

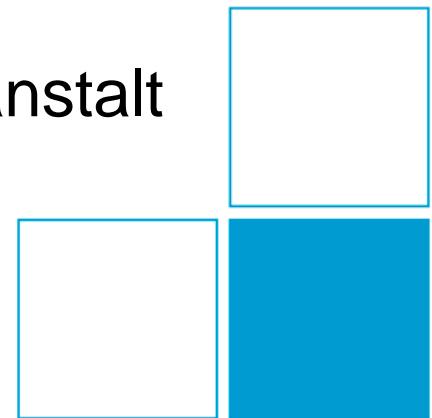


Physikalisch-Technische Bundesanstalt  
Braunschweig und Berlin  
Nationales Metrologieinstitut

# Single electron pumps for the new SI

*H. W. Schumacher*

Physikalisch-Technische Bundesanstalt  
Braunschweig



# Content

- The SI system of units
  - Quantum metrology
  - SI-Revision 2018
- Single electron pumps for metrology
  - precision measurements
  - The self reference quantized current source
  - Semiconductor quantized voltage source
- Single electron pumps beyond metrology
  - Suppression of shot noise
  - Singlet-triplet separation
  - Beam splitter experiments

# PTB: National Metrology Institute

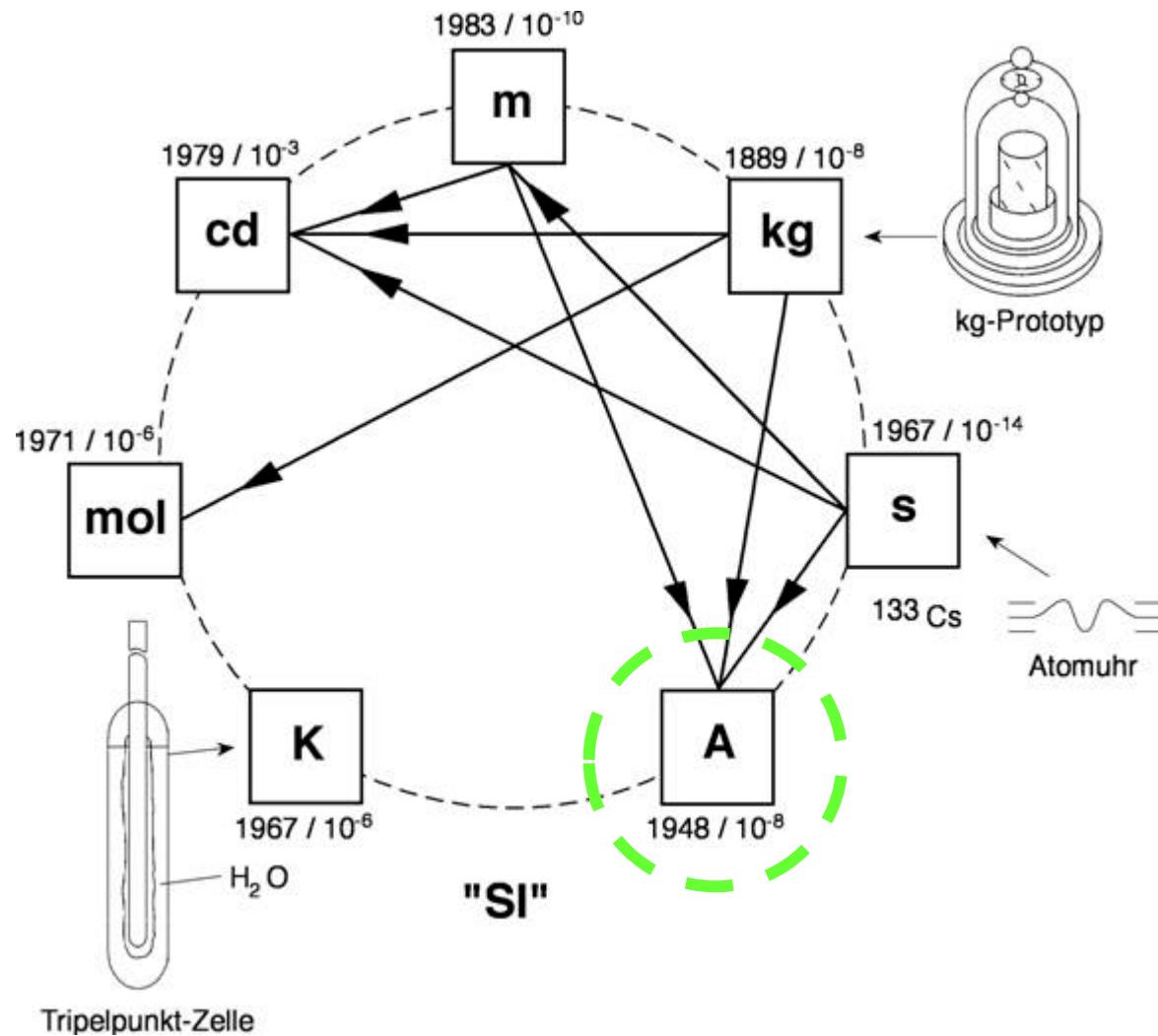


- Founded as PTR in **1887** by **Helmholtz and Siemens**
- World leading National Metrology Institute
- Belongs to Ministry of Economy
- **Braunschweig und Berlin**
- 1800 Employees
  - 800 scientists
  - 100 PhD students

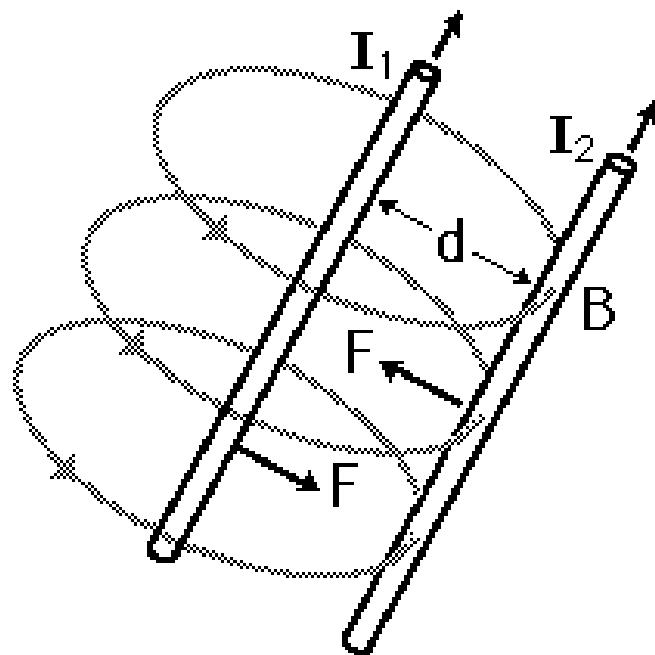
## Duties:

- Realization and dissemination of the units
- Calibration services for industry and society
- Basic research for the realization of the units

# The SI system of units



# Base unit of electrical current: The Ampere

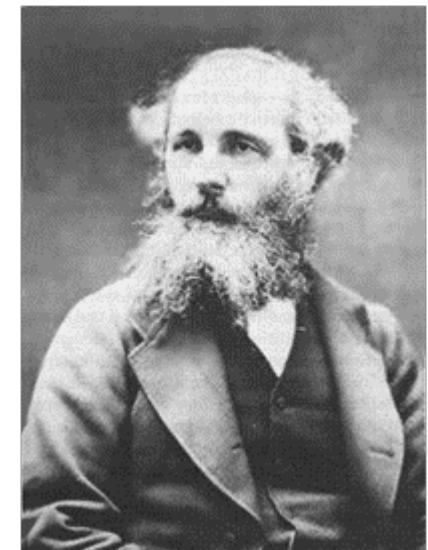


- Definition based on force between two current carrying wires
- Idealized measurement scheme (infinitely long, parallel, negligible cross section...).
- Defines vacuum permeability  $\mu_0$ .
- Time of definition: 1948
- Minimum uncertainty of realization:  $3 \cdot 10^{-7}$

# Quantum metrology

James Clerk Maxwell, 1870:

*„If, then, we wish to obtain standards of length, time, and mass which shall be absolutely permanent, we must seek them not in the dimensions, or the motion, or the mass of our planet, but in the wavelength, the period of vibration, and the absolute mass of these imperishable and unalterable and perfectly similar molecules.“*



# ANNALEN DER PHYSIK.

HERAUSGEGEBEN UND FORTGEFÜHRT DURCH

F. A. GREN, L. W. GILBERT, J. C. POGGENDORFF, G. UND E. WIEDEMANN

VIERTE FOLGE.

BAND 1.

DER GANZEN REIHE 900. BAND.

KURATORIUM:

F. KOHLRAUSCH, M. PLANCK, G. QUINCKE,  
W. C. RÖNTGEN, E. WARBURG.

UNTER MITWIRKUNG

DER DEUTSCHEN PHYSIKALISCHEN GESELLSCHAFT

UND INSbesondere von

**M. PLANCK**

HERAUSGEGEBEN VON

**PAUL DRUDE.**

MIT DREI FIGURENTAFELN.



LEIPZIG 1900.

VERLAG VON JOHANN AMBROSIUS BARTH.

## § 25. Zahlen

Die Werte der universellen Gravitation und der Lichtgeschwindigkeit können sich mit Hülfe der vorliegenden Messungen und der Wienschen Näherung berechnen.

Hr. F. Kurlbaum<sup>1)</sup> hat gefunden, dass die gesamte Energie, welche ein schwarzer Körper bei  $t^0$  C. befindlichen schwarzten Körpern auf einer Fläche  $S$  in  $\lambda$  m abgestrahlt wird:

$$S_{100} - S_0 = 0,01$$

Andererseits beträgt nach (52) die gesamte Energie, welche ein schwarzer Körper in einem Raum ausgesandt, in Richtungen des Halbraumes ausgestrahlt wird:

$$\int K \cos \vartheta d\Omega = K \int_0^{2\pi} d\varphi \int_0^{\pi/2} \cos \vartheta \sin \vartheta d\vartheta = \pi K = \frac{12\pi b(373^4 - 273^4)}{c^2 a^4}.$$

Folglich, wenn das mechanische Wärmeäquivalent zu  $419 \cdot 10^6$  cal gesetzt und der Wert der Constanten im Exponenten der Wien'schen Formel (55) zu 14455 [ $\mu \times$  Celsiusgrad] angenommen wird, im absoluten C.G.S.-Maasse:

$$\frac{12\pi b(373^4 - 273^4)}{c^2 a^4} = 0,01763 \cdot 419 \cdot 10^6$$

oder, da  $c = 3 \cdot 10^{10}$ :

$$(57) \quad \frac{b}{a^4} = 1,278 \cdot 10^{15}.$$

Ferner ist von Hrn. F. Paschen<sup>2)</sup> als Mittel aus seinen besten Beobachtungen der Wert der Constanten im Exponenten der Wien'schen Formel (55) zu 14455 [ $\mu \times$  Celsiusgrad] angegeben worden. Dies ergiebt, bezogen auf Centimeter:

$$a c = 1,4455$$

oder:

$$a = 0,4818 \cdot 10^{-10} [\text{sec} \times \text{Celsiusgrad}]$$

und daraus nach (57):

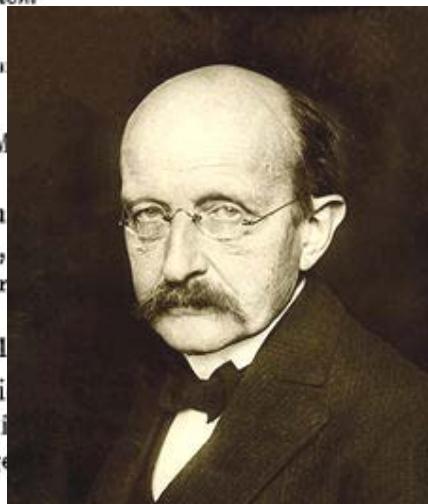
$$b = 6,885 \cdot 10^{-27} [\text{erg} \times \text{sec}].$$

## § 26. Natürliche Maasseinheiten.

Alle bisher in Gebrauch genommenen physikalischen Maasssysteme, auch das sogenannte absolute C.G.S.-System, verdanken

1) F. Kurlbaum, Wied. Ann. 65, p. 754. 1898.

2) F. Paschen, Sitzungsber. d. k. Akad. d. Wissensch. zu Berlin, p. 419. 1899.



# Quantenmetrologie

## § 26. Natürliche Maasseinheiten.

Alle bisher in Gebrauch genommene **physikalischen Maassysteme** verdanken ihren Ursprung insofern dem Zusammentreffen **zufälliger Umstände**, als die Wahl der jedem System zu Grunde liegenden Einheiten nicht nach allgemeinen, notwendig für alle Orte und Zeiten bedeutungsvollen Gesichtspunkten, sondern wesentlich mit Rücksicht auf die speziellen Bedürfnisse unserer irdischen Cultur getroffen ist. So sind die **Einheiten der Länge und der Zeit aus den gegenwärtigen Dimensionen und der gegenwärtigen Bewegung unseres Planeten hergeleitet** worden.

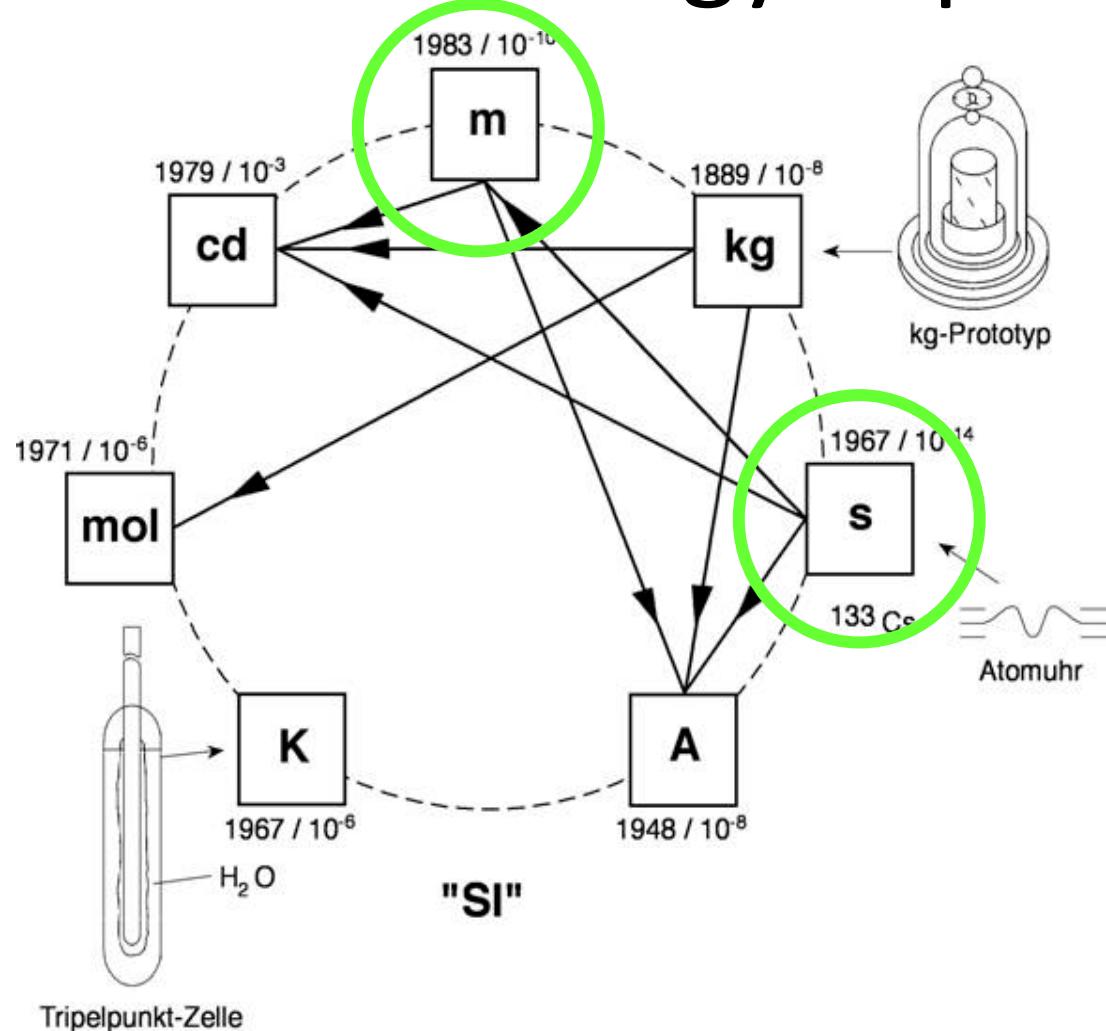
# Quantenmetrologie

Dem gegenüber dürfte es nicht ohne Interesse sein zu bemerken, dass mit Zuhilfenahme der beiden in dem Ausdruck (41) auftretenden **Constanten  $h$  und  $k$**  die Möglichkeit gegeben ist, Einheiten für Länge, Masse, Zeit und Temperatur aufzustellen, welche, unabhängig von speziellen Körpern oder Substanzen, ihre Bedeutung für alle Zeiten und für alle, auch ausserirdische und aussermenschliche Culturen notwendig behalten und welche daher als „**„natürliche Maasseinheiten“** bezeichnet werden können.

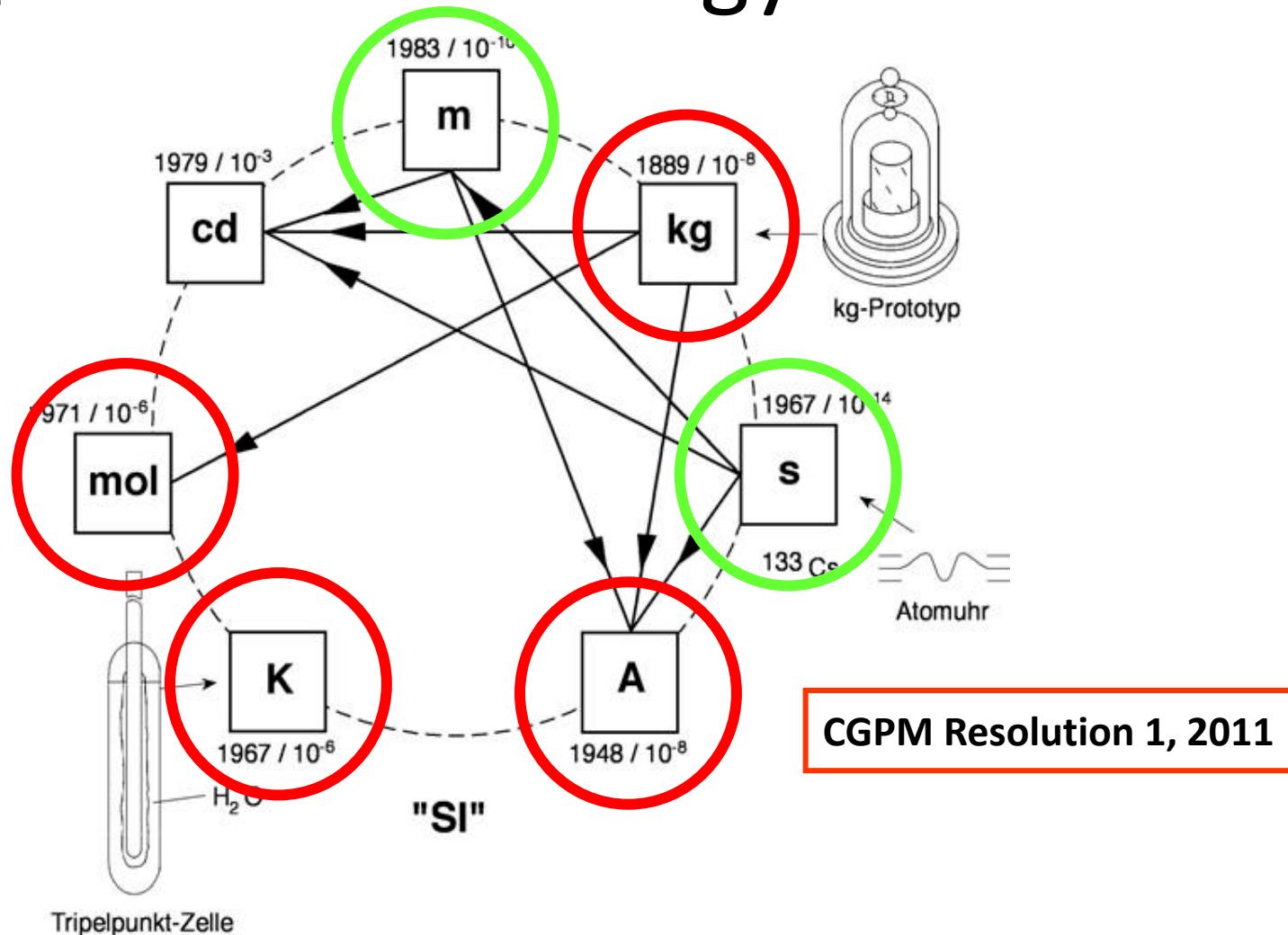
Quantenmetrologie basierend auf **Naturkonstanten**.

