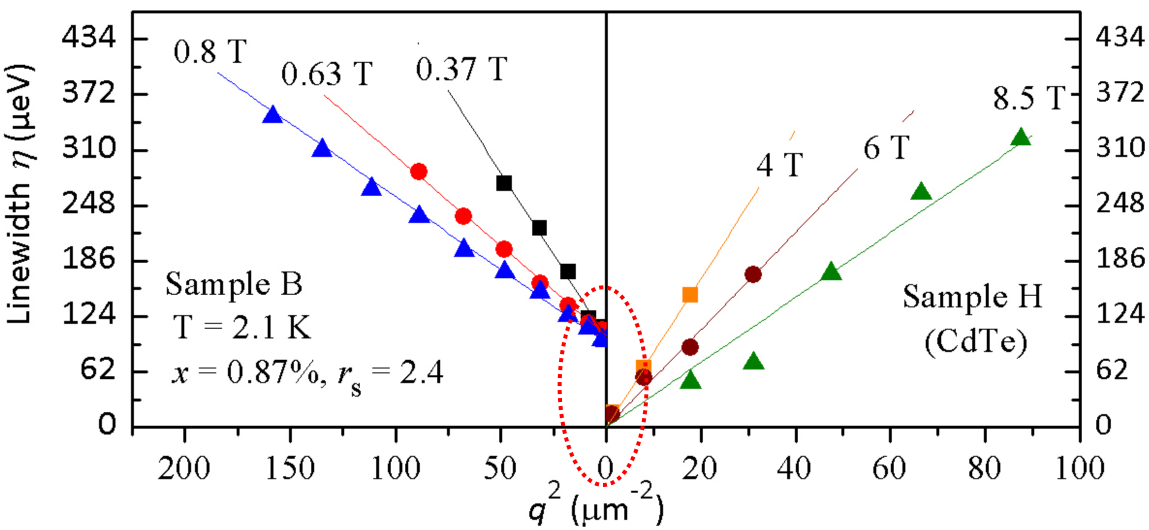




$$\hbar\omega_{SFW} = Z - \underbrace{\frac{1}{|\zeta|}\,\frac{Z}{Z^{*}-Z}\,\frac{\hbar^{2}}{2m^{*}}}_{\text{Spin-wave stiffness}}\,q^{2}$$

*Theory : F. Perez. Phys. Rev. B (2009)*
*F. Perez & P. Kossacki « Physics of DMS », Springer 2010*
*C. Akuh-Leh, F. Perez et al. Phys. Rev. B (2011)*

# Spin wave damping




$$\eta = \eta_0 + \eta_2 q^2$$

Due to Mn

Intrinsic (Coulomb)

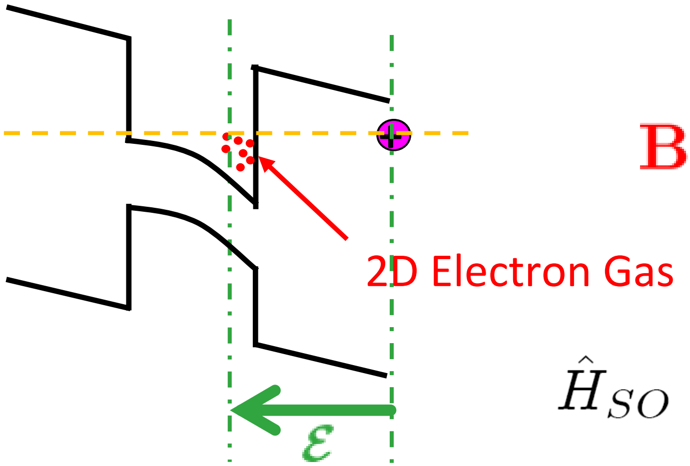
Universal linearity Damping vs Frequency :

$$\eta = \tilde{\eta}_0 + \frac{2m^*}{\hbar}\frac{\eta_2}{S_{\mathrm{SW}}}\omega$$

*Theory : E. Hankiewicz et al. PRB (R) (2008)*

*Experiment : J. Gomez et al. PRB (R) (2010)*

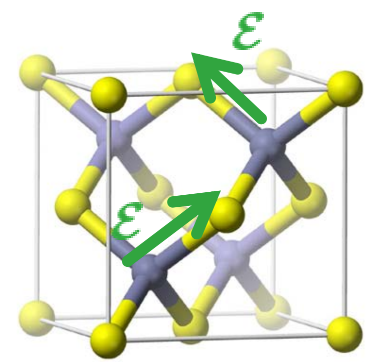
# Reminder : SPIN-ORBIT in the CONDUCTION BAND of a QW




