Spin current swapping and spin Hall effect in a 2DEG

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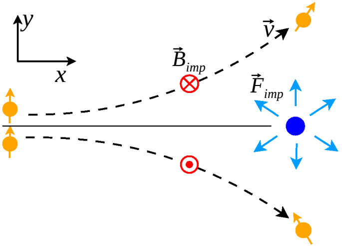
My aim today

1. Spin-charge and spin-spin couplings: spin current swapping vs Hanle spin Hall effect
2. Spin Hall effect due to skew-scattering by phonons
3. Spin Hall effect in systems with striped spin-orbit coupling

Spin current swapping: Lifshits and Dyakonov, PRL **103**, 186601 (2009)

Spin-orbit coupling: $H_{so} = -\lambda^2 \boldsymbol{\sigma} \times \nabla V_{imp}(\mathbf{r}) \cdot \mathbf{p}$



- λ Effective Compton wavelength
- Effective magnetic field $\mathbf{B}_{imp} \sim \lambda^2 m \mathbf{F}_{imp} \times \mathbf{v} \propto \hat{\mathbf{z}}$
- Spin precesses around the z axis yielding a x component
- Primary spin current J_x^y
- Secondary spin current J_y^x

$$J_i^a = \kappa \left(J_a^i - \delta_{ia} \sum_l J_l^l \right)$$

For dimensionless coupling $\kappa = \lambda^2 k_F^2$, k_F^{-1} being the only relevant length

Spin current swapping as a scattering effect

Scattering amplitude in the presence of spin-orbit coupling

$S_{\mathbf{pp}'} = A + B\hat{\mathbf{p}} \times \hat{\mathbf{p}}' \cdot \sigma$
The density matrix changes upon scattering $\rho_{\mathbf{p}} \to \rho_{\mathbf{p}'} = S_{\mathbf{pp}'} \rho_{\mathbf{p}} S^*_{\mathbf{pp}'}$

Various processes

1. Standard scattering $\propto |A|^2 + |B|^2$
2. Elliott-Yafet spin relaxation $\propto 2|B|^2$
3. Mott skew-scattering or spin-charge coupling (SCC) $\propto AB^* + A^*B$
4. Spin current swapping (SCS) $\propto AB^* - A^*B$

Comment

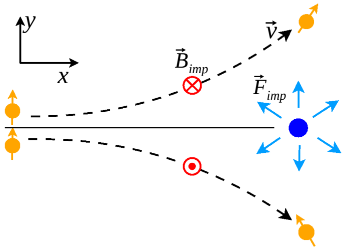
Spin current swapping more robust than skew-scattering because already exists at the level of Born approximation when A real, B imaginary

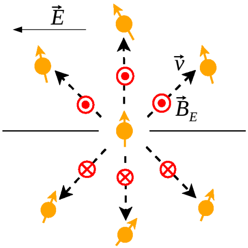
How to observe the effect?

Non trivial question

In principle, generation of a primary spin current in response to the conjugated spin vector potential A_x^y

In practice, application of an electric field E_x to drift a spin polarization S^y, $J_x^y \propto S^y E_x$





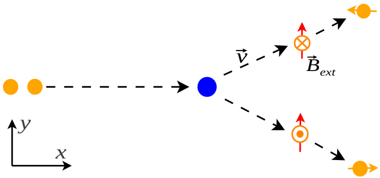
$\mathbf{B}_{imp} \sim \lambda^2 m \mathbf{F}_{imp} \times \mathbf{v}$

$\mathbf{B}_E \sim -\lambda^2 m e \mathbf{E} \times \mathbf{v}$

However, the effective magnetic field generated by the electric field is equal and opposite to the one generated by impurities ($\langle \nabla V_{imp} \rangle = \mathbf{E}$ in the steady state)

The effect of a magnetic field along y

- Apply an eletric field E_x
- Primary spin current J_x^y
- The spin Hall effect generates J_y^z in response to E_x
- The spin polarization along z precesses around the external magnetic field: $J_y^z \to J_y^x$
- The "precessed" spin current cannot be distinguished from the "swapped" one