M. Planck, Ann. Physik 1, 69-122 (1900)

# Quantum metrology in present SI



# Quantum metrology in new SI



# The new SI

## CGPM -Resolution 1 :

"The International System of Units, the SI, will be the system of units in which:

1. the ground state hyperfine splitting frequency of the caesium 133 atom ... *is exactly 9 192 631 770 reciprocal second,*
2. the speed of light in vacuum  $c$  *is exactly 299 792 458 metre per second*
3. the Planck constant  $h$  *is exactly  $6.62606 \times 10^{-34}$  joule second ,*
4. the elementary charge  $e$  *is exactly  $1.60217 \times 10^{-19}$  coulomb*
5. the Boltzmann constant  $k$  *is exactly  $1.380 6 \times 10^{-23}$  joule per kelvin,*
6. the Avogadro constant  $N_A$  *is exactly  $6.022 14 \times 10^{23}$  reciprocal mole,*

# The new SI

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4. the elementary charge  $e$  *is exactly  $1.60217 \times 10^{-19} \text{ A} \cdot \text{s}$*
5. the Boltzmann constant  $k$  *is exactly  $1.380 6 \times 10^{-23} \text{ joule per kelvin},$*
6. the Avogadro constant  $N_A$  *is exactly  $6.022 14 \times 10^{23} \text{ reciprocal mole},$*

*X* from CODATA value at time of redefinition

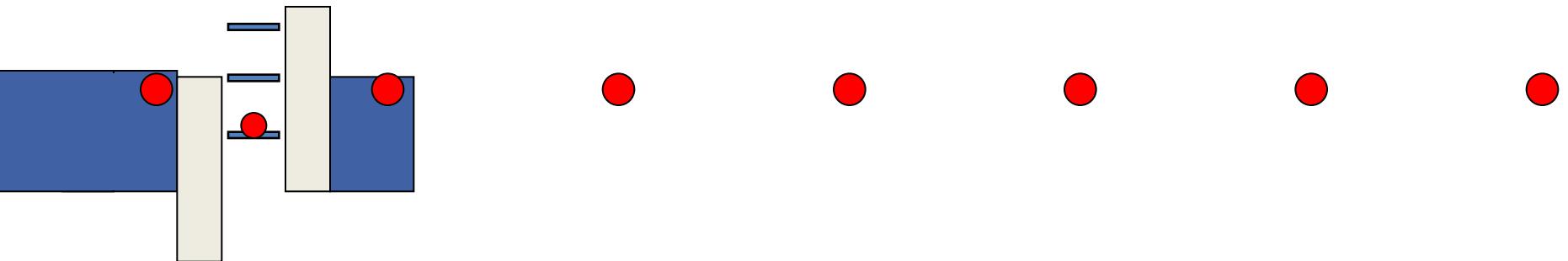
**Redefinition planned for 2018!**

# The new Ampere

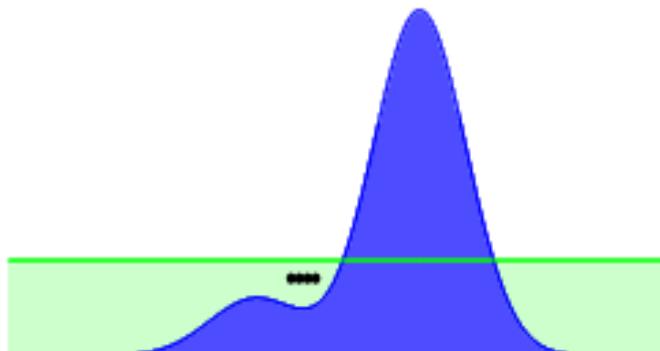
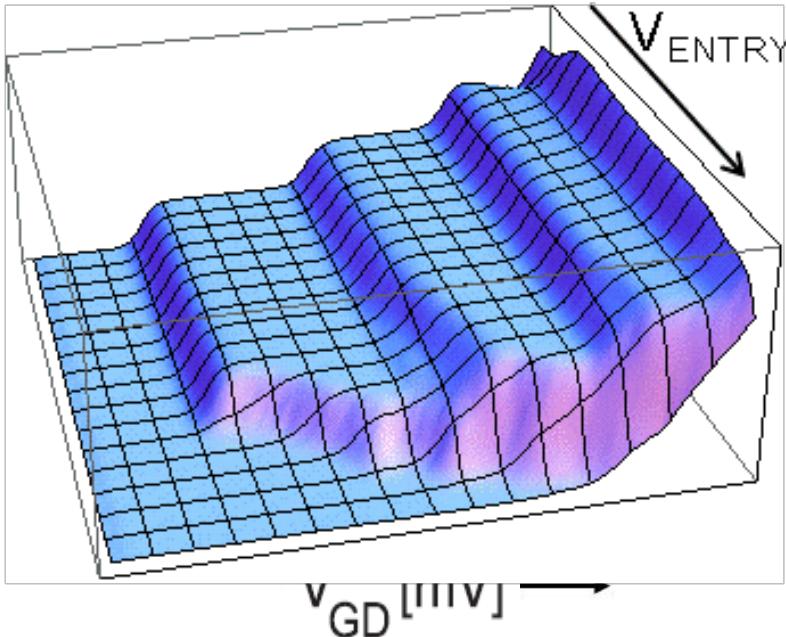
## Direct realization:

Generate a quantized current  $I = e \cdot f$  by pumping electrons one by one through a nanostructure with frequency  $f$

$f$ : Pumping frequency;     $e$ : electron's charge



# Semiconductor single electron pumps



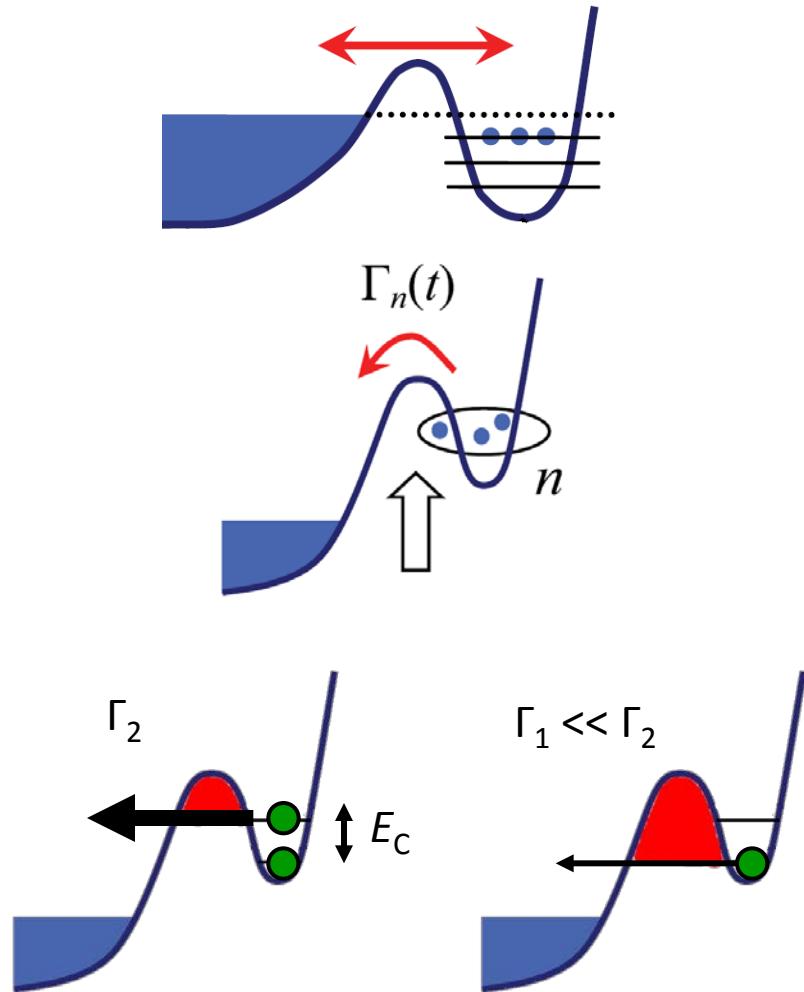
- Semiconductor channel
- Topgates define quantum dot
- Modulation of entrance barrier
- Capture electron on source
- Eject to drain
- Quantised current:  $I = e \cdot f$
- Advantage:
- High frequency → high currents
- Simple fabrication
- Low error rate

Problem:

- Rare **pump errors** are **inevitable**

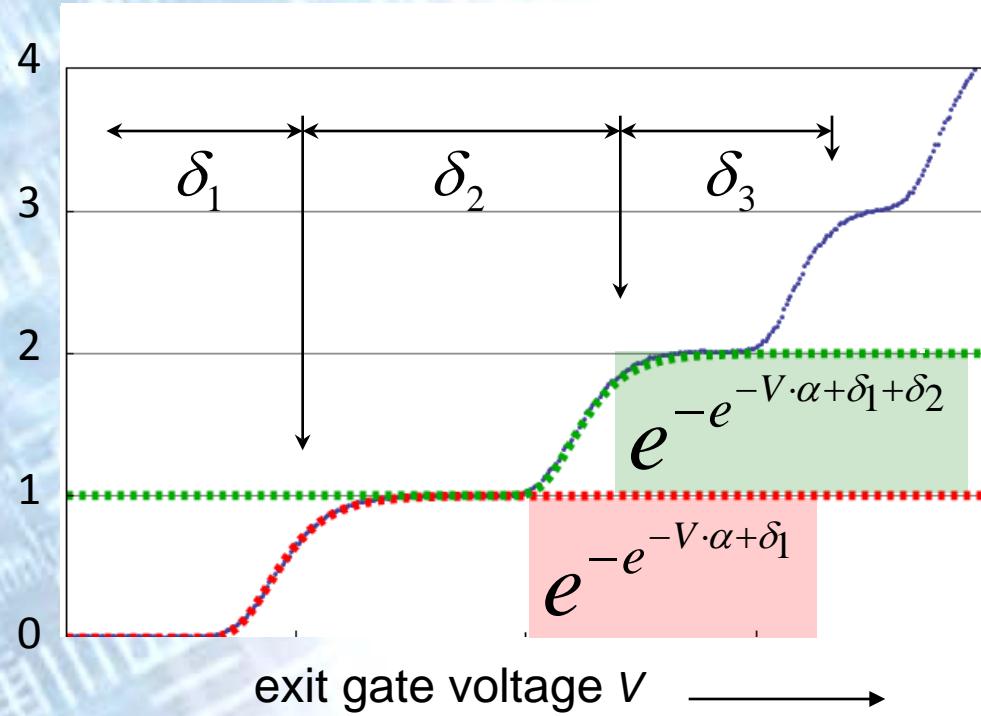
# Decay cascade model

- Electrons loaded from source into QD
- Barrier and dot rise  
→ backtunneling of excess electrons
- Barrier more transparent for higher electron number
  - time scale separation
  - **n-electron state frozen** by further barrier rise
- Calculate pumped current

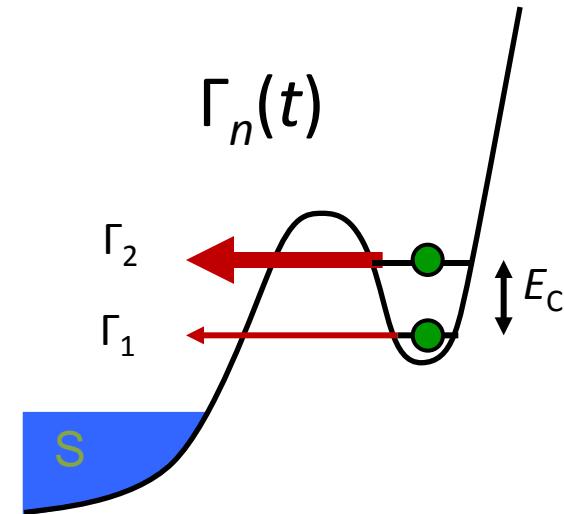


# Decay cascade model

$\delta$ -fitting for uncertainty estimate



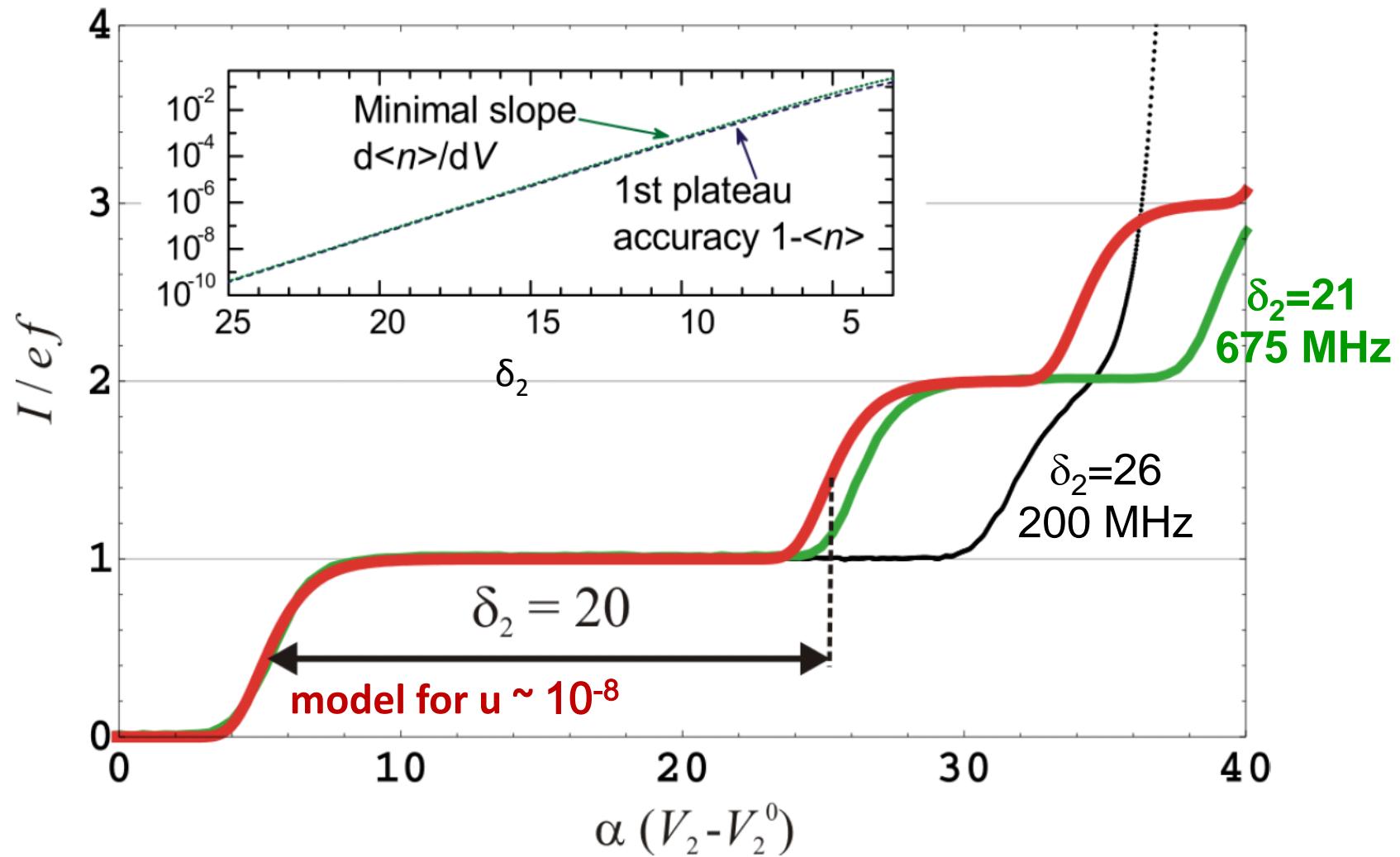
n-electron error  $\approx \exp(\delta_{n+1})$   
 $u \sim 10^{-8}$  for  $\delta > 20$

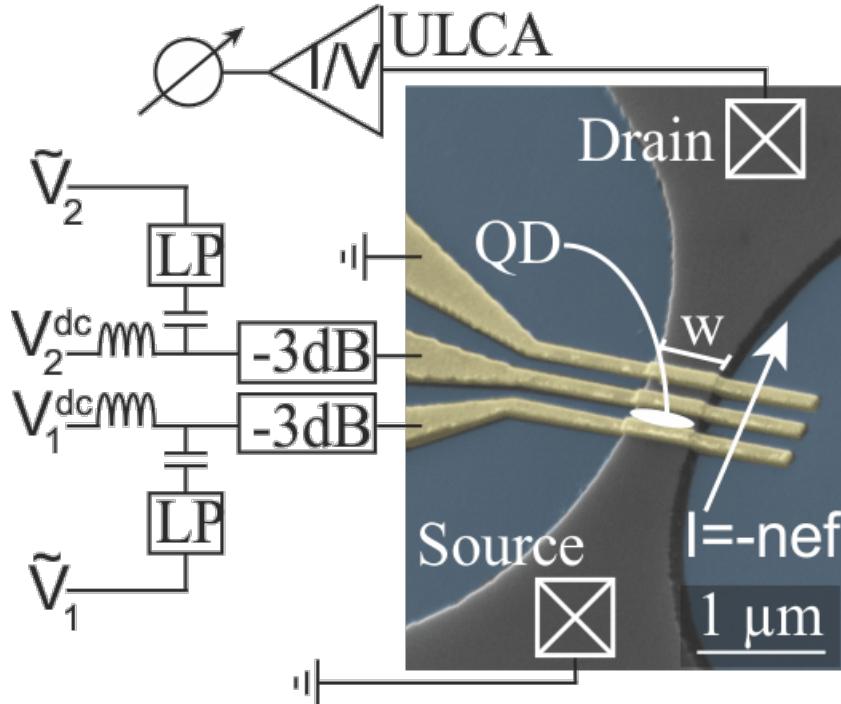


$$I/ef = \sum_n e^{-e^{-V\cdot\alpha+\Sigma_n \delta_n}}$$

$$\delta_n \approx \ln\left(\frac{\Gamma_n}{\Gamma_{n-1}}\right) \propto E_C$$

# Error estimate with decay cascade model



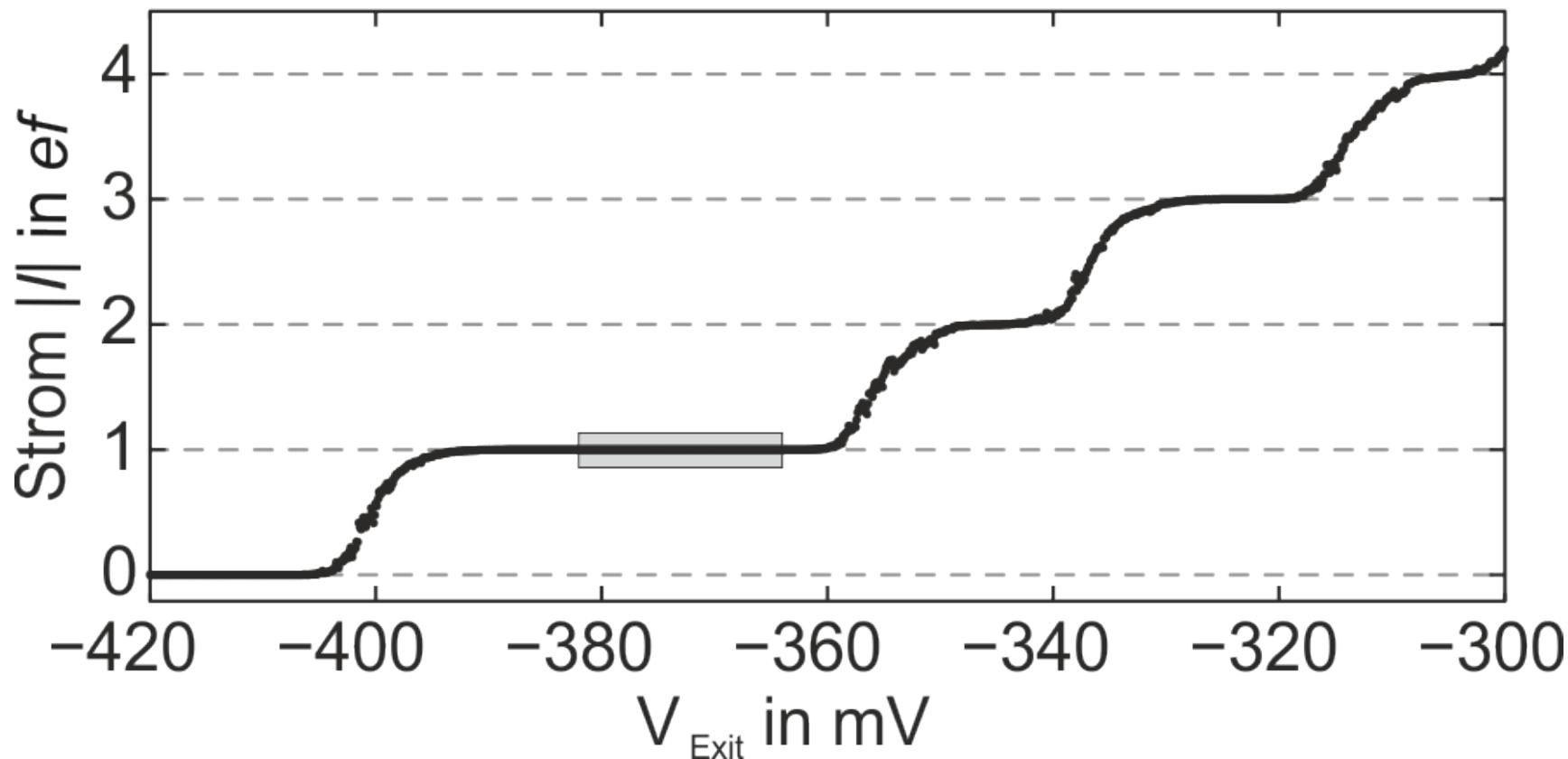


Highly stable precision current amplifier

- Traceable with uncertainty < **0.06 ppm**
- Stability: Drift < **0.1 ppm/week**, Temperatur coefficient ~ **0.2 ppm/K**
- Low noise **2.4 fA/ $\sqrt{\text{Hz}}$**
- Measure 100 pA in 10 h with uncertainty below **0.1 ppm**