$$\mathbf{B}_{\mathrm{SO}} = -\frac{1}{c^2}\mathbf{v} \times \mathcal{E}$$

$$\hat{H}_{SO} = \mathbf{B}_{SO}(\mathbf{k}) \cdot \frac{\hat{\sigma}}{2}$$

Blende de Zinc



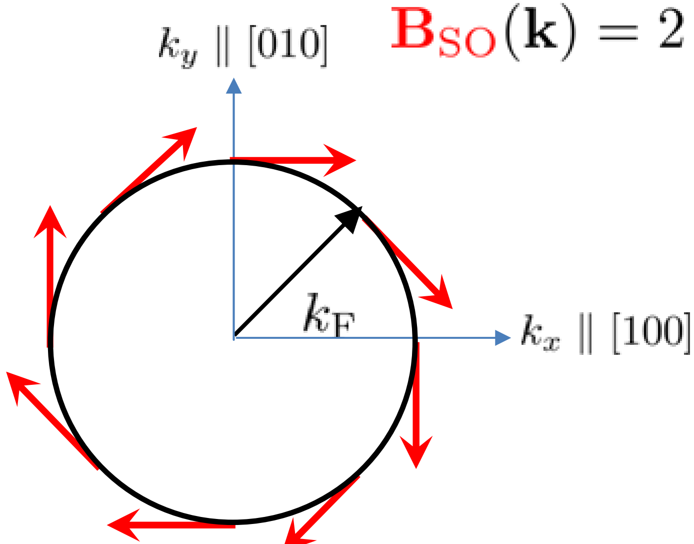

Structural Inversion Asymetry

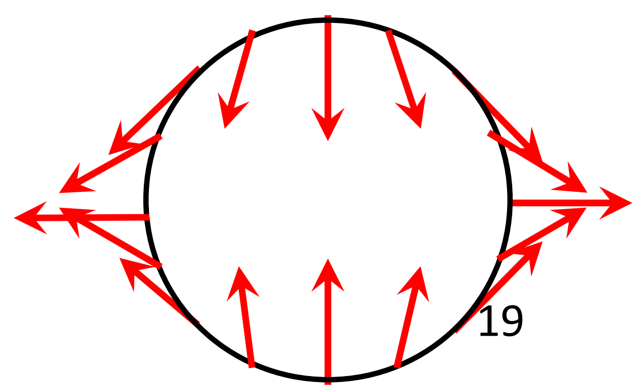
**Rashba term**

Bulk Inversion Asymetry

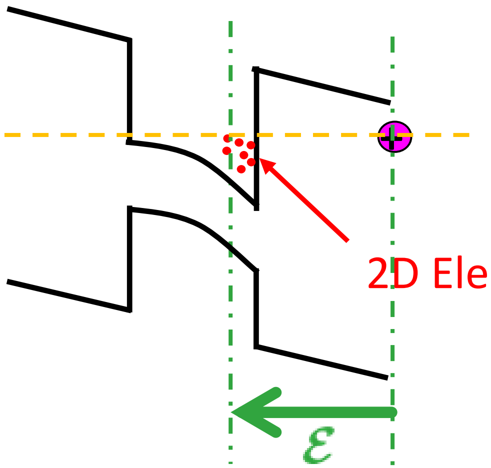
**Dresselhaus term**

$$\mathbf{B}_{\mathrm{SO}}(\mathbf{k}) = 2\,\alpha \overbrace{\begin{pmatrix} k_y \\ -k_x \end{pmatrix}} + 2\,\beta \overbrace{\begin{pmatrix} k_x \\ -k_y \end{pmatrix}}$$



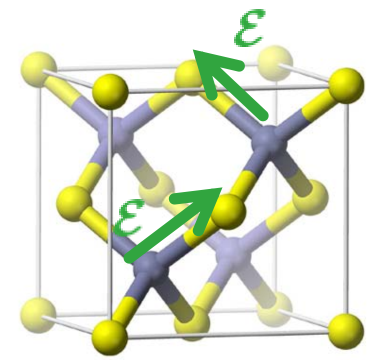



# Reminder : spin-orbit fields in quantum wells




$$\mathbf{B}_{\mathrm{SO}} = -\frac{1}{c^2}\mathbf{v} \times \mathcal{E}$$

Blende de Zinc



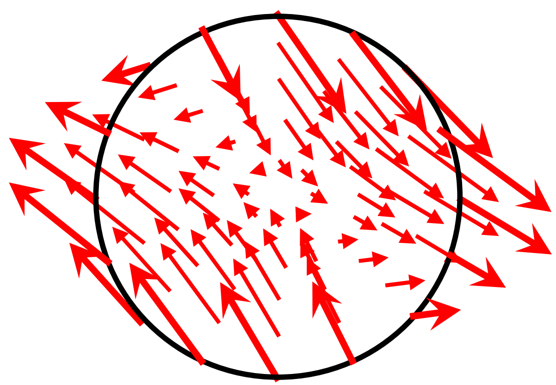

Asymetric **potential**

**Rashba effect**

Asymetric **cell**

**Dresselhaus effect**

$$\mathbf{B}_{\mathrm{SO}}(\mathbf{k}) = 2\,\overbrace{\alpha \begin{pmatrix} k_y \\ -k_x \end{pmatrix}} + 2\,\overbrace{\beta \begin{pmatrix} k_x \\ -k_y \end{pmatrix}}$$



$C_{2V}$ Symetry

Linear with **k**

# Spin-Orbit and Spin Waves from first principles : Twisted Spin Waves

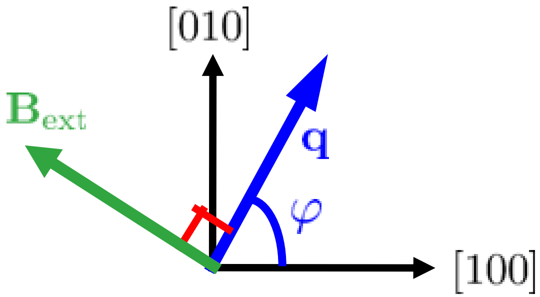
$$\hat{H}_{2DEG} = \underbrace{\underbrace{\underbrace{\sum_i \frac{\hat{\mathbf{p}}_i^2}{2m^*}}_{\hat{H}_K} + \underbrace{Z \sum_i \frac{1}{2}\hat{\sigma}_{z,i}}_{\hat{H}_Z} + \hat{H}_{\text{Coulomb}}}_{\text{Spin Waves}} + \underbrace{\sum_i \mathbf{B}_{SO}\left(\mathbf{k}_i\right) \cdot \frac{\widehat{\sigma_i}}{2}}_{\hat{H}_{SO}}}_{\text{Twisted Spin Waves}}$$