The Hanle spin Hall effect

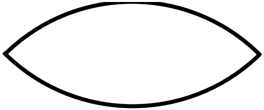
Diagrammatic theory of the Hanle spin Hall effect

A model calculation (See PRB **92**, 035301 (2015))

- 2DEG with density of states $N_0 = m/2\pi$ and density $n = k_F^2/2\pi$
- Applied uniform magnetic field along x with Zeeman energy Δ
- Standard white-noise disorder $\langle V(\mathbf{r})V(\mathbf{r}')\rangle = \frac{1}{2\pi N_0 \tau}$ with τ the scattering time and $D = v_F^2$ the diffusion coefficient
- Apply uniform electric field E_x and evaluate the Kubo formula

Charge and primary spin current

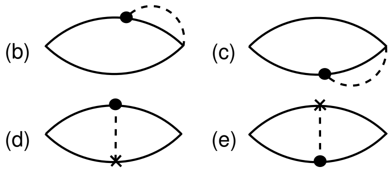
$$
\begin{aligned}
J_x &= \sigma_{xx} E_x, \ \sigma_{xx} = 2e^2 N_0 D \\
J_x^x &= \sigma_{xx}^x E_x, \ \sigma_{xx}^x = \frac{(-e)}{4\pi}\Delta\tau
\end{aligned}
$$



Note: the primary spin current is the algebraic sum of the number currents of the two spin populations

To lowest order in the spin-orbit coupling

- side-jump-like diagrams (b and c) as those considered in the SHE (Tse and Das Sarma PRL **96**, 056601 (2006));
- vertex corrections diagrams (d and e).



- • spin-independent impurity potential
- × spin-orbit coupling due to impurity potential

$$J_x^x = \sigma_{xx}^x E_x, \; \sigma_{xx}^x = \frac{(-e)}{4\pi}\Delta\tau$$

$$J_y^y = \sigma_{yx}^y E_x, \; \sigma_{yx}^y = en\lambda^2 \frac{\Delta\tau}{1+\Delta^2\tau^2}$$

The effect of an exchange field and the Hanle SHE (Ka et al. PRB **92**, 035201 (2015))

- What about the "apparent" swapping?

$$\kappa = \frac{\sigma^y_{yx}}{\sigma^x_{xx}} = -2k_F^2\lambda^2 \frac{1}{1+\Delta^2\tau^2} \rightarrow_{\Delta\rightarrow 0} -2k_F^2\lambda^2 \quad (1)$$

  which is in agreement with LD's prediction provided $\sigma^x_{xx} \neq 0$, which is not the case in the present situation

- The side-jump contribution to the SHE is

$$J^z_y = \sigma^z_{yx} E_x, \; \sigma^z_{yx} = en\lambda^2 \quad (2)$$

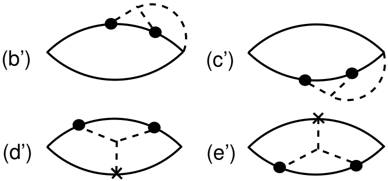
  and σ^y_{yx} can be interpreted as the Hanle effect of the spin polarization associated to the spin current in the SHE. Notice that the momentum relaxation time τ enters the expression of the precession factor

- The question arises about what happens when considering higher order, in the impurity potential, diagrams? What about the HSHE from skew-scattering?