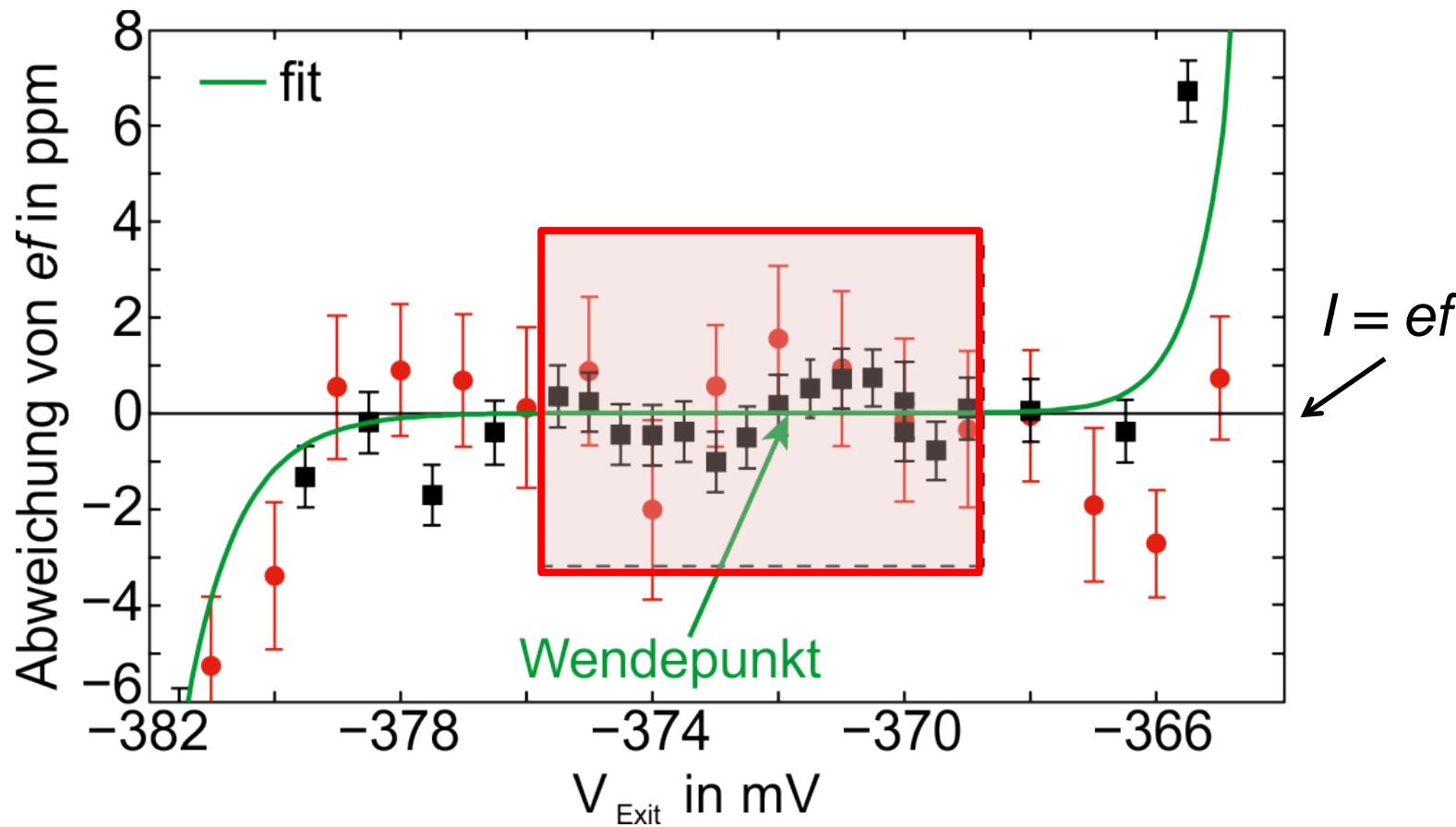


D. Drung et al., Rev. Sci. Instrum. 86, 024703 (2015)



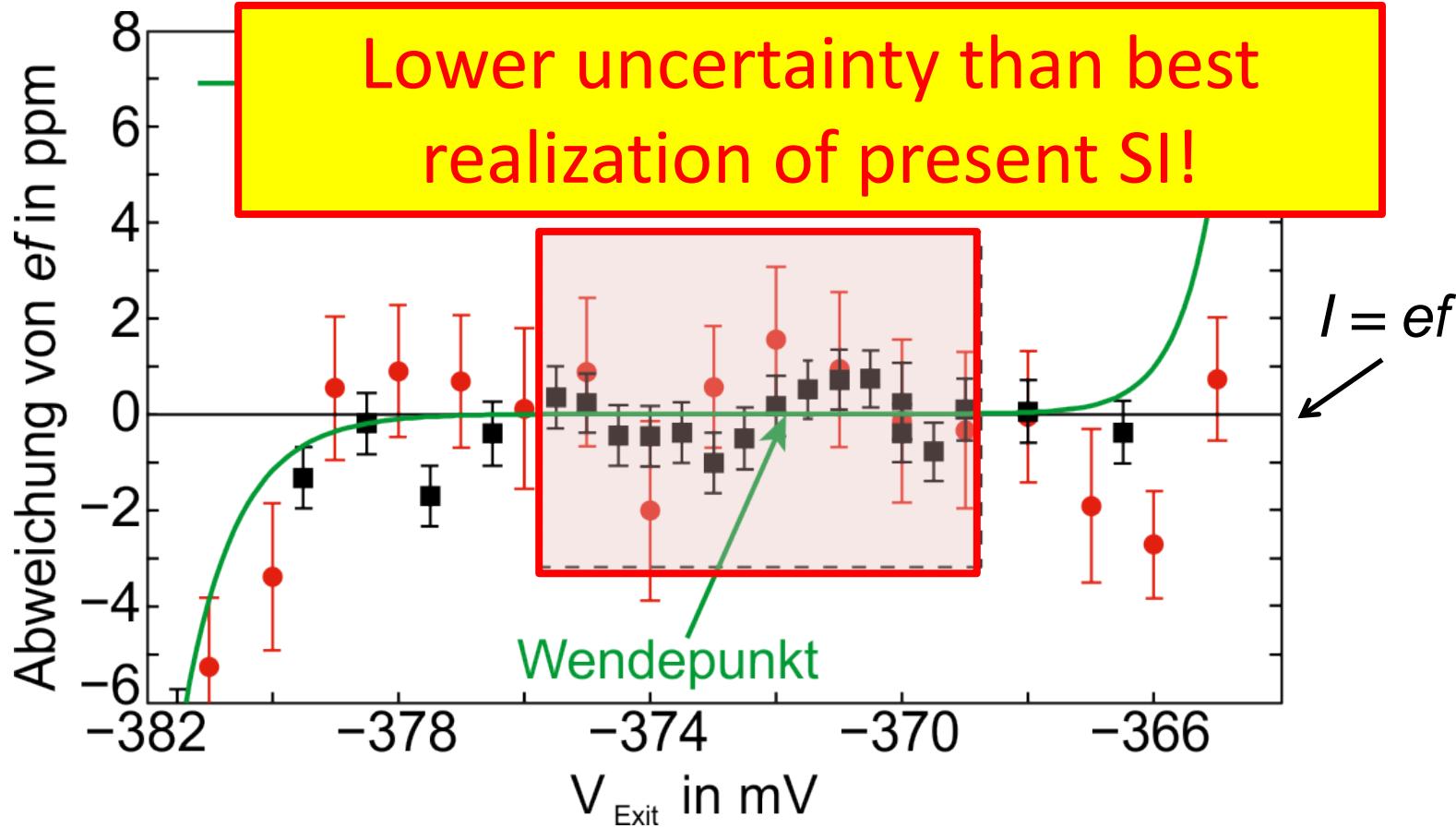
$$B = 16 \text{ T}$$

$$f = 545.45 \text{ MHz} \rightarrow |I| \approx 87 \text{ pA}$$



$$I_{\text{pump}} - ef = (-0.06 \pm 0.2) \text{ ppm}$$

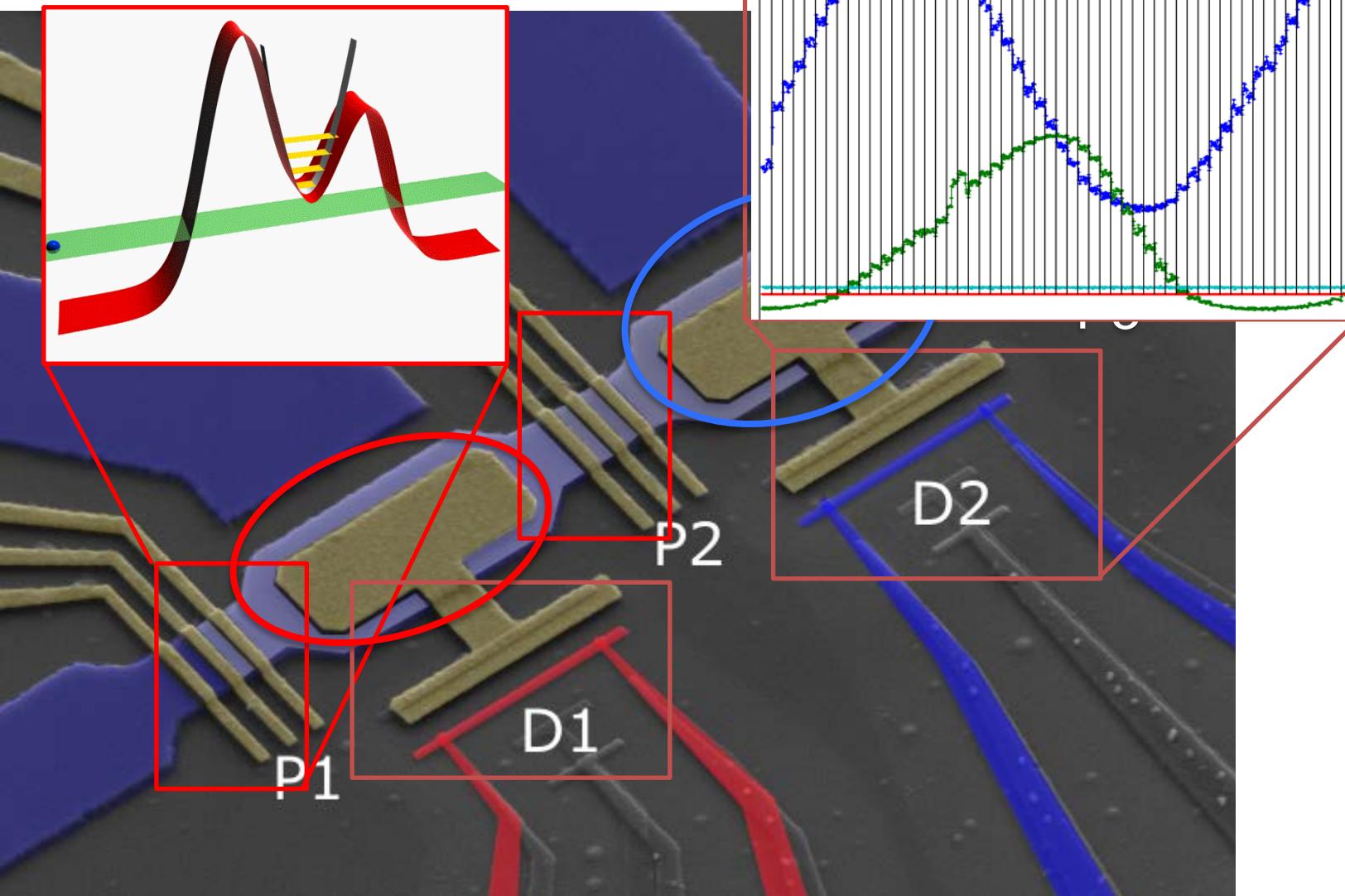
Stein et al, APL 107, 103501 (2015)



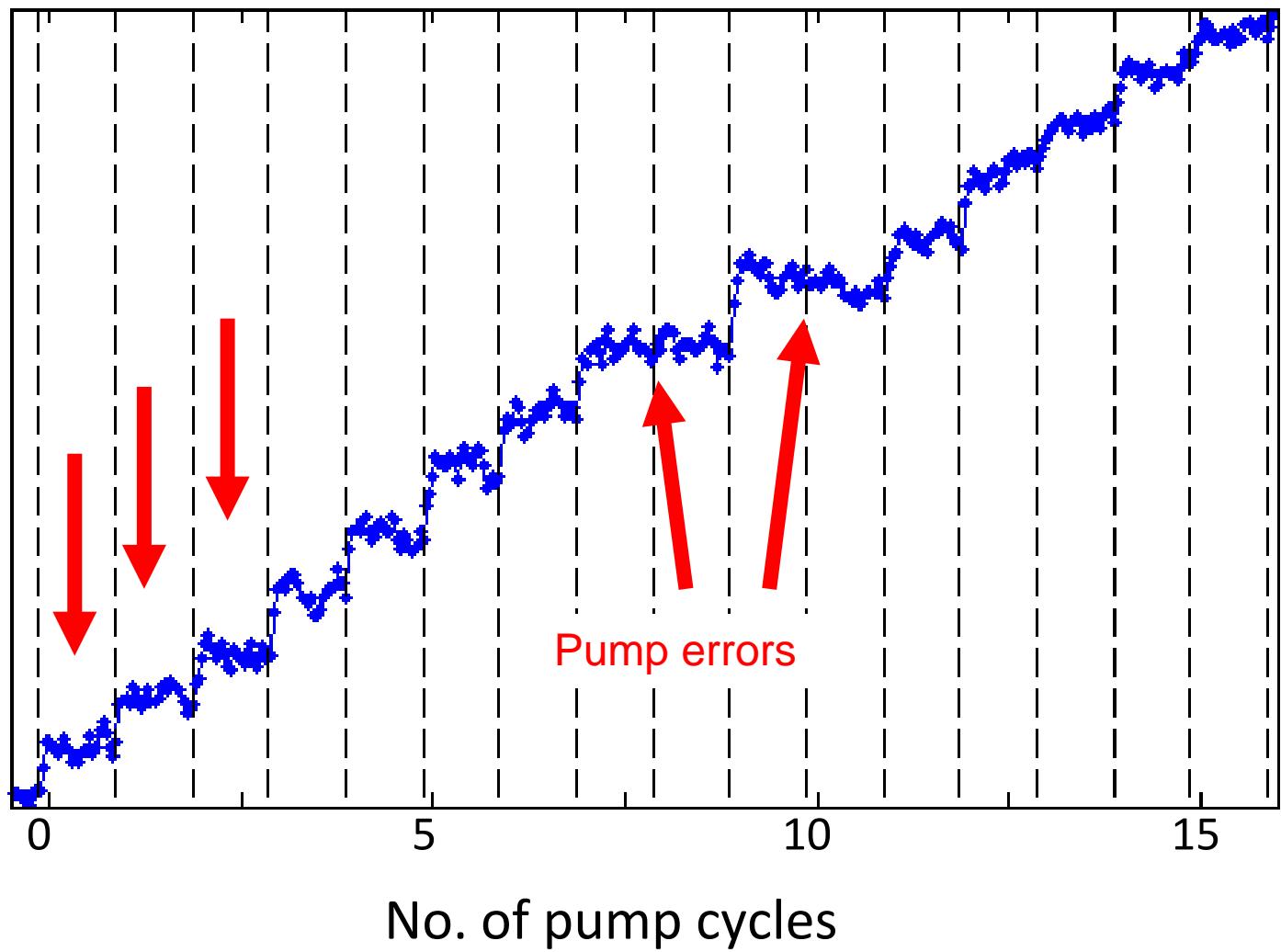
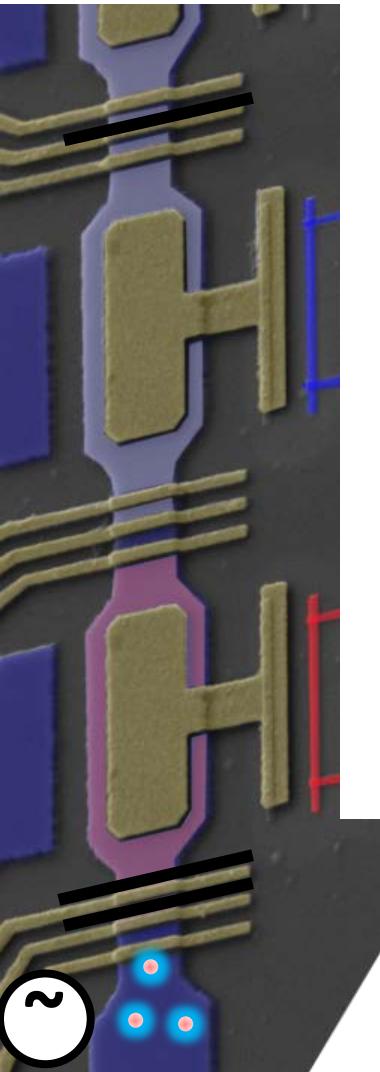
$$I_{\text{pump}} - ef = (-0.1 \pm 0.16) \text{ ppm}$$

Stein et al, APL 107, 103501 (2015)