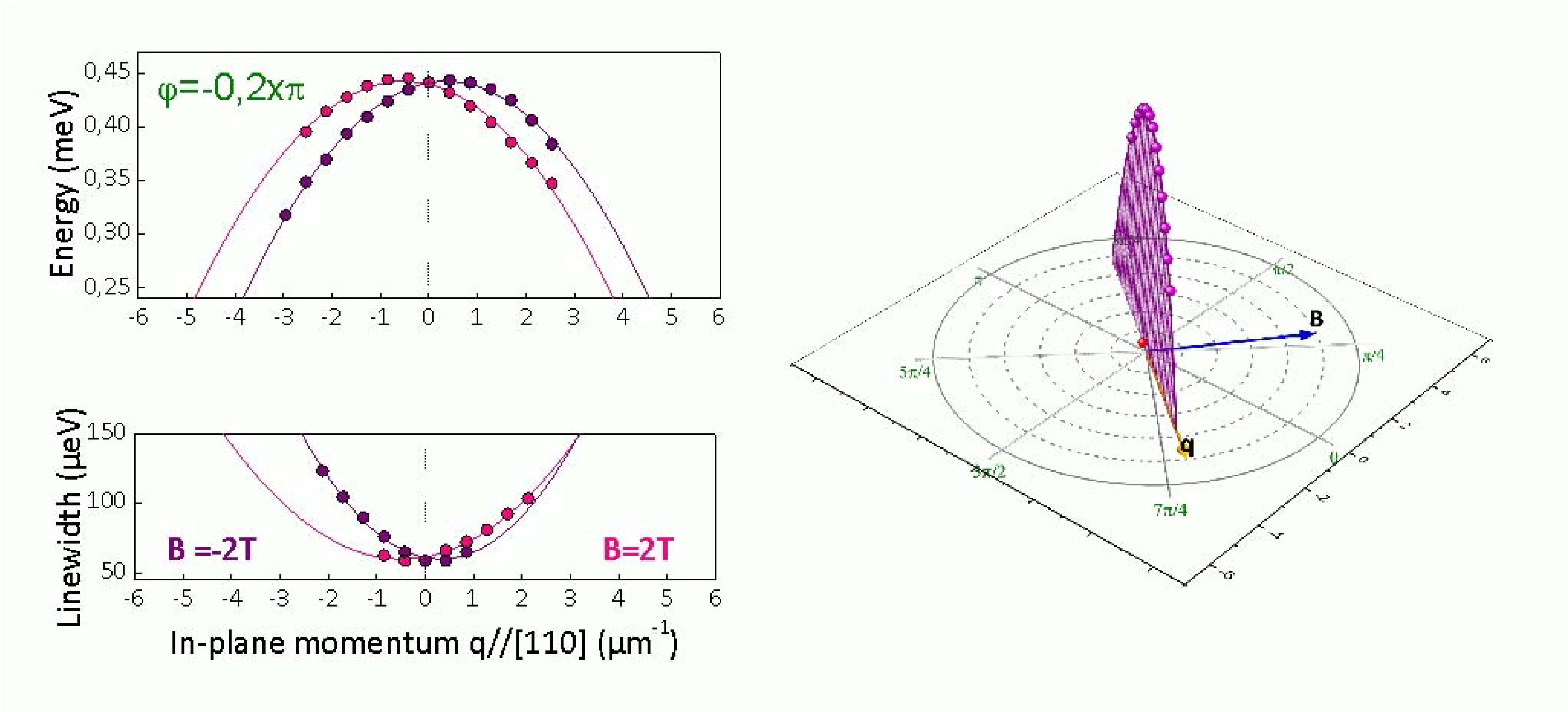
Previous steps :

F. Baboux, F. Perez *et al.*, Phys. Rev. Lett. **109**, 166401 (2012) ;
F. Baboux, F. Perez *et al.*, PRB Rapid Comm. **87**, 121303 (2013)
Theory : C. Ullrich et al., PRB 2002 & 2003 ; I. d'Amico

# Twisted Spin Waves

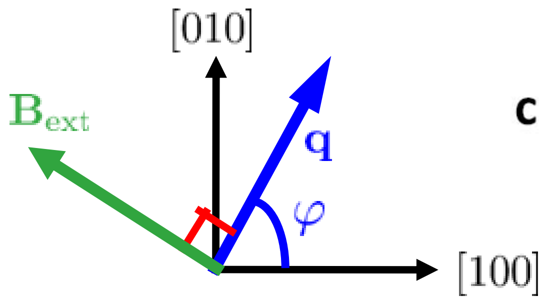
## (1) EXPERIMENTAL FACTS

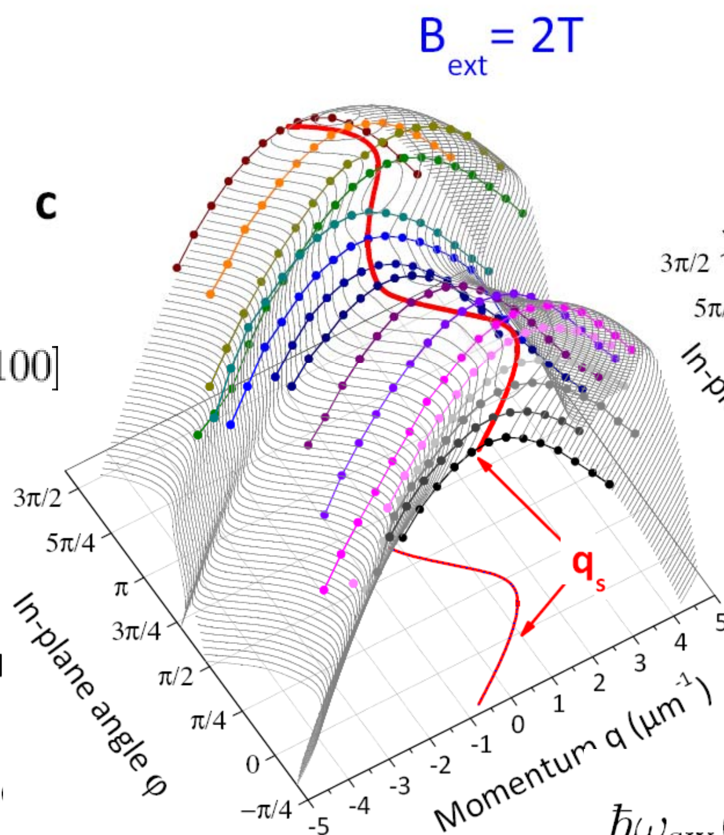


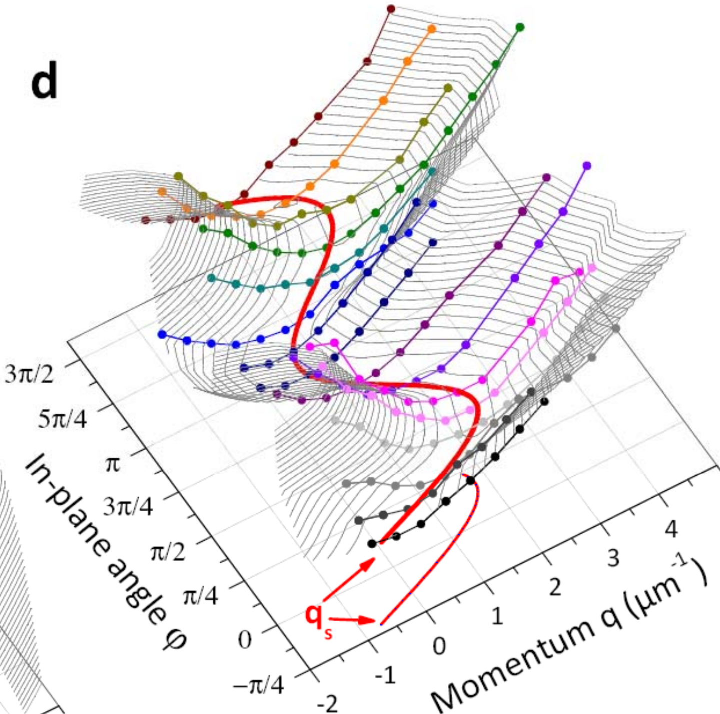



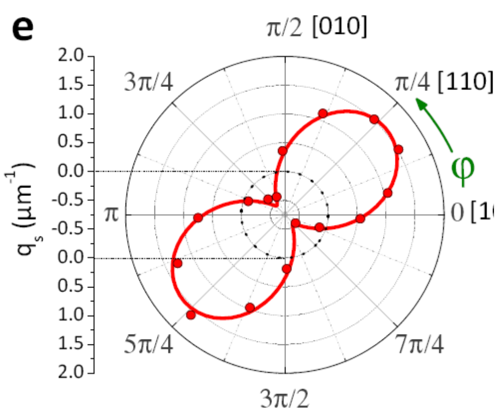
# Twisted Spin Waves

## (2) SPIN-WAVE STIFFNESS CONSERVATION



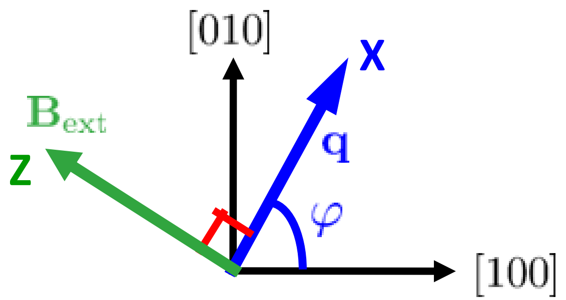




$$\hbar\omega_{\rm sw}(\mathbf{q}) = Z + S_{\rm sw}\frac{\hbar^2}{2m^*}q^2 + i\eta_q \; .$$

$$\hbar\omega_{\rm sw}^{\rm SO}(\mathbf{q}) = Z + S_{\rm sw}\frac{\hbar^2}{2m^*}|\mathbf{q}-\mathbf{q}_s|^2 + i\eta_{\mathbf{q}-\mathbf{q}_s}$$

# Twisted Spin Waves

**(3) THEORY (F. Perez, C. Ullrich, G. Vignale, I d'Amico):**




Rotated frame :

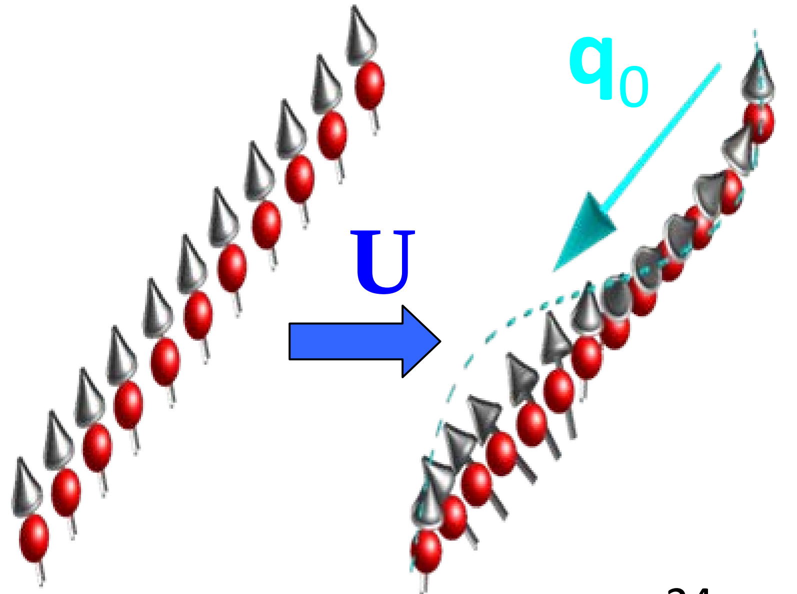
$$\hat{H}_{SO} = -\hbar\mathbf{q}_0 \cdot \hat{\mathbf{J}}^Z_{\mathbf{q}=\mathbf{0}} + \hbar\mathbf{q}_1 \cdot \hat{\mathbf{J}}^X_{\mathbf{q}=\mathbf{0}}$$

First order 2nd order

$$\mathbf{q}_{0/1} = \frac{2m^*}{\hbar^2}[(\alpha +/- \beta\sin 2\varphi)\,\mathbf{e}_{x/z} + \beta\cos 2\varphi\mathbf{e}_{z/x}].$$

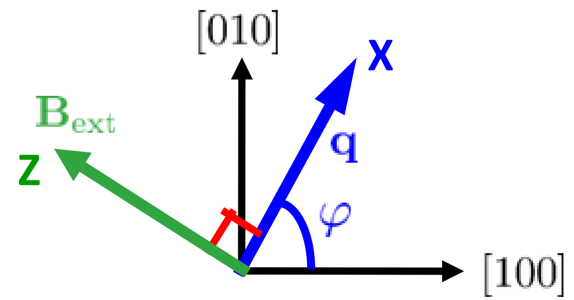
Unitary transformation (twist operator) :

$$\hat{U} = e^{-i\mathbf{q}_0 \sum_i \mathbf{r}_i \hat{\sigma}_{zi}/2}$$



# Twisted Spin Waves

## (3) THEORY (F. Perez, C. Ullrich, G. Vignale, I d'Amico):




Rotated frame :

$$\hat{H}_{SO} = -\hbar \mathbf{q}_0 \cdot \hat{\mathbf{J}}^{Z}_{\mathbf{q}=\mathbf{0}} + \hbar \mathbf{q}_1 \cdot \hat{\mathbf{J}}^{X}_{\mathbf{q}=\mathbf{0}}$$