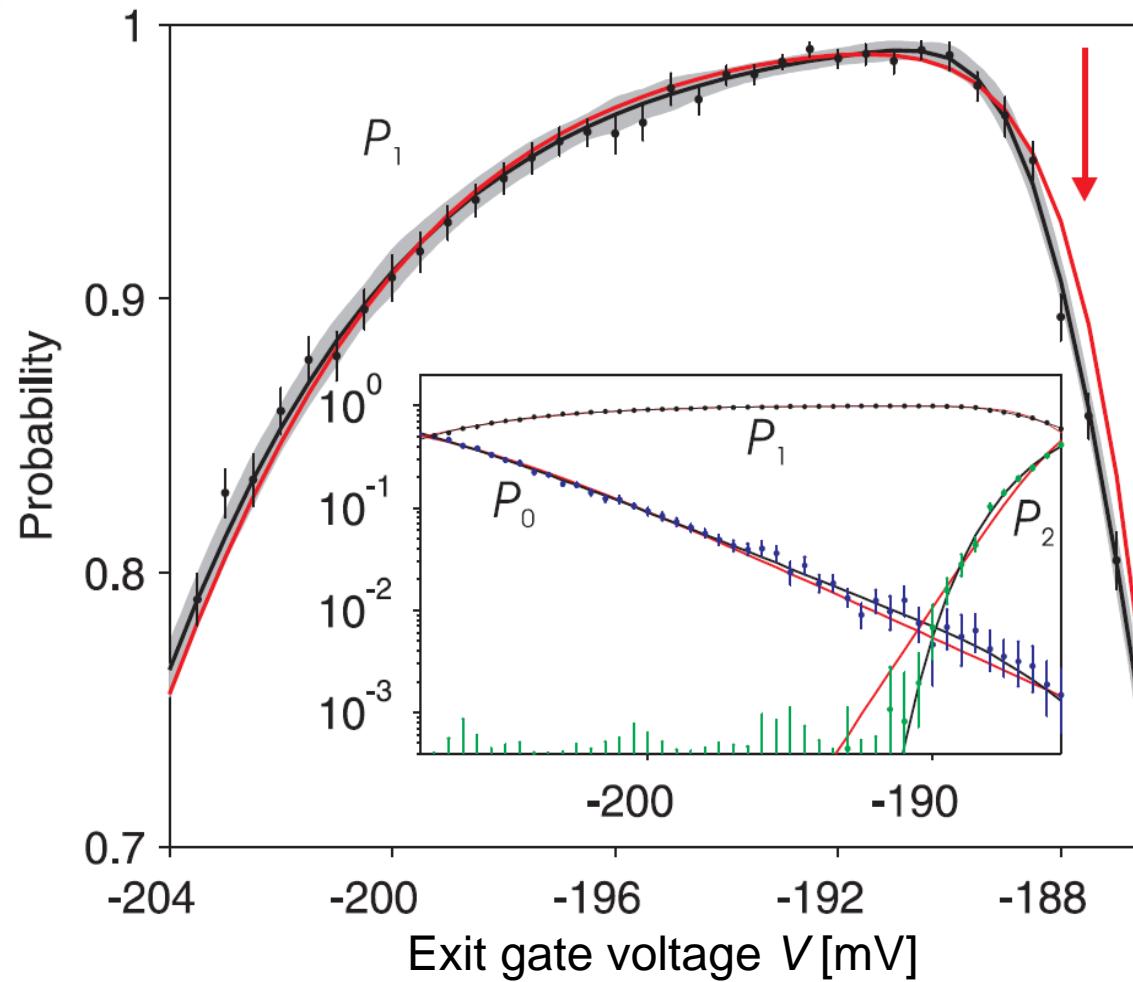
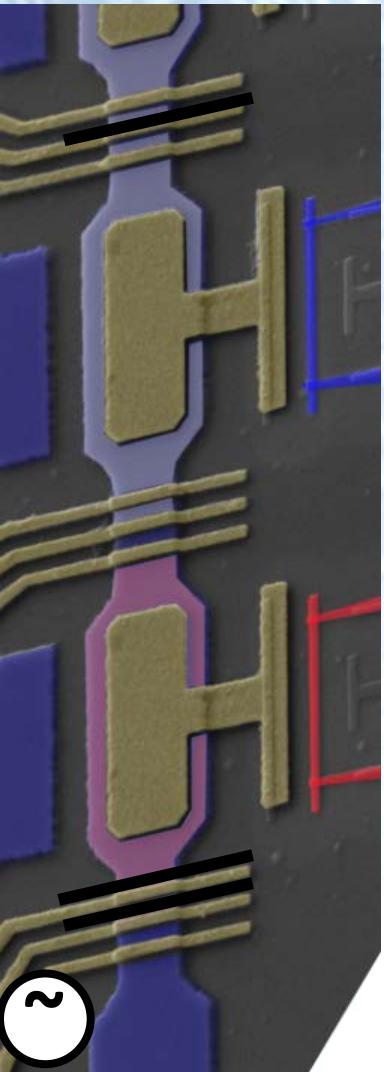
# Self referenced quan



# Single electron detection



# Counting statistics of pumping



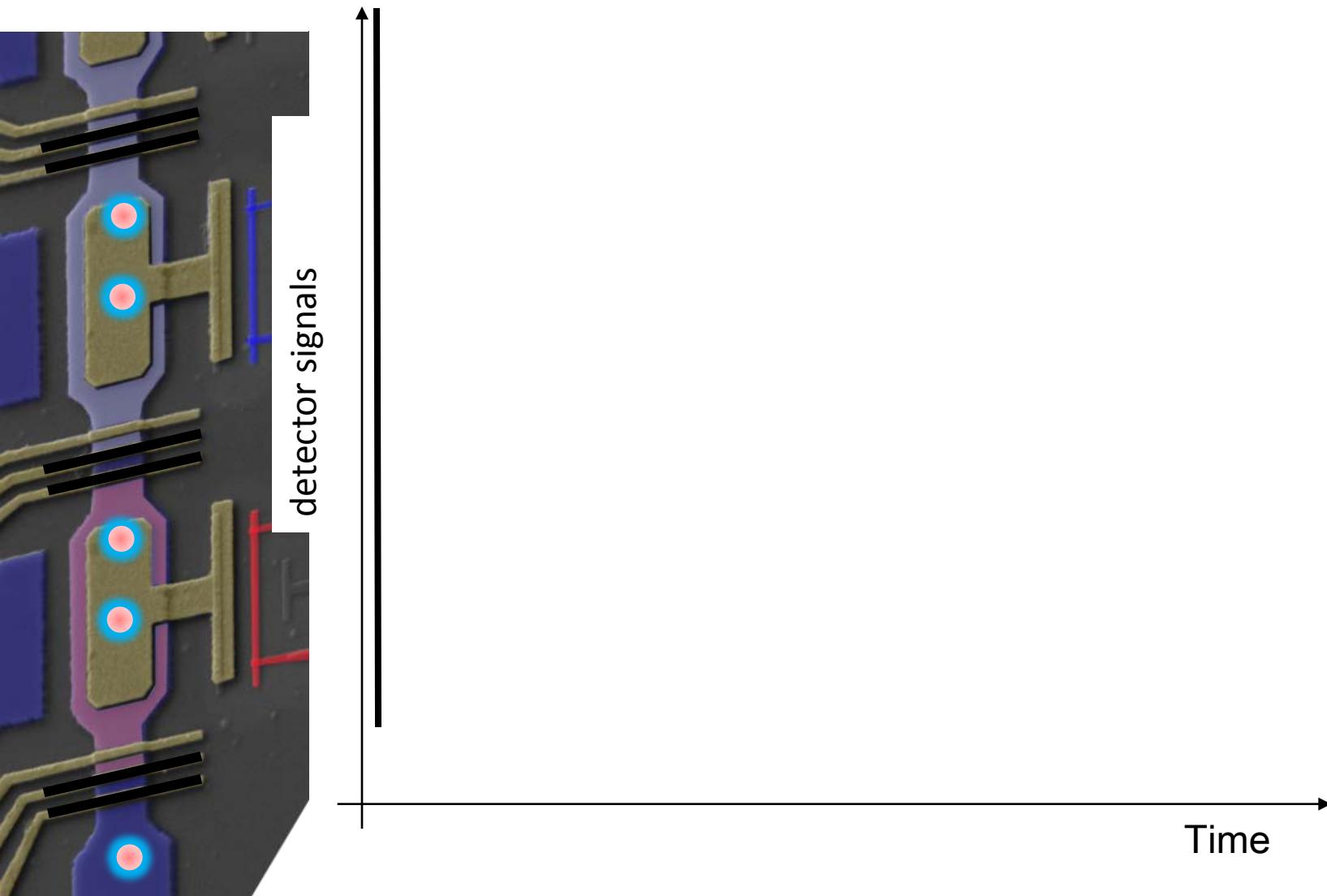
Statistical data agrees with decay cascade model



# In-situ error detection

No errors:

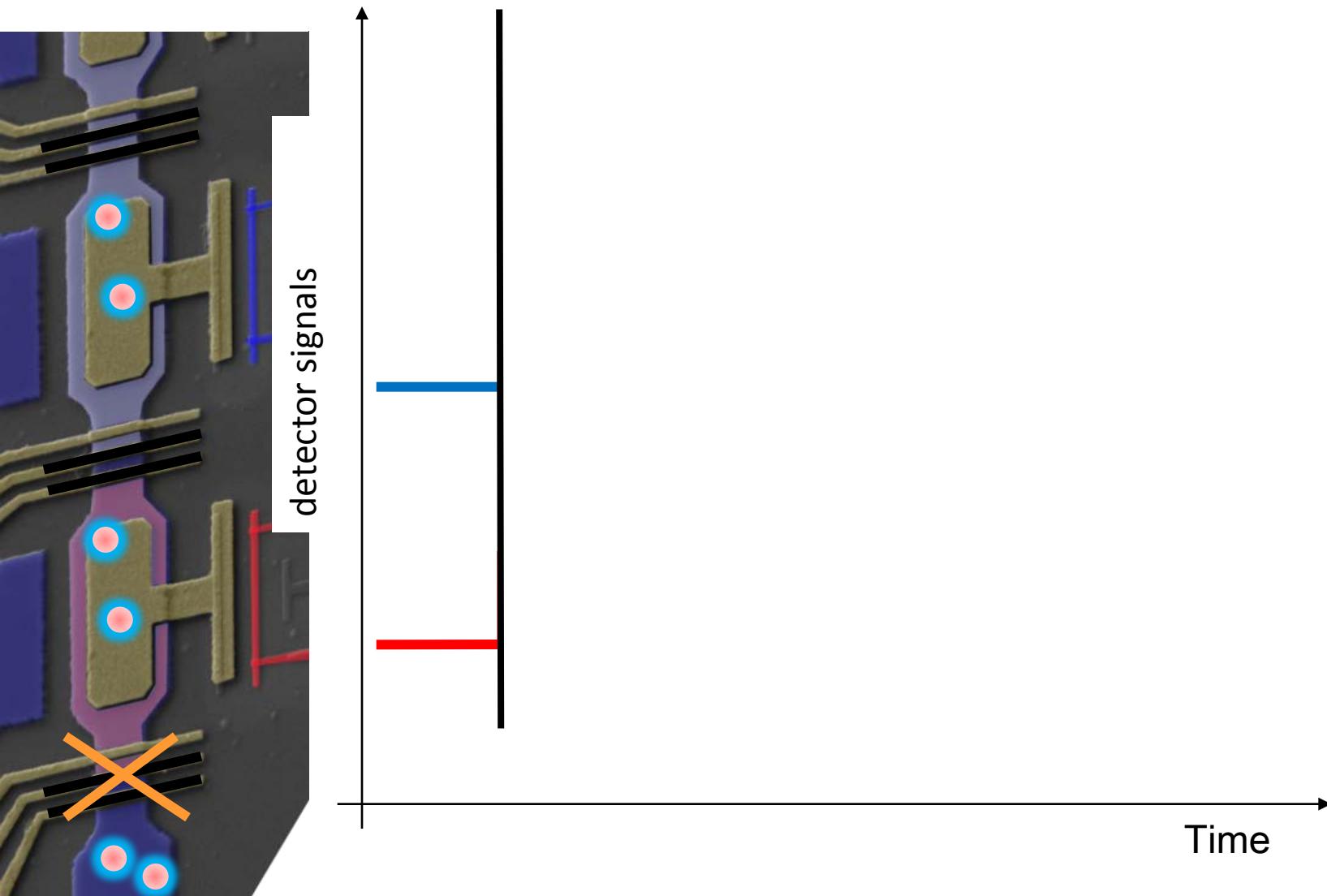
Constant detector signal



# In-situ error detection

Fehler P1:

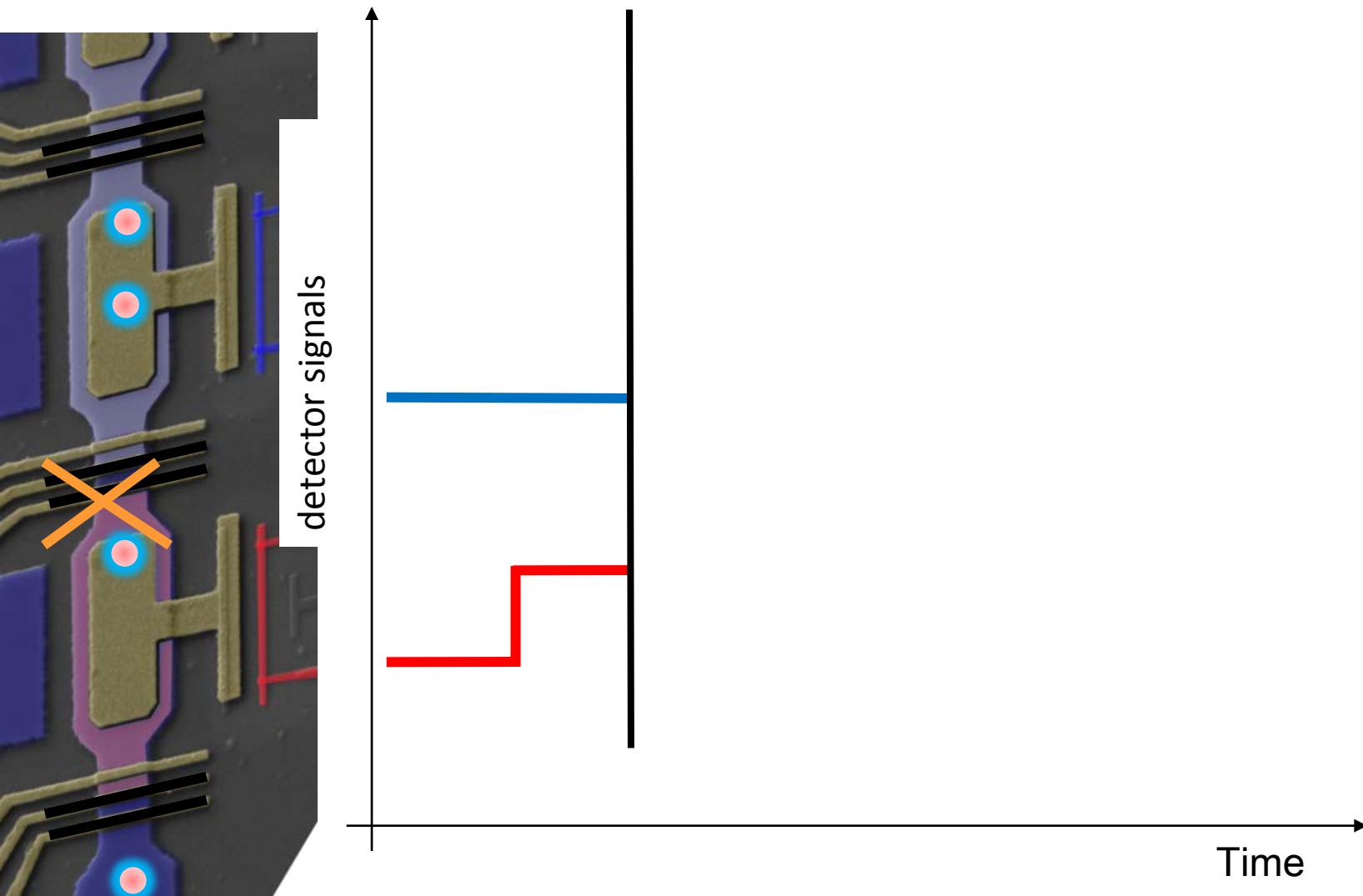
D1: 1 one electron less



# In-situ error detection

Error P2:

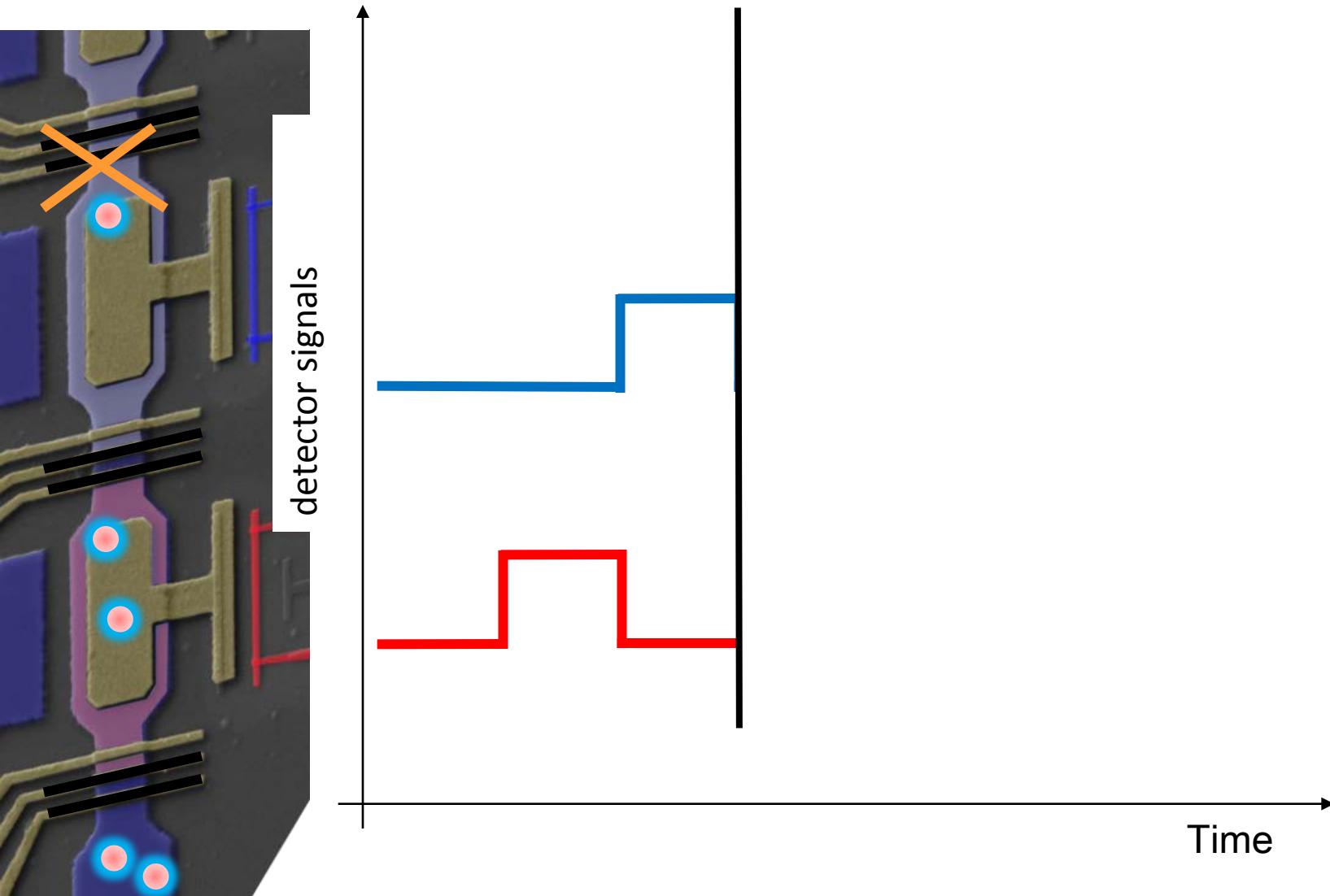
D1: one electron more  
D2: one electron less