First order 2nd order

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Unitary transformation (twist operator) :

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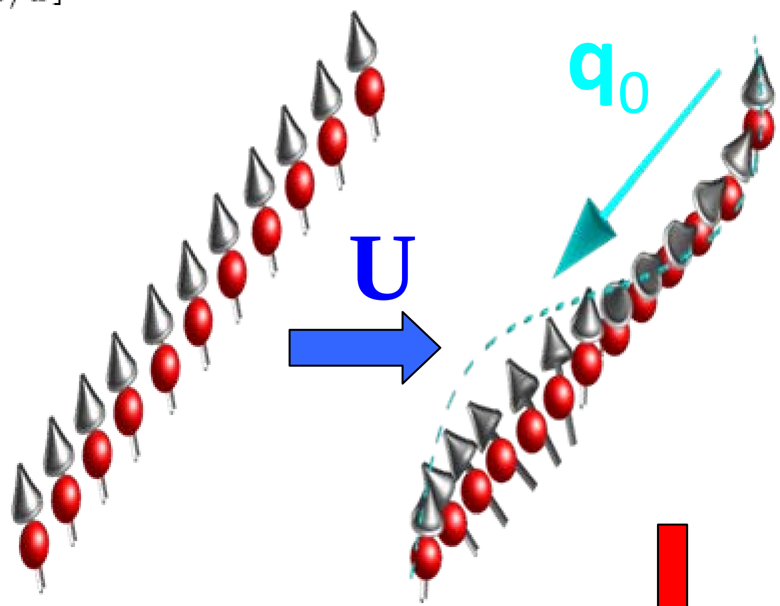
$$\hat{H}' = \hat{U}\hat{H}\hat{U}^{+} = \hat{H}^{SO=0} - \sum_i \frac{\hbar^2 q_0^2}{2m^*}$$

$$\hat{U}\hat{S}_{+,\mathbf{q}}\hat{U}^{\dagger} = \hat{S}_{+,\mathbf{q}+\mathbf{q}_0}$$

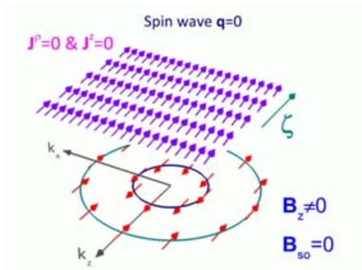


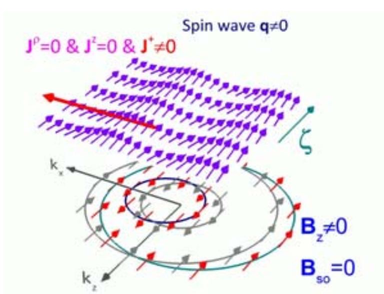

$$i\hbar \frac{d}{dt}\hat{S}_{+,\mathbf{q}} = [\hat{S}_{+,\mathbf{q}}, \hat{H}] = \hat{U}^{\dagger}[\hat{S}_{+,\mathbf{q}+\mathbf{q}_0}, \hat{H}']\hat{U}$$

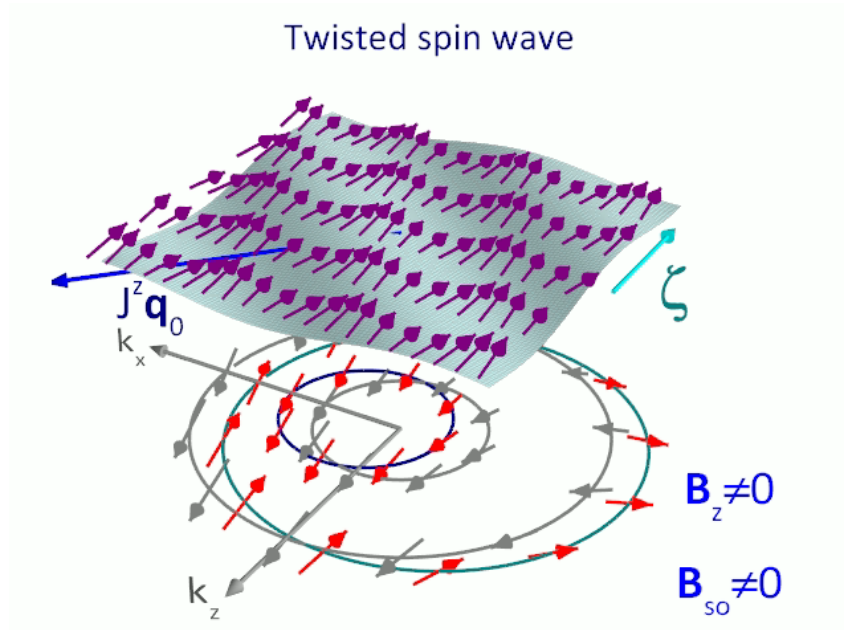
$q_s = -q_0$

# Twisted Spin Waves

## (4) INTERPRETATION: OSCILLATING INVERSE SPIN-GALVANIC EFFECT



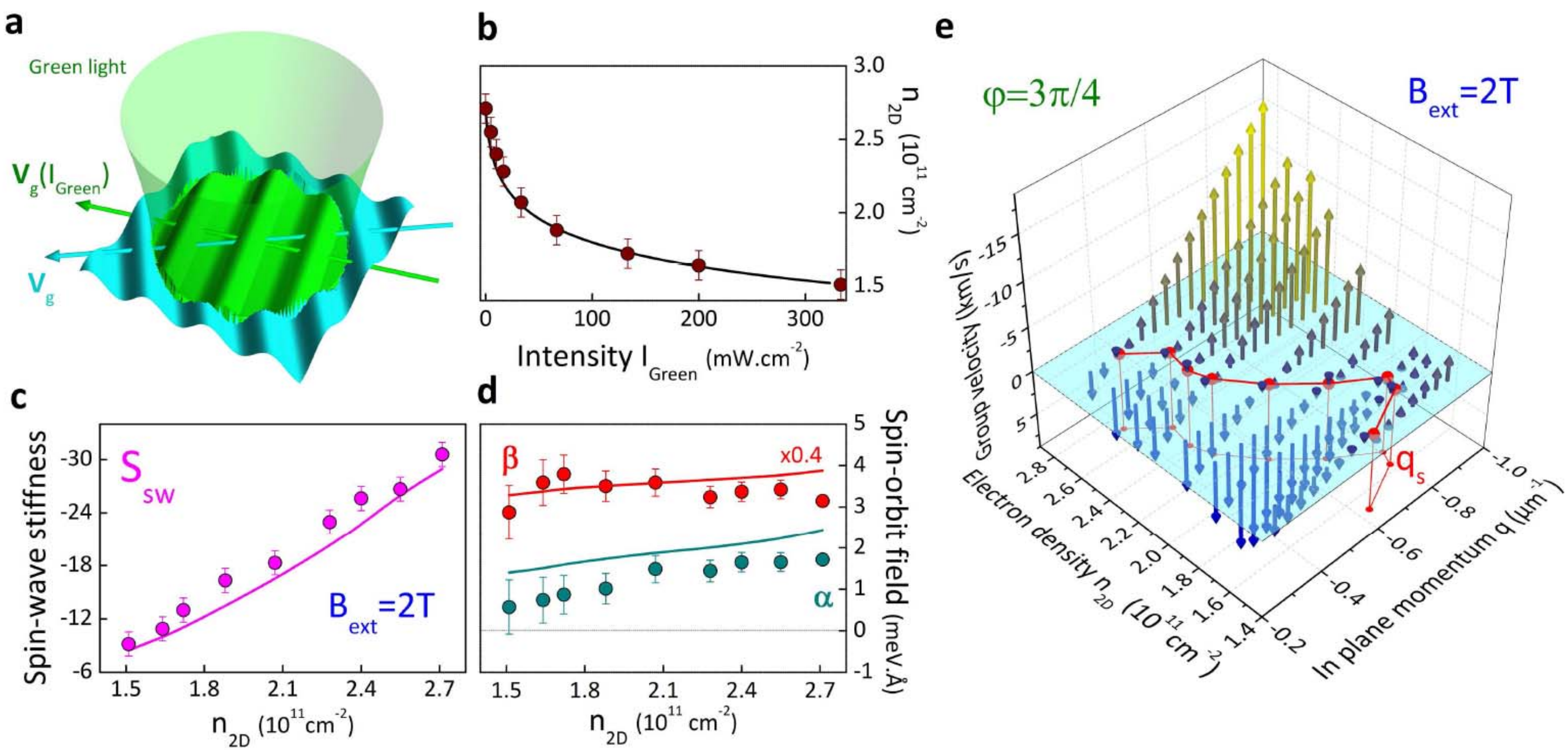
# Twisted Spin Waves : control of the group velocity

**Group velocity :**

$$\mathbf{v}_{g,\mathbf{q}} = S_{\mathrm{sw}}\hbar\mathbf{q}/m^* \Rightarrow \mathbf{v}_{g,\mathbf{q}} = S_{\mathrm{sw}}\hbar(\mathbf{q}+\mathbf{q}_0)/m^*$$

no spin-orbit

Twisted Spin Wave






F. Perez, F. Baboux, C. Ullrich, G.Vignale, I. d'Amico , submitted to PRL 2016

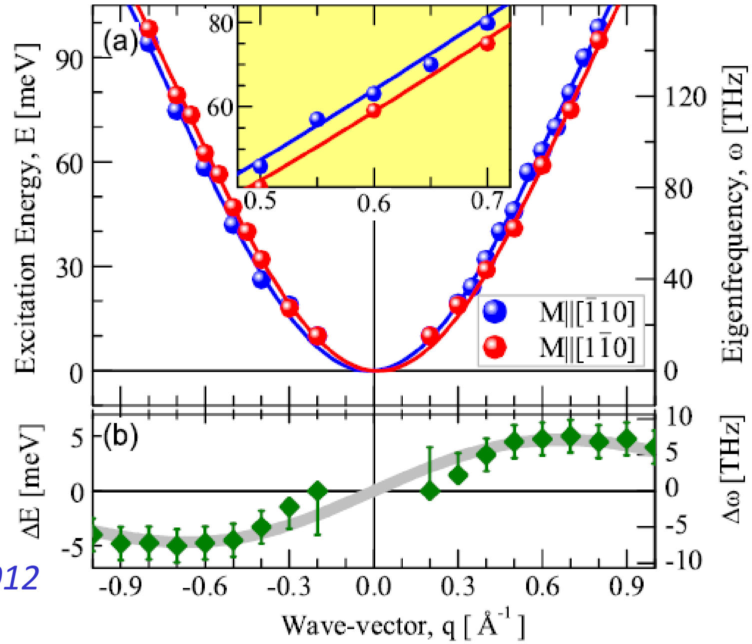
# SCOPE

- Spin-wave control by spin-orbit interaction
- Twisted spin waves (our work)
- Comparison with chiral spin waves in conducting ferromagnet : DMI
- Perspectives

# Chiral Spin waves in conducting ferromagnet

**Two Fe atomic layers**
**Grown on W(110)**

*Kirschner et al., Phys. Rev. Lett. 2012*




$$\hat{H} = \underbrace{-\sum_{i\neq j} J_{ij}\hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j}_{\text{Heisenberg, Symetric exchange}} + \underbrace{\sum_{i\neq j} \mathbf{D}_{ij} \cdot \hat{\mathbf{S}}_i \times \hat{\mathbf{S}}_j}_{\text{Dzyaloshinski-Moriya Antisymetric exchange}}$$