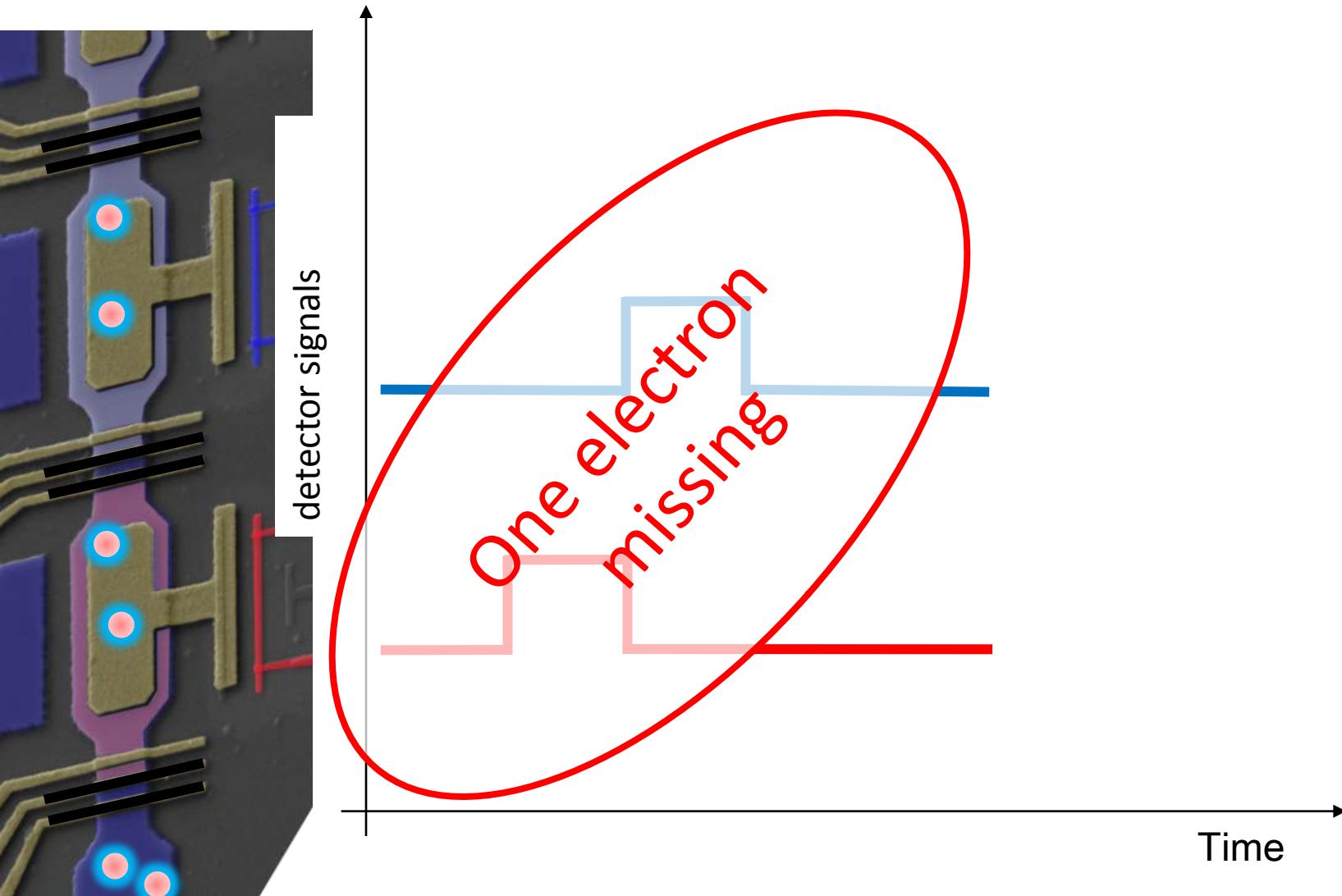
# In-situ error detection

Error P3:

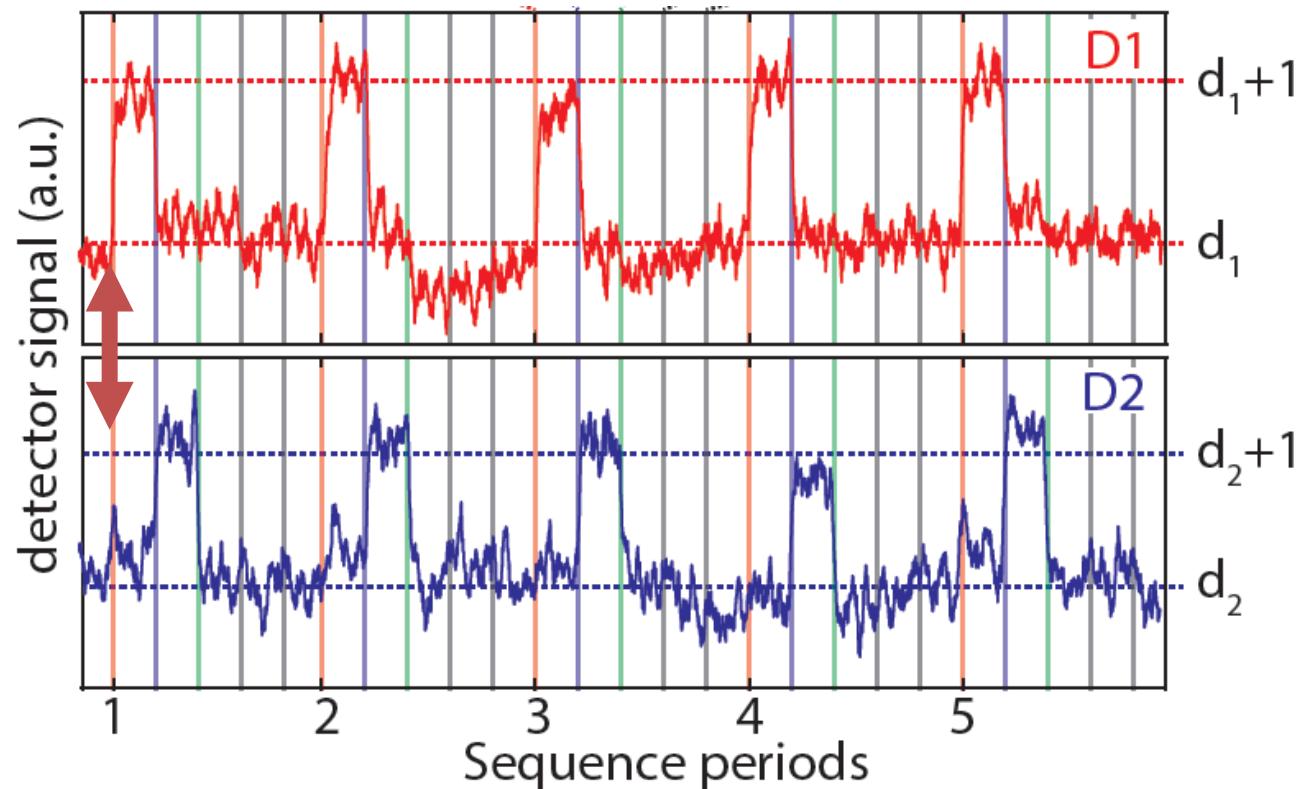
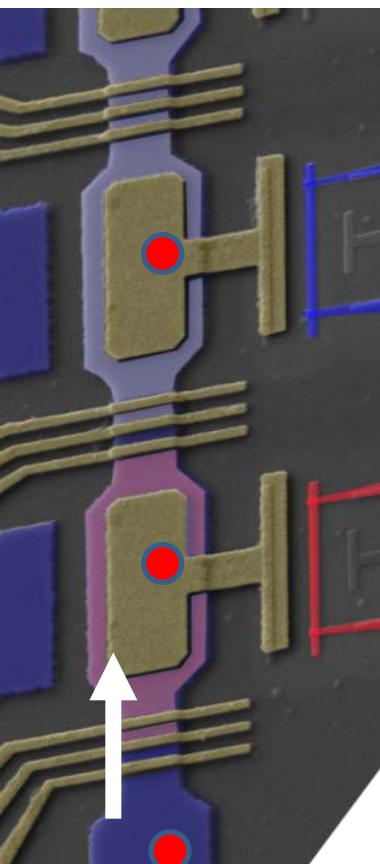
D2: one electron more



# In-situ error detection

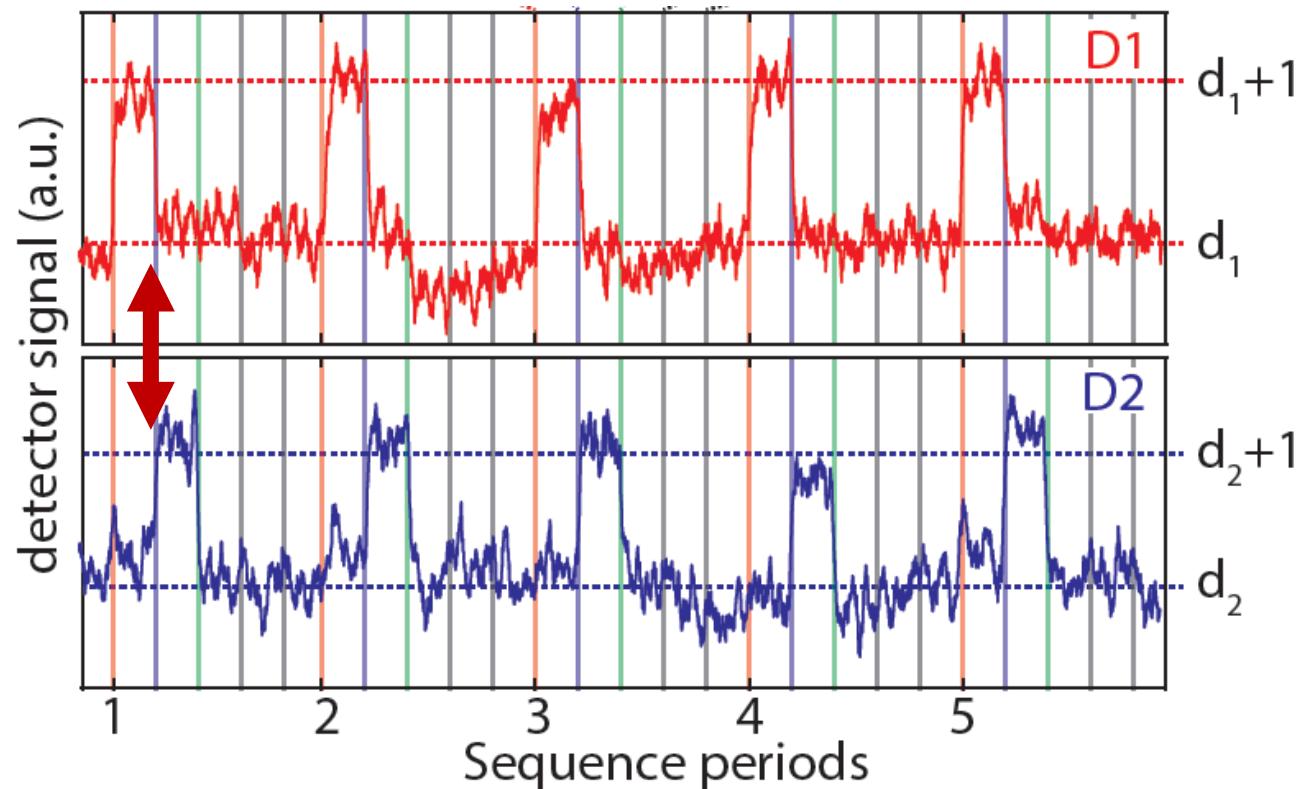
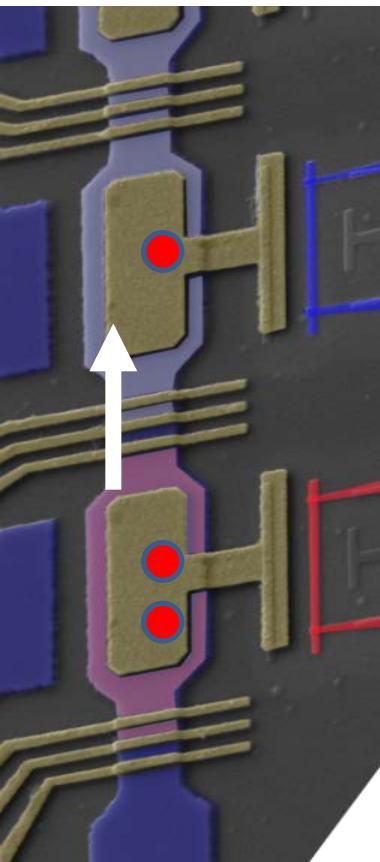


# Experiment: Serielles und simultanes Pumpen



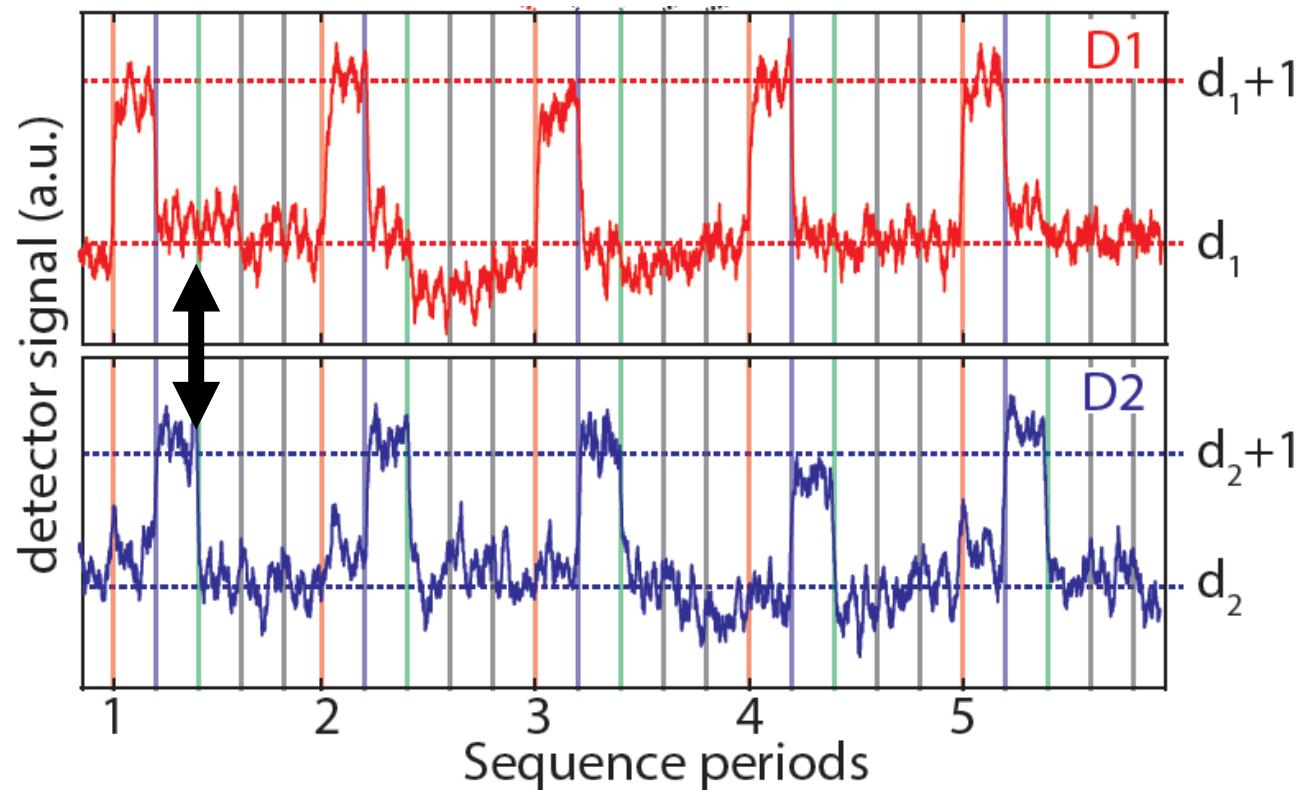
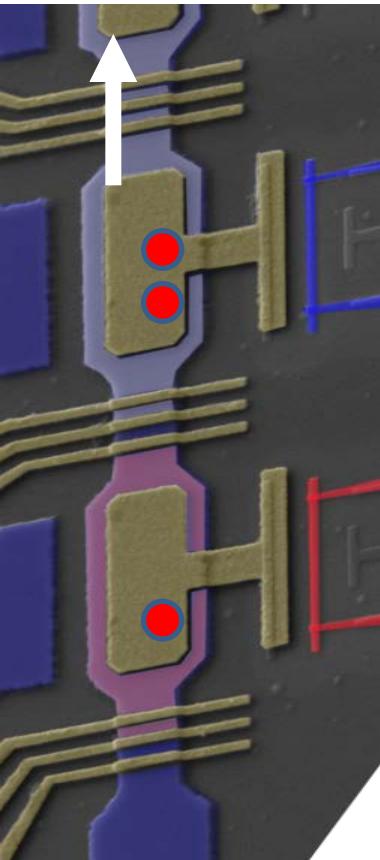
# Experiment:

## Serial and simultaneous pumping



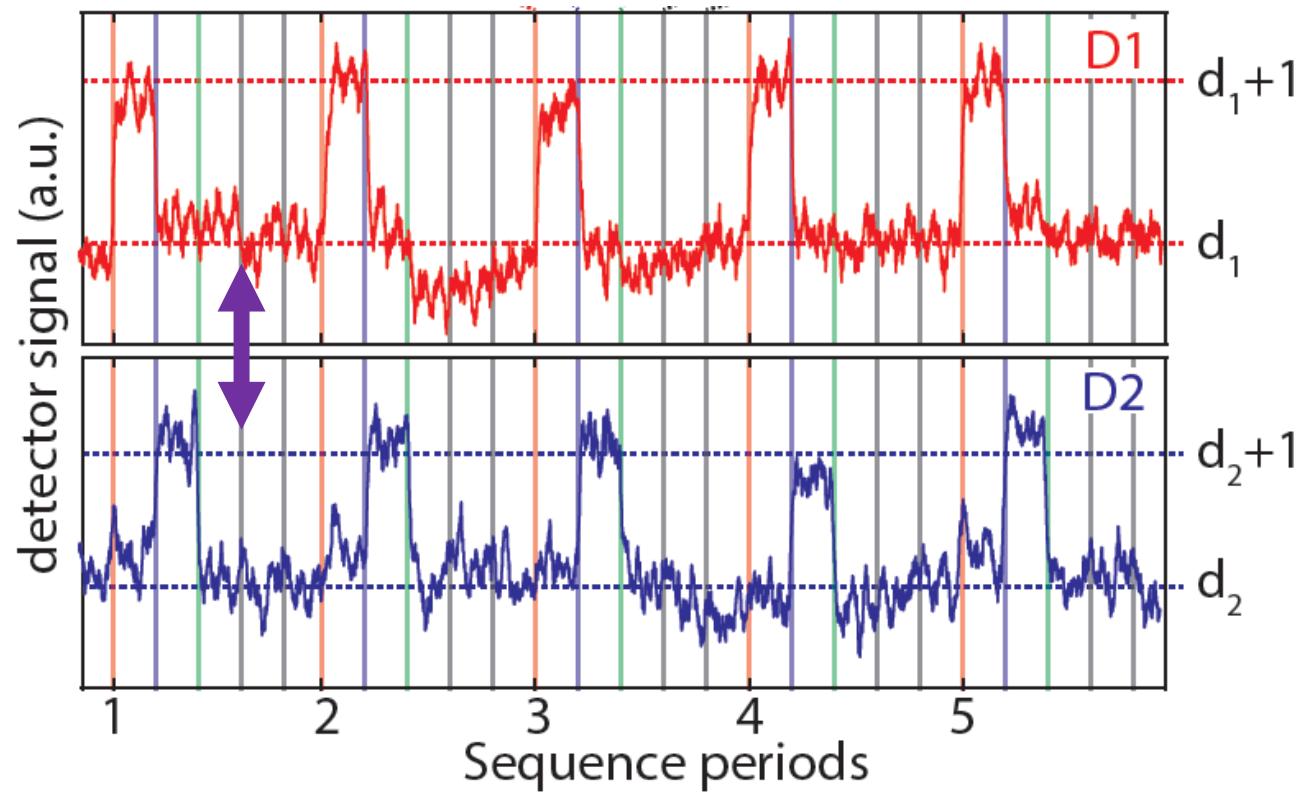
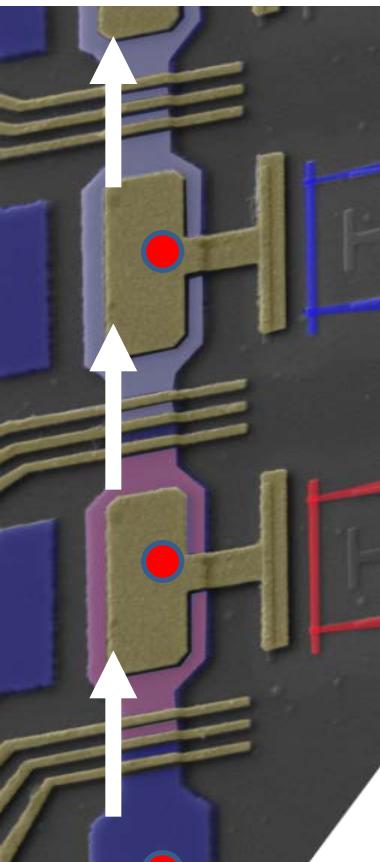
# Experiment:

## Serial and simultaneous pumping



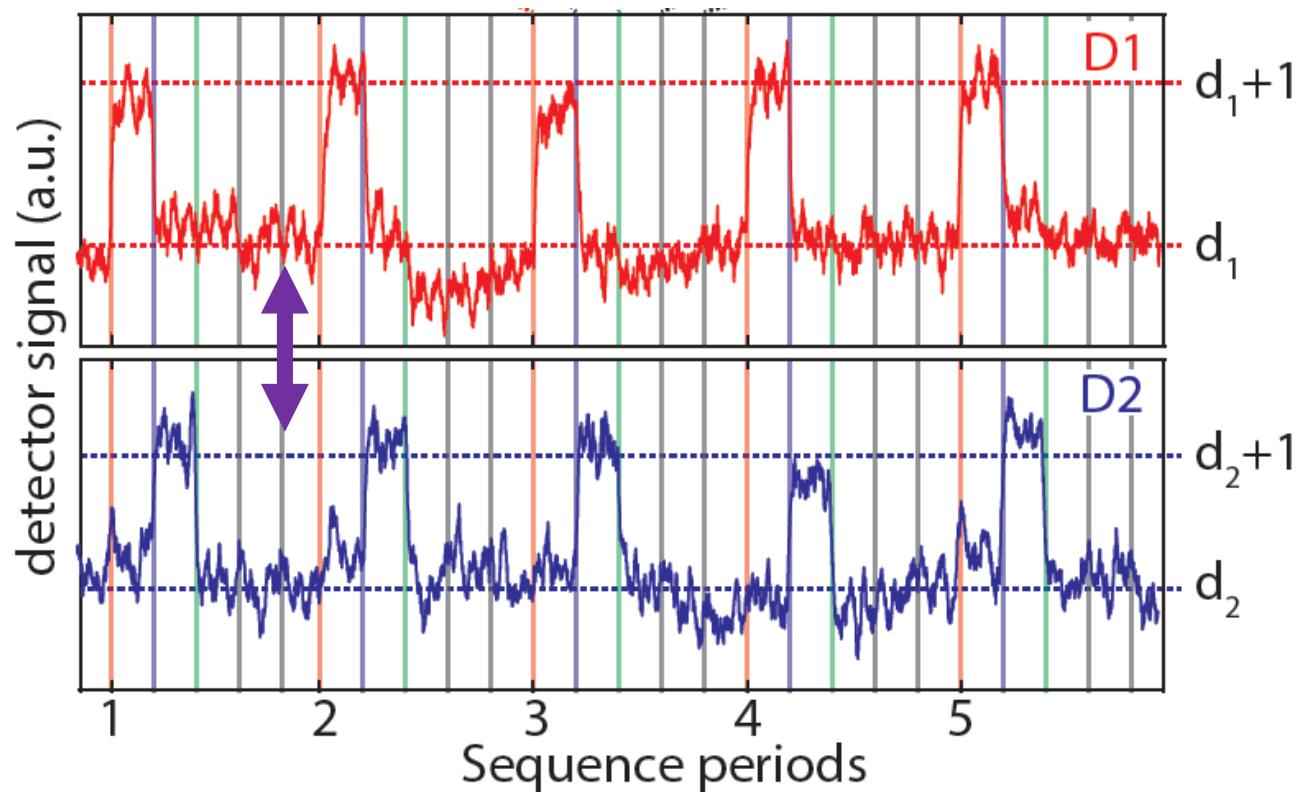
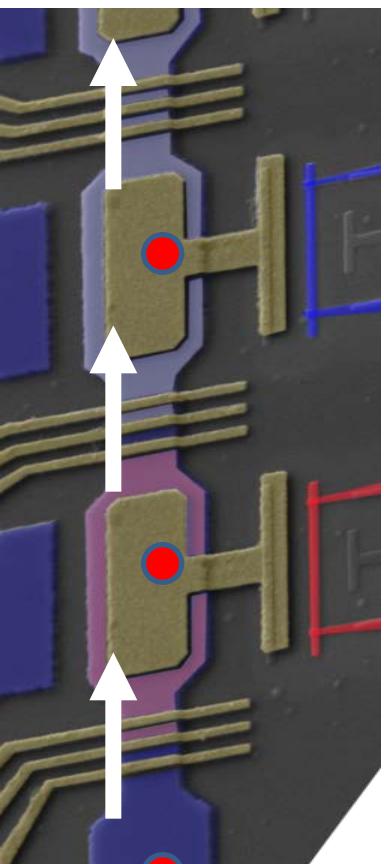
# Experiment:

## Serial and simultaneous pumping

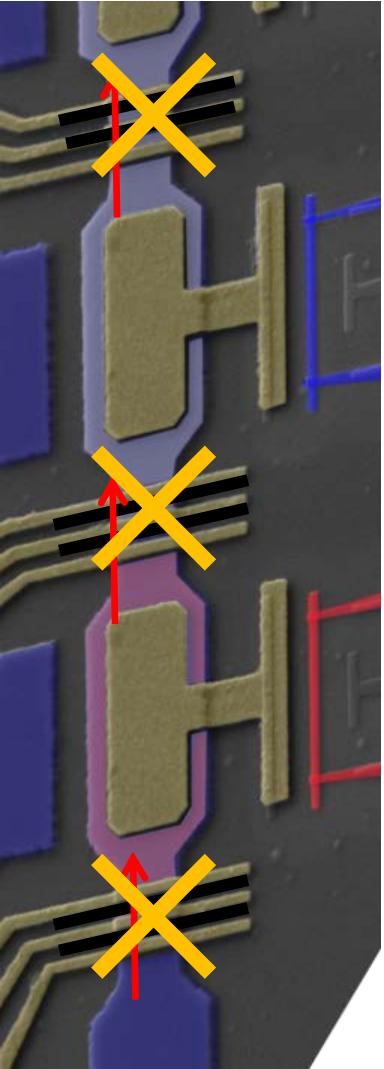


# Experiment:

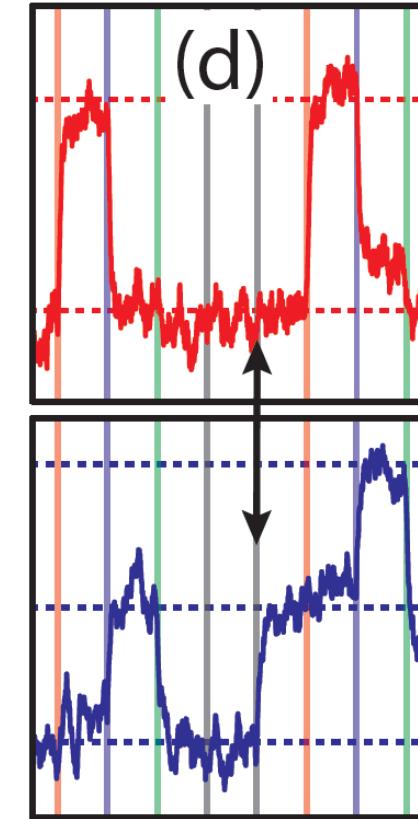
## Serial and simultaneous pumping



# Experiment: Detection of pump errors

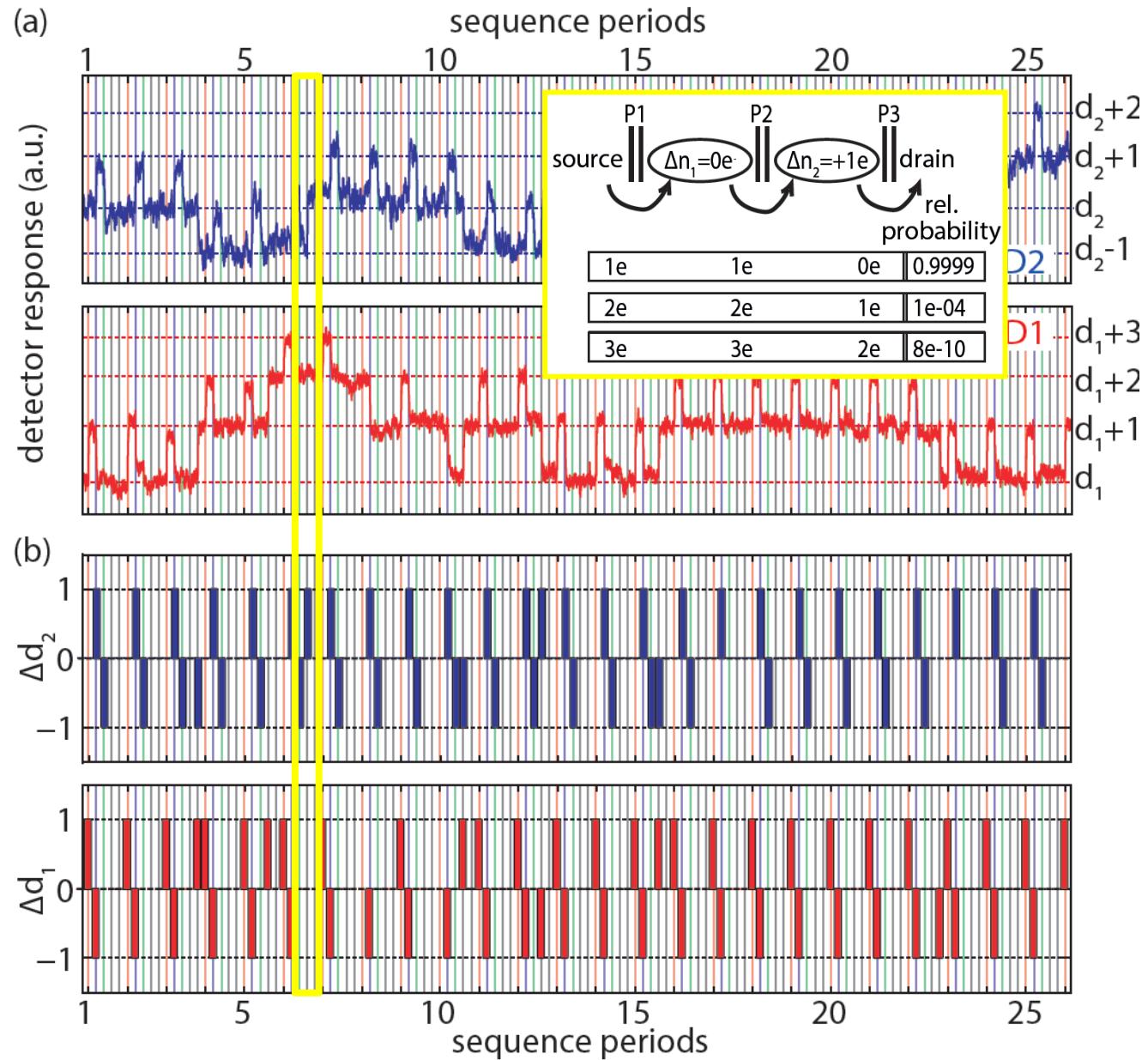
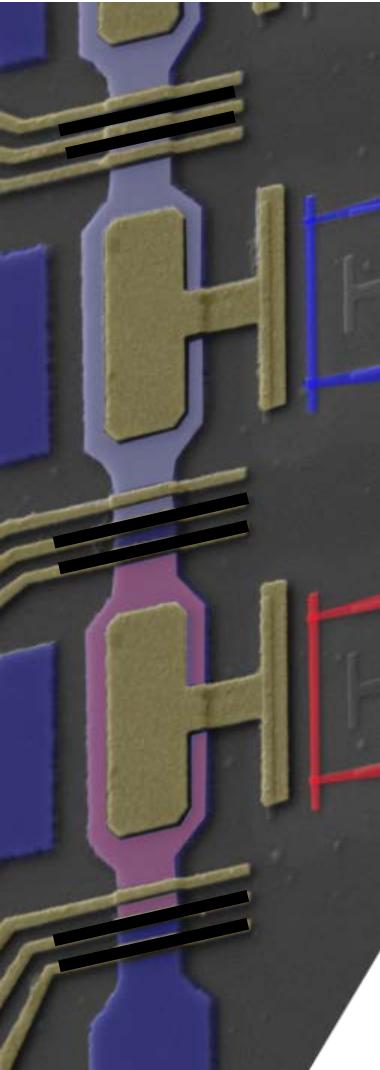


detector signal (a.u.)

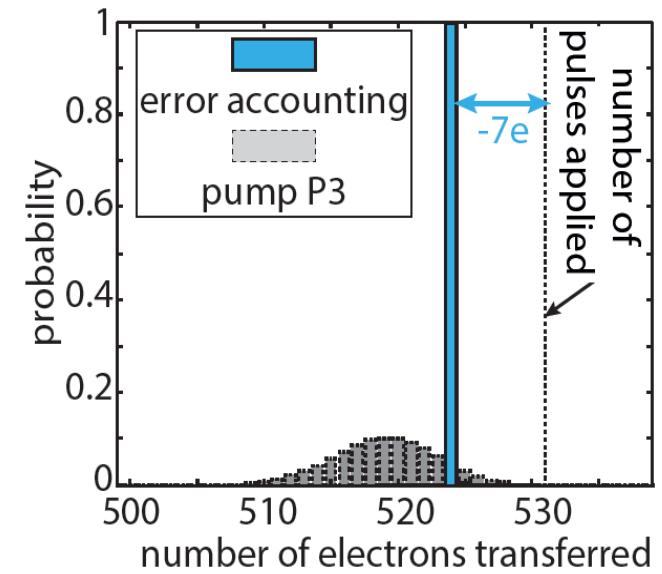
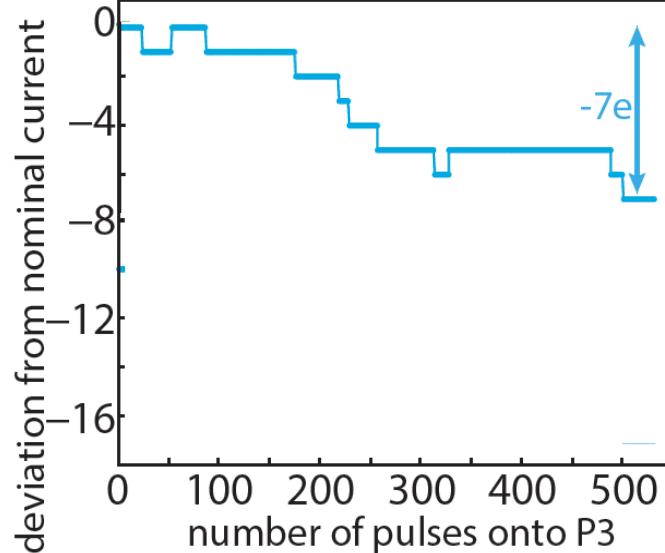
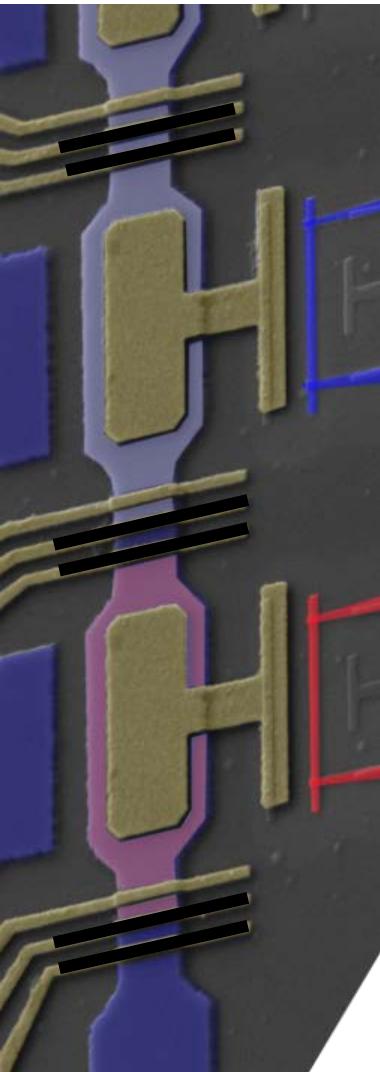


Error by P3 (simultaneous pumping)

# Statistical analysis of pump errors



# Statistic analysis of current uncertainty



- Self reference quantized current source
- Uncertainty reduced by factor of 50

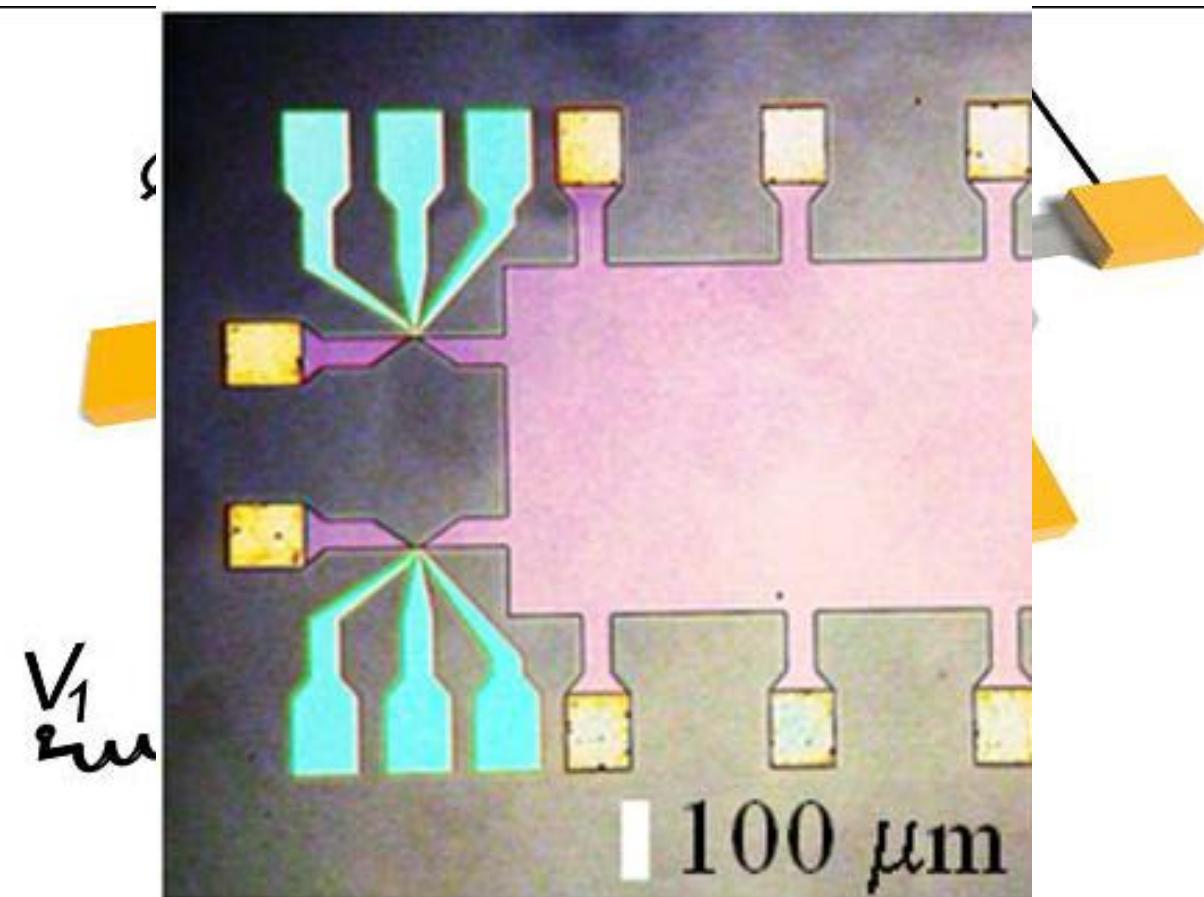
# Outlook: Achievable current and uncertainty

$$\text{corr. error rate} \sim \left( \frac{\text{Detection time}}{\text{Time between errors}} \right)^{\text{No. islands}/2} \times \left( \text{error rate} \right)^{\text{No. islands}/2}$$