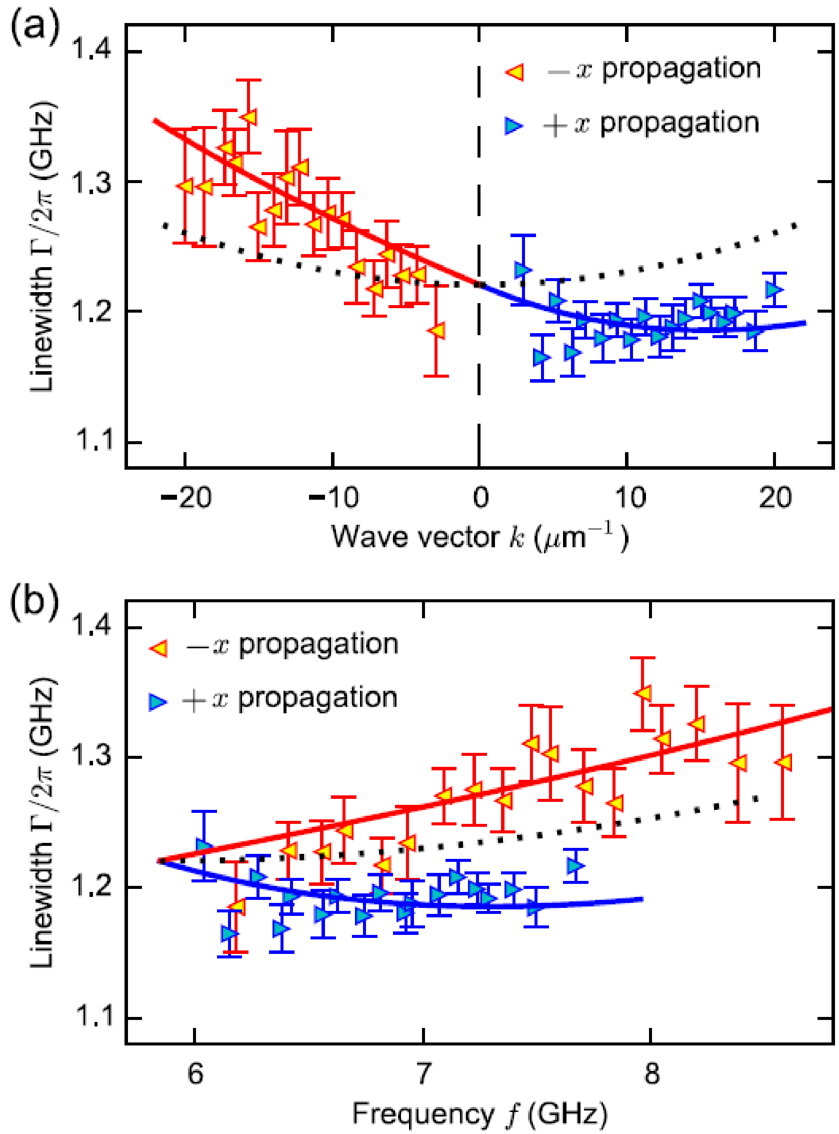
**Chiral linear shift :** $$\Delta E = \pm 2\left[3D_1^z \sin\frac{qa}{2} + D_2^z \sin qa\right]$$

$J_1 = 7.5$ meV, $J_2 = 4.5$ meV $\qquad 3D_1^z = 0.9$ meV, $D_2^z = 0.5$ meV

# Chiral damping in conducting ferromagnet

**Pt/Co/Ni**

*Kai Di et al., Phys. Rev. Lett. 2015*

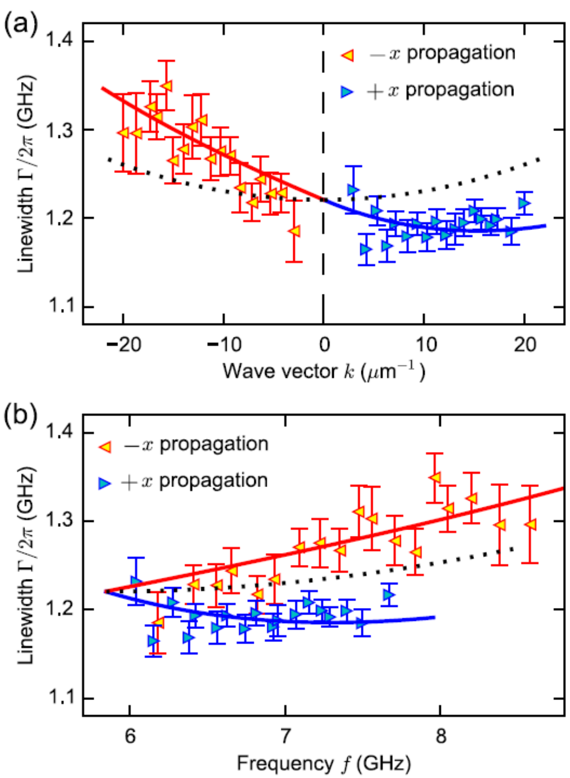


# DMI vs Twisted spin-waves

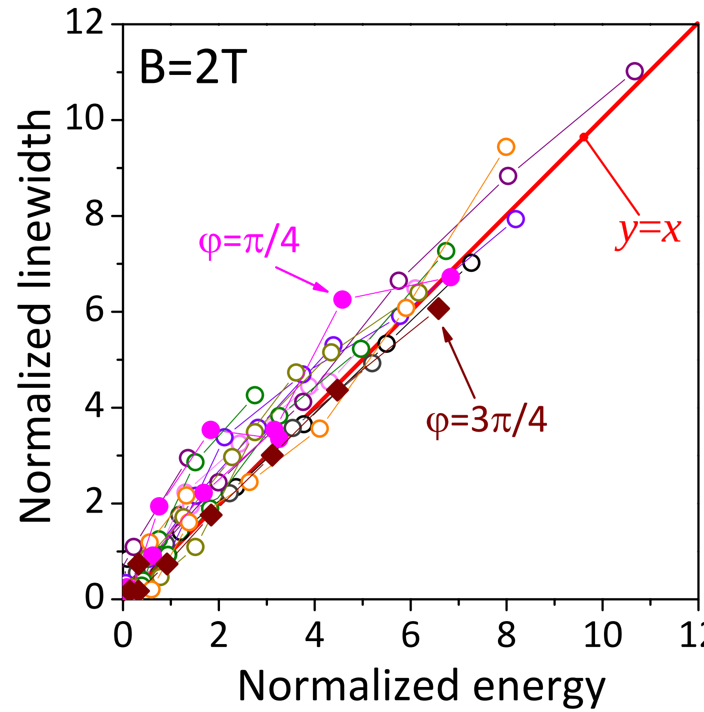
$$\eta_q = \eta_0 + \eta_2 \left(q - q_s\right)^2 \qquad \hbar\omega = Z + S_{\mathrm{sw}} \frac{\hbar^2}{2m^*} \left(q - q_s\right)^2$$

**Pt/Co/Ni**
**DMI**
**Semi-localized spins**
*Kai Di et al., Phys. Rev. Lett. 2015*

$$\Rightarrow \quad \eta = \tilde{\eta}_0 + \frac{2m^*}{\hbar} \frac{\eta_2}{S_{\mathrm{sw}}} \omega$$



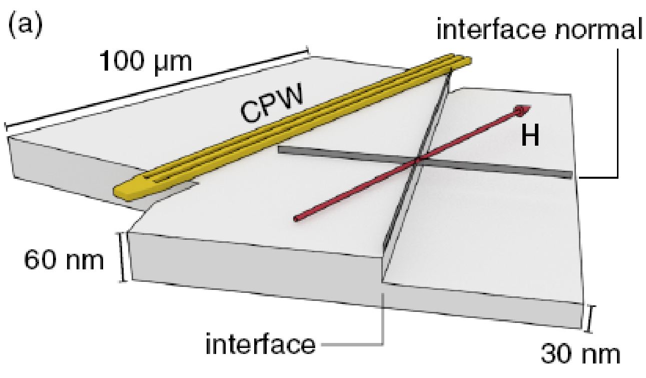

$$\frac{\eta - \eta_0}{\eta_2}$$



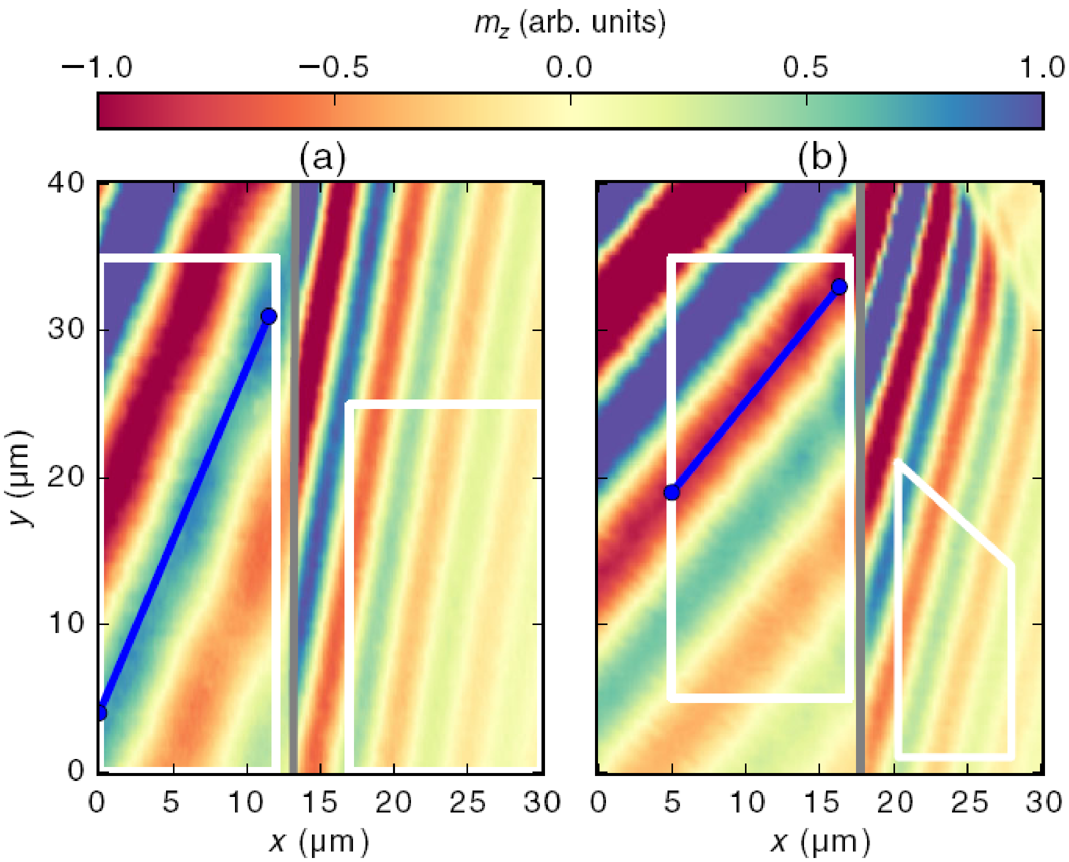

$$\frac{\hbar\omega - Z}{S_{\mathrm{sw}} \frac{\hbar^2}{2m^*}}$$

# Perspectives : Snell's law for spin waves

**Thickness interface (Permalloy)**




**TRMOKE**




*J. Stigloher et al., Phys. Rev. Lett. July 2016*

# Summary

- We have presented the discovery of a new type of spin waves, the **spin-orbit twisted spin waves [F. Perez et al. PRL 117, 137204 (2016)]**

- SOTSWs exist in a magnetized, Galilean invariant system, subject to spin-orbit interaction. Their dispersions experience a chiral shift in wavevector space in a vectorial form that we predict from a rigorous many-body theorem, and then verify in detail their dependence on the angle and other parameters.

- SOTSWs have **the same spin-wave stiffness as SP2DEG spin waves.** This gives rise to the possibility to control their group velocity.

- The SOTSW's velocity can be engineered towards applications (refraction law…).

# Contributors

**Raman group**

**F. Baboux[1]** (PhD), **J. Gomez[2]** (Post-doc), **supervision : F.Perez**

Institut des NanoSciences de Paris, CNRS UMR7588

**Theory**

**C. A. Ullrich, G. Vignale**

University of Missouri, USA

**I. D'Amico**

University of York, UK

**Growth group**

**V. Kolkovsky, Grzegorz Karczewski, Tomek Wojtowicz**

IFPAN, Polish Academy of Sciences, Warsaw