- Pumping frequency  $f \simeq 1 \text{ GHz}$
- Initial error rate  $\simeq 1 \text{ ppm}$
- Detector bandwidth  $\simeq 50 \text{ kHz}$
- 5 pumps in series

$$\rightarrow I > 160 \text{ pA}, \quad u_{\text{ges}} < 10^{-8}$$

# Semiconductor quantized voltage source



An "integrated quantized circuit" (IQC)

# Semiconductor quantized voltage source

SET-pump:

$$I = n e f$$

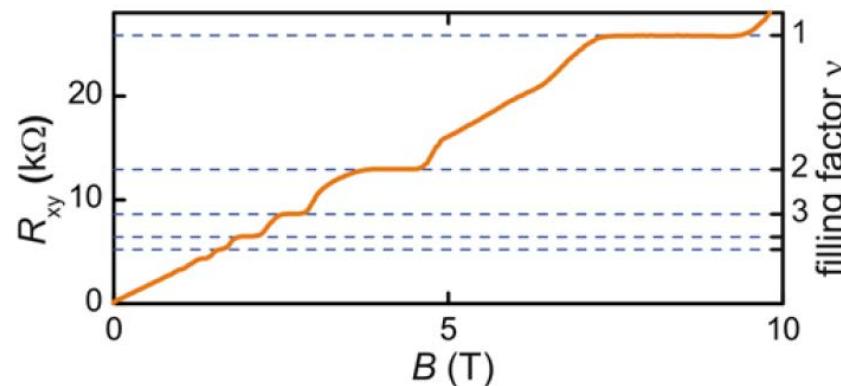
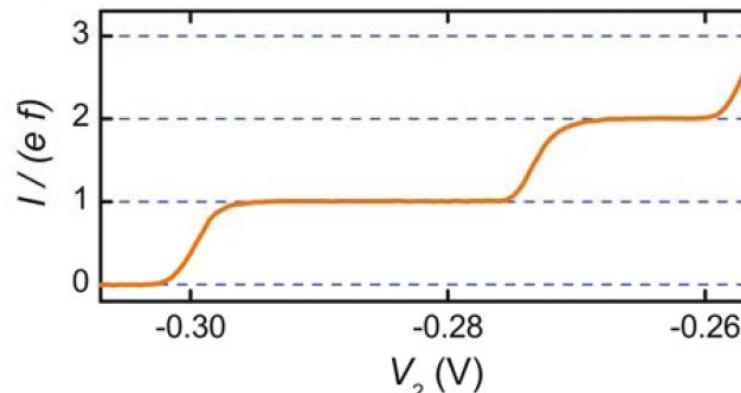
+

QHE

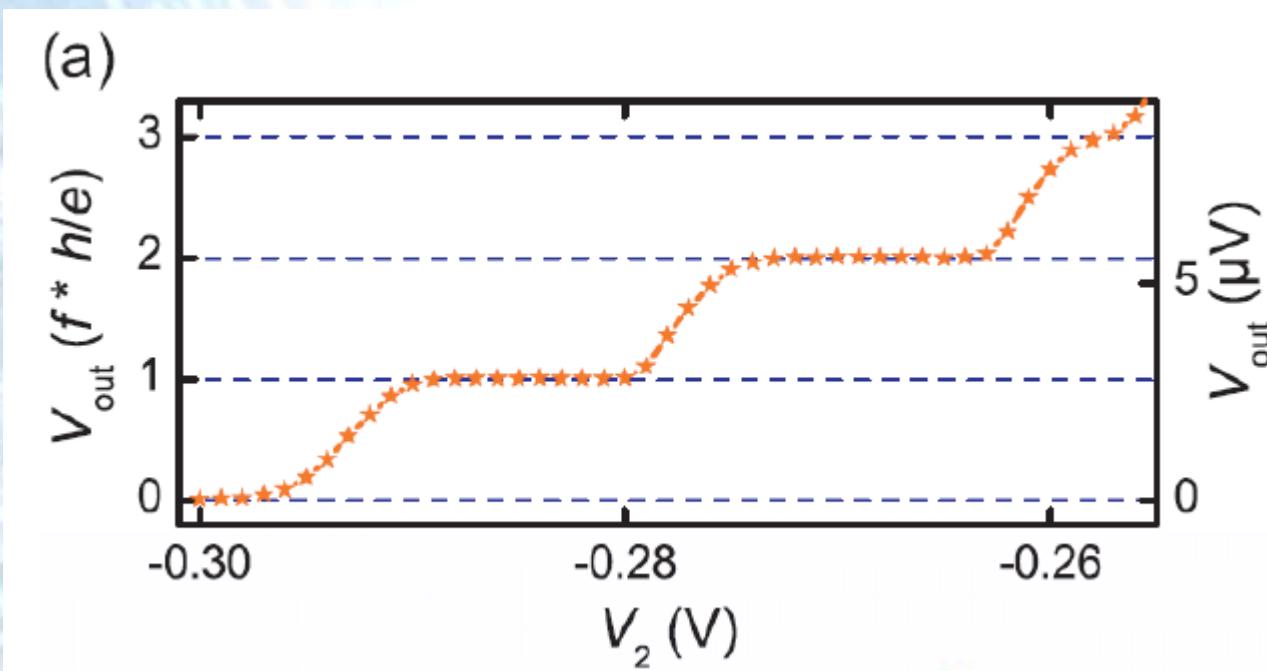
$$V = I^* h / (\nu e^2)$$

=

$$V_{\text{out}} = (n/\nu) \cdot (h/e) \cdot f$$



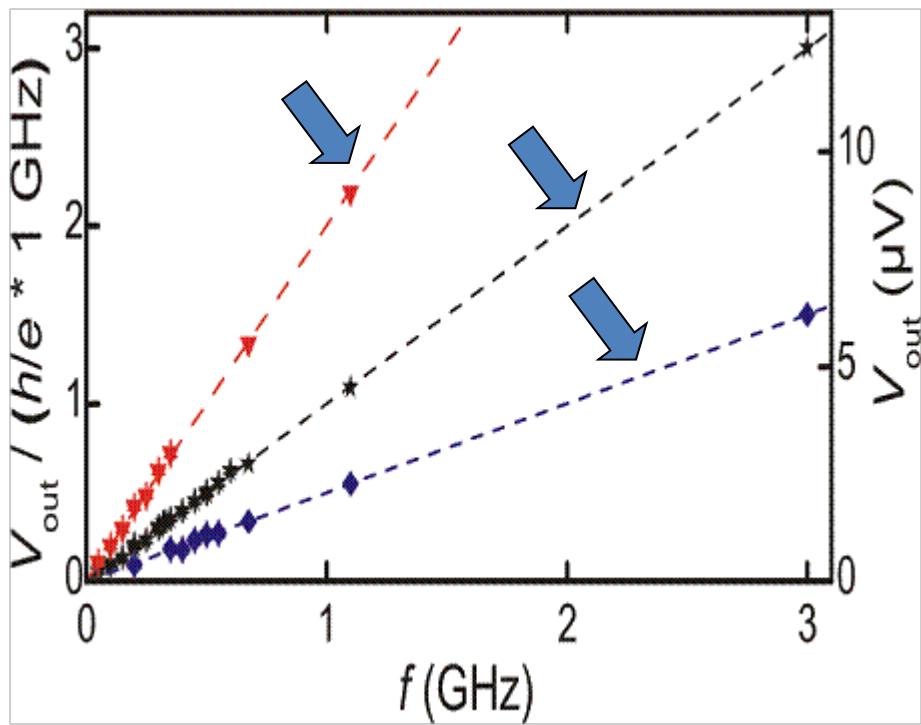
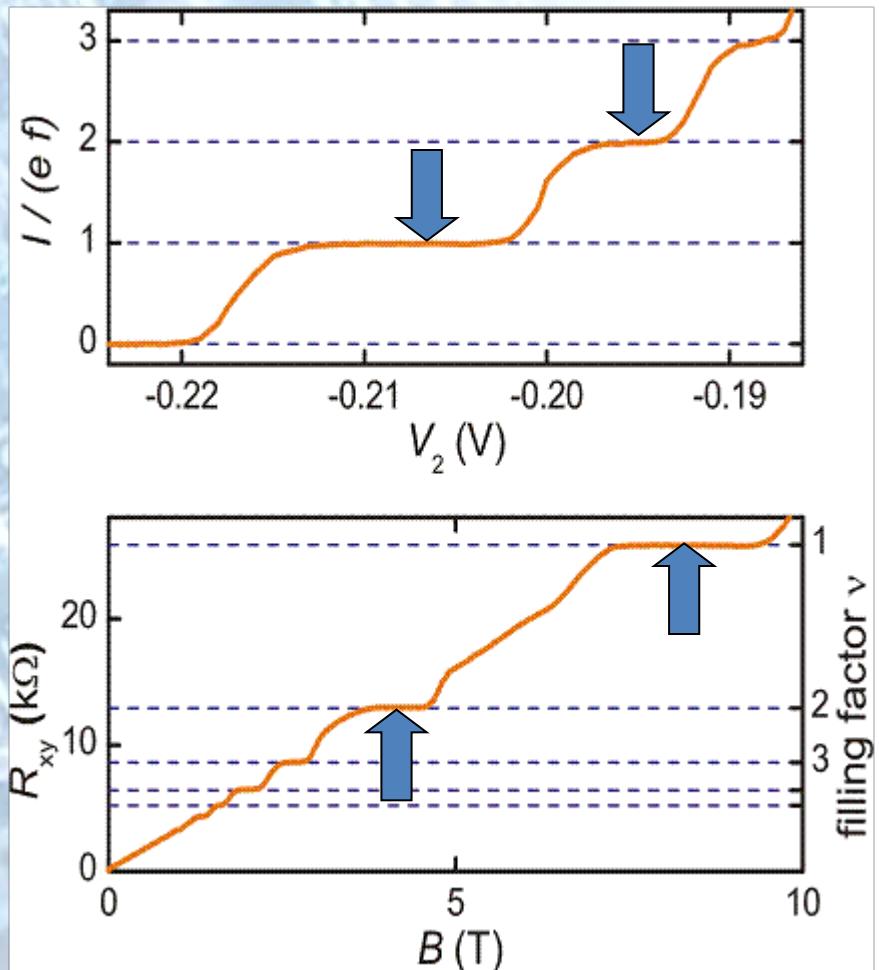
# Quantized voltage



$$V_{\text{out}} = (n/v) \cdot (h/e) \cdot f$$

$v=1$ ,  $f = 675 \text{ MHz}$

# Quantized voltage vs. frequency

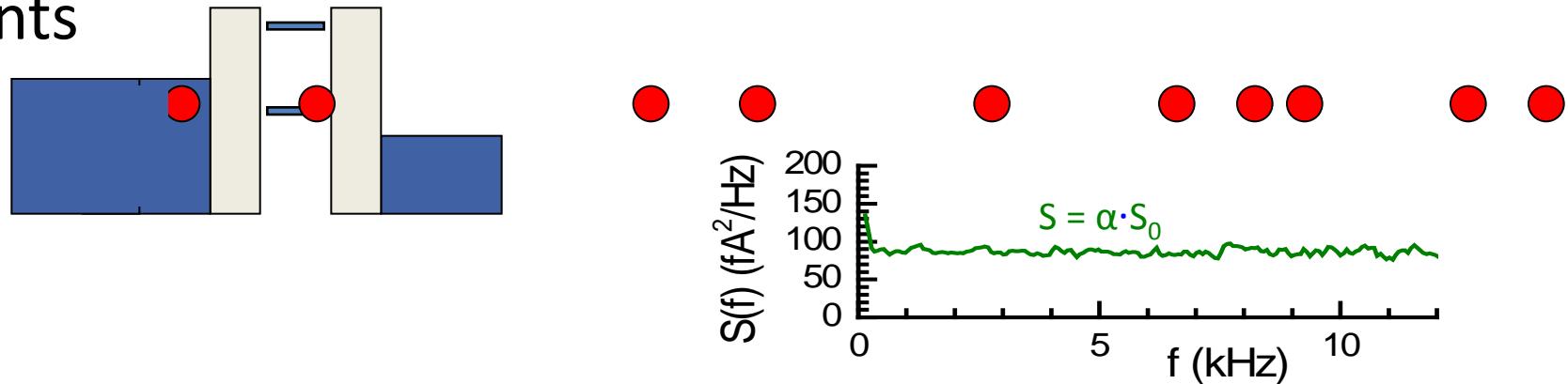


$$V_{out} = n/\nu * h/e f$$

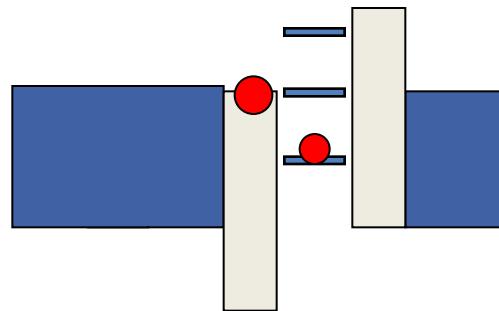
# Beyond metrology: Noise suppression

# Putting electrons on a string

- Shot noise caused by randomness of tunnelling events

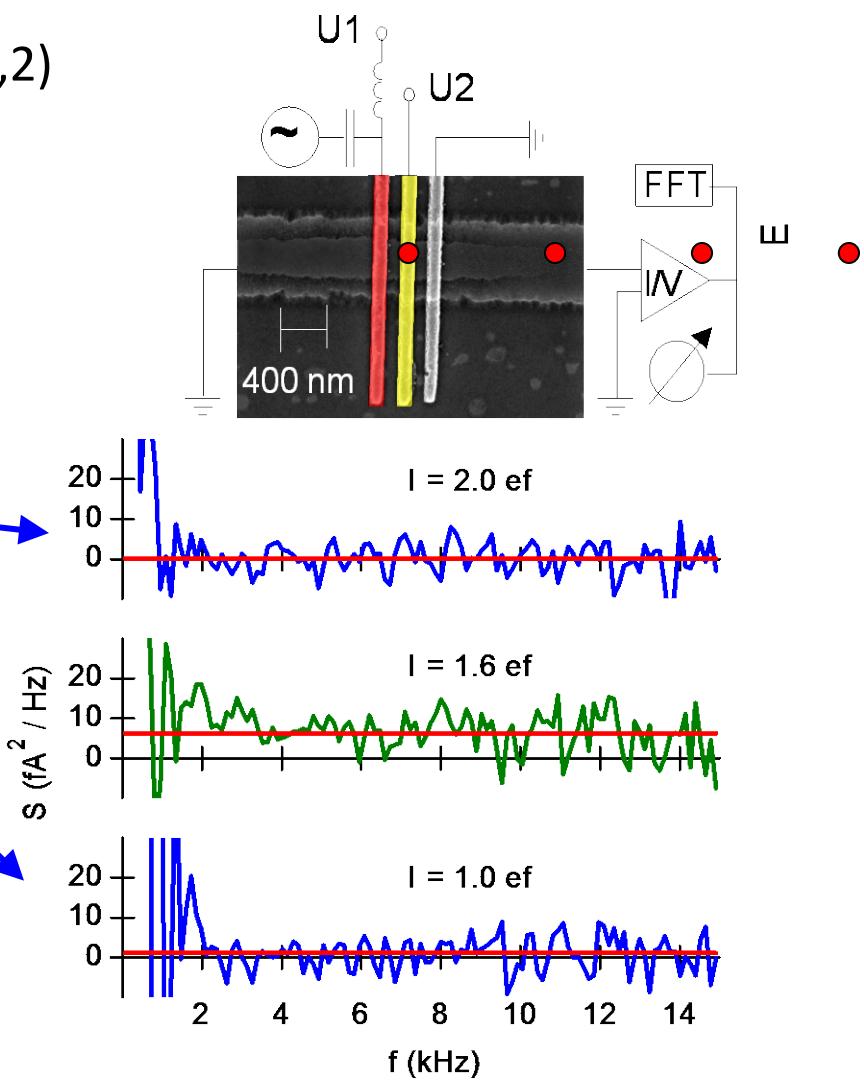
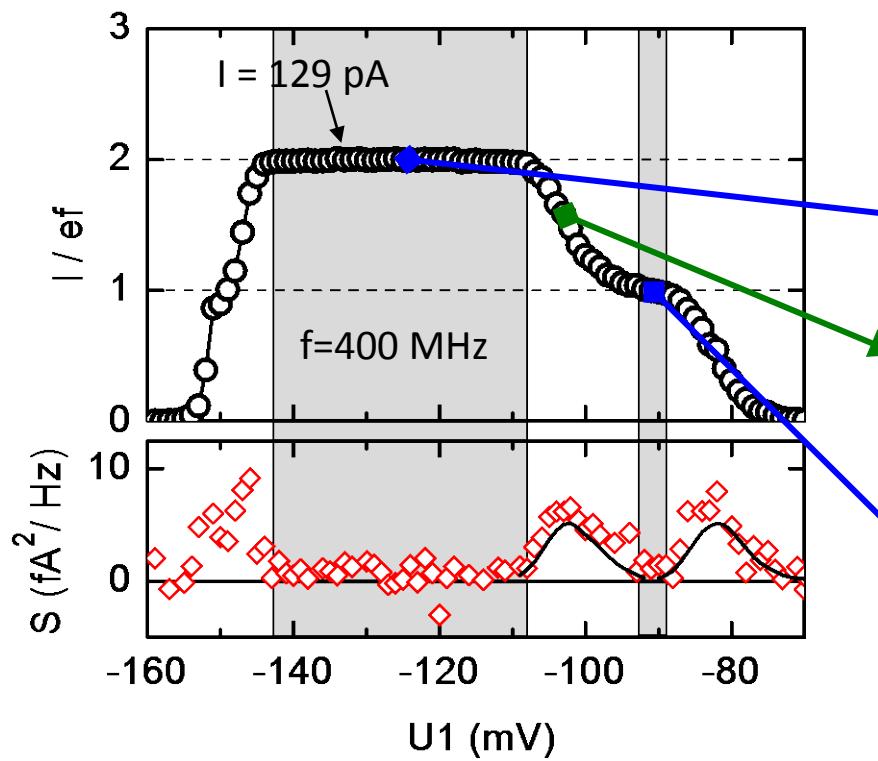


- Noise in a single electron pump?
  - Electrons at well defined times → no noise expected



# Noise suppression in an electron pump

- Quantized current plateaus  $I = n \cdot e \cdot f$  ( $n = 1, 2$ )
- Noise suppressed for quantized pumping
- Noise in plateau transitions



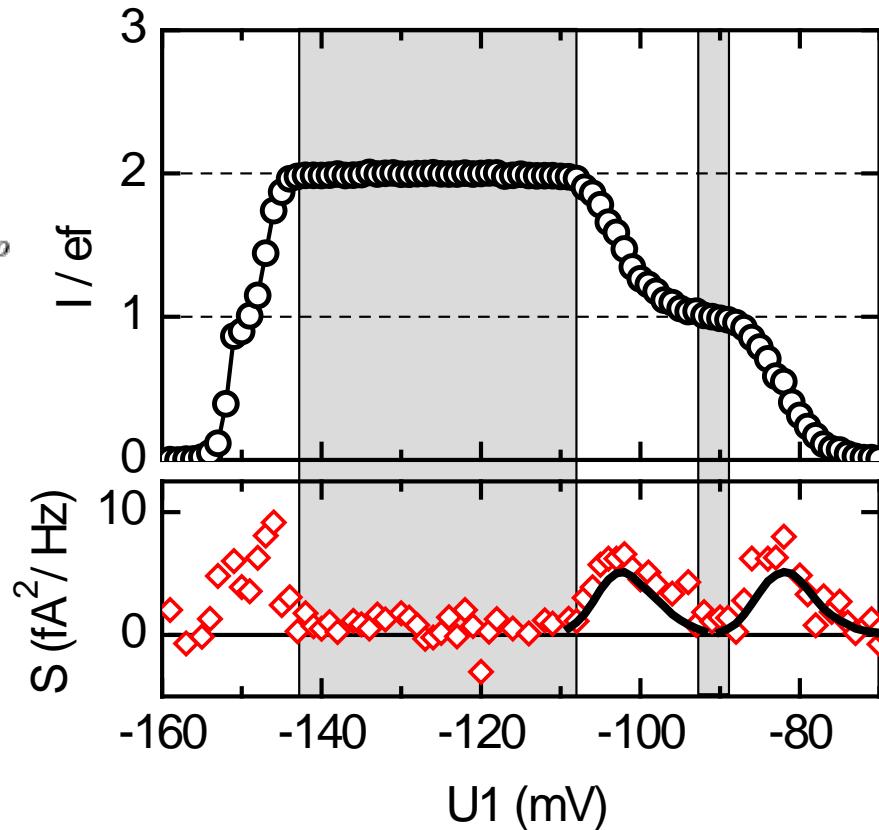
APL 92, 082112 (2008)

# Noise in plateau transitions

- Probabilities for pumping  $n$  electrons per cycle:  $p_n$
- 3 probabilities dominate
- Shot noise (assuming  $p_{n-1}+p_n+p_{n+1}=1$ )

$$S = 2 [p_{n-1} + p_{n+1} - (p_{n-1} - p_{n+1})^2] e^2 f_p$$

- Test for transition between plateaus:  
either  $n$  or  $n+1$  electrons
  - nonzero shot noise
  - can determine  $p_n$  and  $p_{n+1}$  from current
  - computed noise power agrees with data



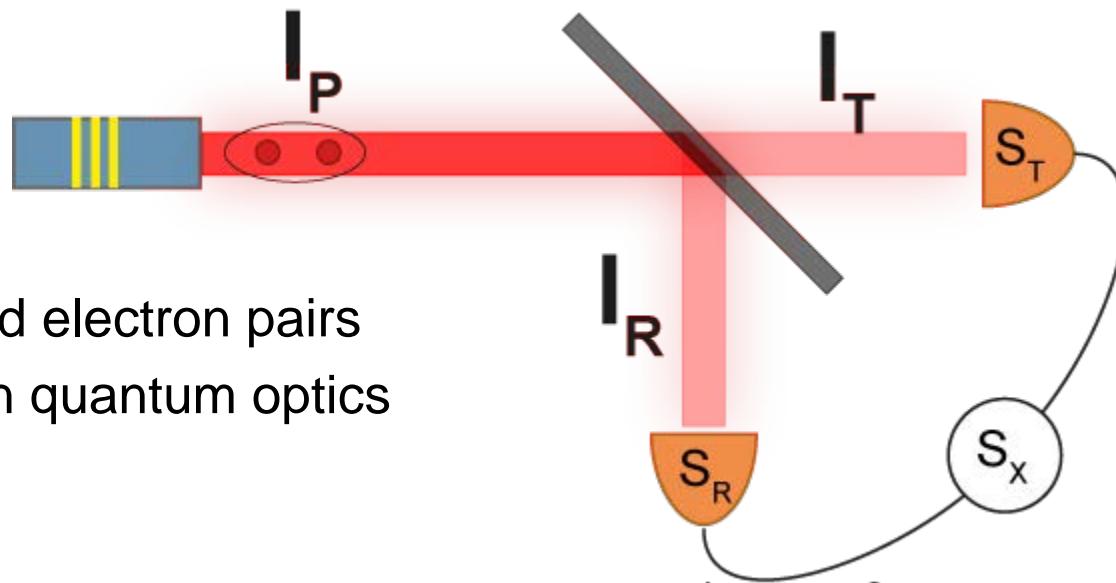
APL 92, 082112 (2008)

# Beyond metrology: On demand source of specific quantum states?

# Beyond metrology: Beam splitter experiments and pair partitioning

# Partitioning of on-demand electron pairs

On demand electron pairs  
for electron quantum optics



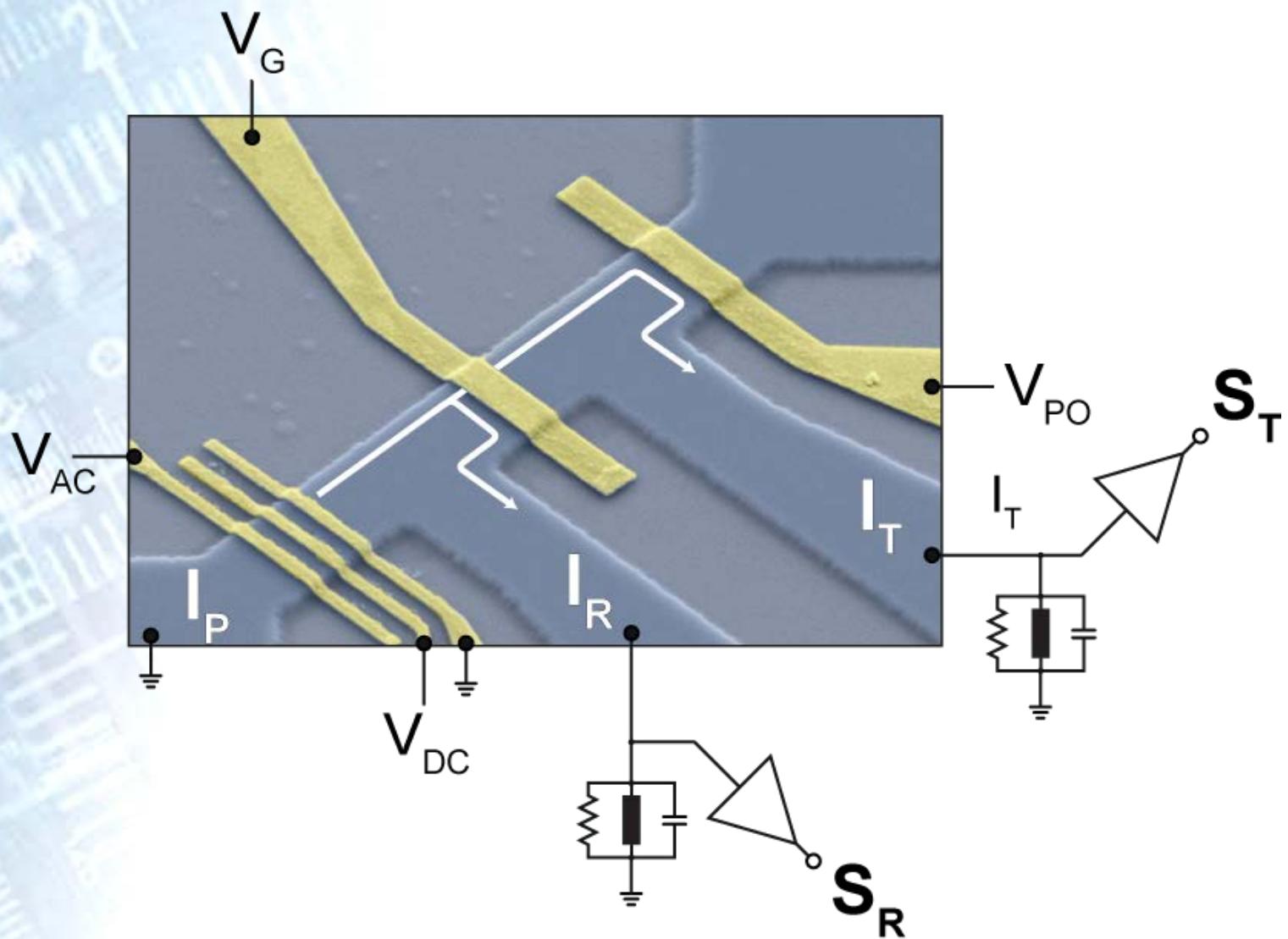
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Hannover

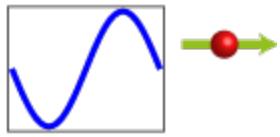


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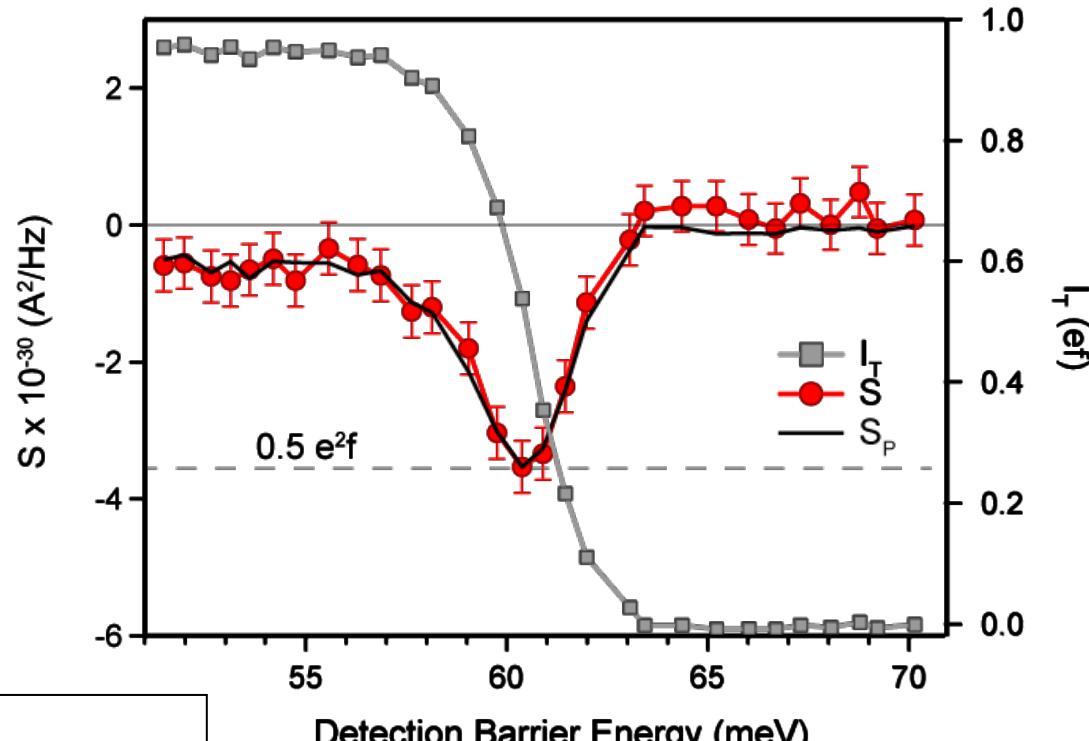
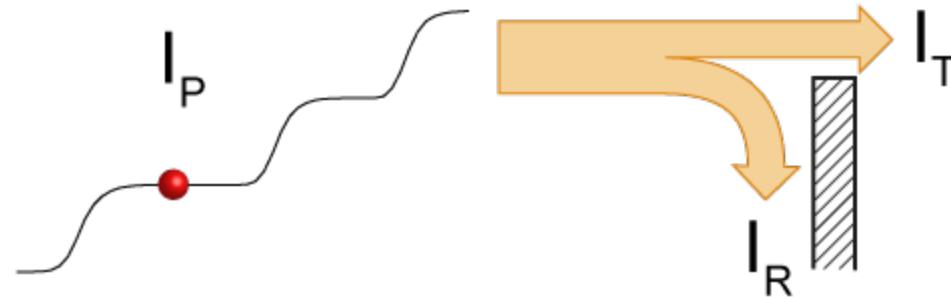
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# Setup





# On-demand source of single electrons

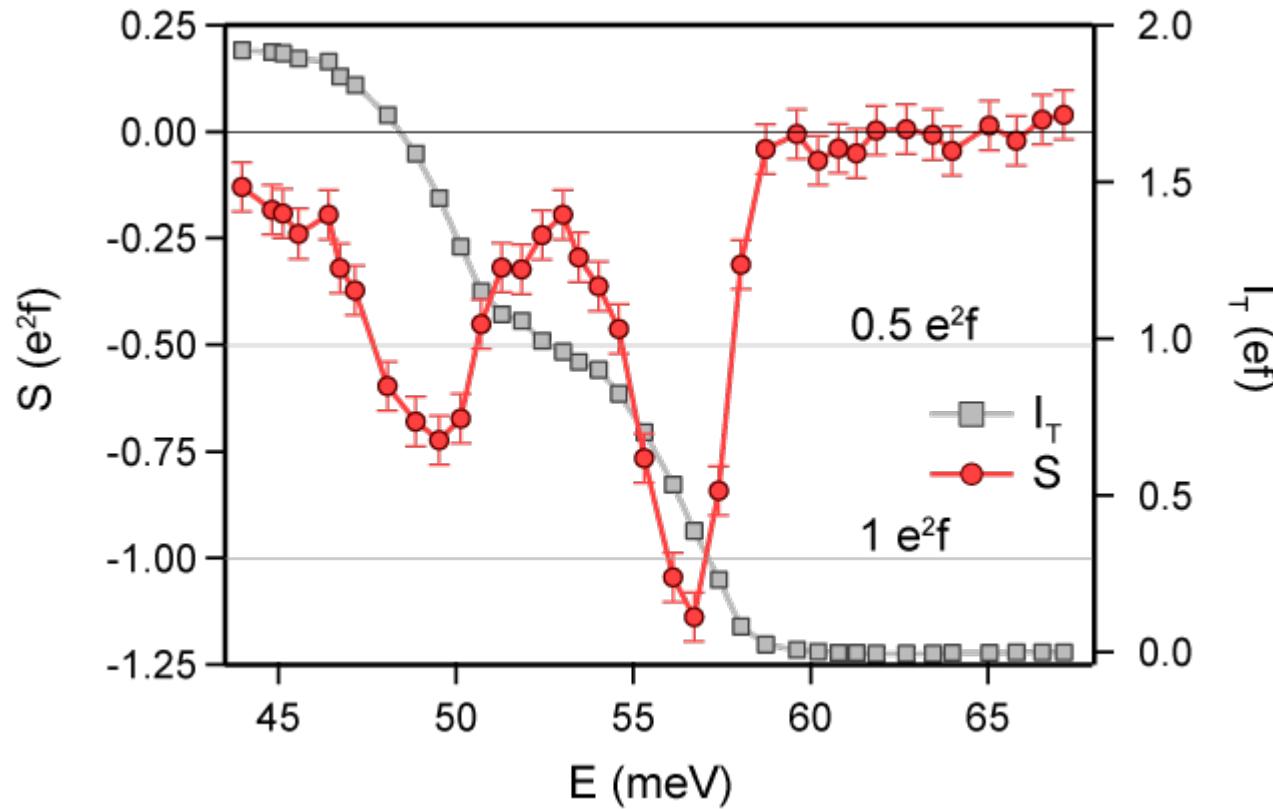
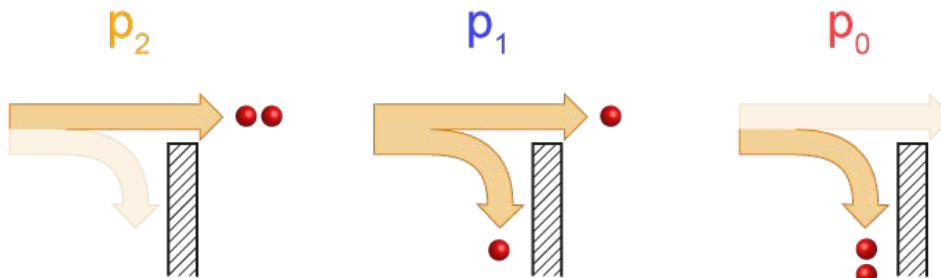


$$T(E) = I_T / I_P$$

$$S_p = 2e (ef) T (1 - T)$$

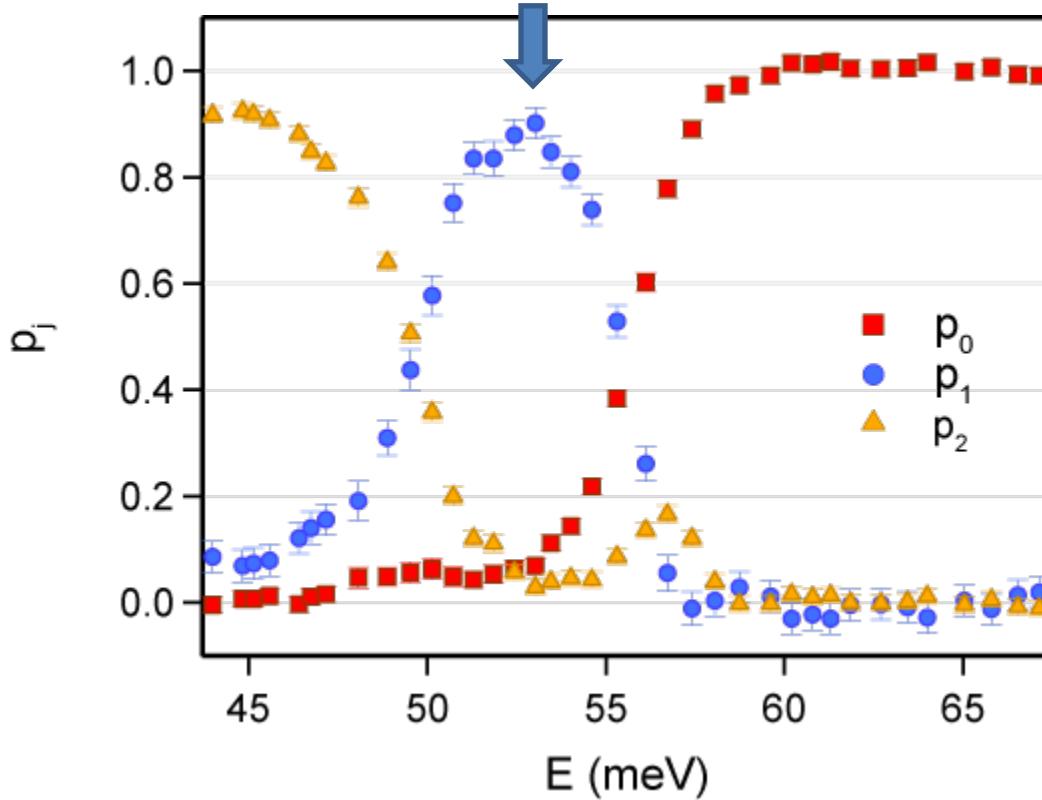
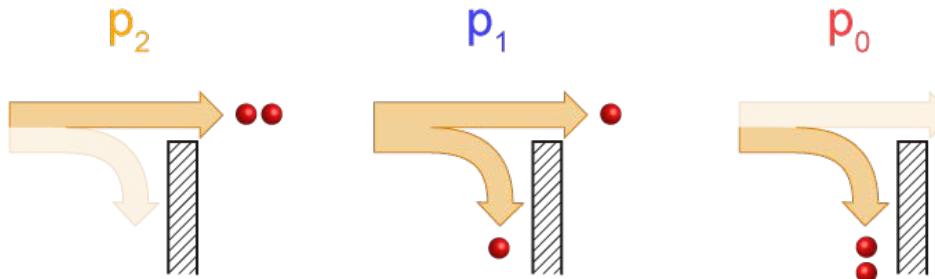


# Partitioning of electron pairs





# Partitioning of electron pairs



Reliable partitioning of on-demand electron pairs:

Prospects for electron quantum optics experiments

Ubbelohde et al, Nature Nano. 10, 46 (2014)

# What you should keep in mind



- Redefinition of SI base units in 2018
  - Expected scale change:  $< 10^{-7}$
- Semiconductor quantized current sources
  - Suitable as direct realization:  $u < 2 \cdot 10^{-7}$
  - Self referenced current source under development
- Beyond metrology:
  - noise less currents
  - partitioning
  - on-demand source of quantum states

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