Higher order terms and skew-scattering HSHE

Higher (third) order diagrams

- • spin-independent impurity potential
- $\times$ spin-orbit coupling due to impurity potential



Key observations

- New diagrams have the same structure as "parent" diagrams with the renormalization of the scattering amplitude $v_0 \to v_0 + \delta v^{R(A)} \equiv v^{R(A)}$
- $v^{R(A)} = v_0 \mp \mathrm{i}\pi N_0 v_0^2$
- Diagrams can be classified in two classes:
  1. $\propto v^R + v^A$, scattering time renormalization, not present at this (third) order
  2. $\propto v^R - v^A \sim AB^* + A^*B$ yields the skew-scattering contribution to the HSHE

How can we observe Spin Current Swapping (SCS)

SU(2) point of view

The potential in the spin-orbit Hamiltonian includes also the contribution due to the applied electric field $V(\mathbf{r}) = V_{imp}(\mathbf{r}) + e\mathbf{r}\cdot\mathbf{E}$

- Effective spin-dependent vector potential $H_{so,E} = \mathbf{p}\cdot\mathbf{A},\ \mathbf{A} = \mathbf{A}^a\sigma^a/2$
- Only components $A_x^z = 2em\lambda^2 E_y,\ A_y^z = -2em\lambda^2 E_x$
- Covariant derivative in drift-diffusion equation $(\nabla_i O)^a = \partial_i O^a - \varepsilon^{abc} A_i^b O^c$
- Spin current $J_i^a = -\frac{e\tau}{m} S^a E_i - D(\nabla_i S)^a + \kappa\left(J_a^i - \delta_{ia} J_j^j\right) - \theta_{SH}\varepsilon_{ija}J_j$

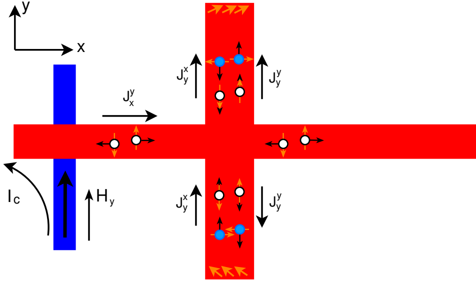
SCS with non-uniform conditions

$$
\begin{aligned}
J_x^x &= (J_x^x)^{drift} + (J_x^x)^{diff} - \kappa(J_y^y - (J_y^y)^{drift}) \\
J_y^y &= (J_y^y)^{drift} + (J_y^y)^{diff} - \kappa(J_x^x - (J_x^x)^{drift})
\end{aligned}
$$

Only the diffusion part of the primary spin current contributes to spin current swapping

Suggested experimental set up

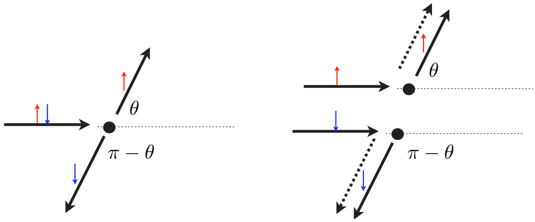
- Inject a spin current from the FM electrode (blue) into the PM system (red)
- Spin primary diffusion currents flow in the horizonatal arm (J_x^y) and vertical arm (J_y^y)
- Spin secondary current J_y^x flows in the vertical arm
- S^x spin polarization accumulates at the ends of the vertical arm with opposite sign and can be detected either by ISHE or Faraday rotation



Why phonon skew scattering?

Spin Hall angle

$$\theta^{sH} = \frac{e\sigma^{sH}}{\sigma}$$



$T = 0$

- $\sigma^{sH}_{sj} =$ **constant**
- $\sigma^{sH}_{ss} =$ **constant** $\times$ **mobility** $\propto \sigma$

$T \neq 0$

- $\tau_{imp} \rightarrow \tau_{e-ph}$
- $\sigma^{sH}_{ss}, \sigma \sim \mathbf{1/T}$

Conventional analysis (Is it enough?)

- in high mobility samples skew scattering dominates, θ^{sH} **T**-independent
- On the contrary, if $\theta^{sH} \sim$ **T**, the side-jump mechanism dominates

Vila et al. PRL **99**, 226404 (2007); Niimi et al. PRL **106**, 126601 (2011); Isasa et al. PRB **91**, 024402 (2015); Hankiewicz et al. PRL **97**, 266601 (2006).

Review of the Debye model for electron-phonon coupling

- Start from elasticity theory with displacement field $\mathbf{u}(\mathbf{r},t)$
- After quantization introduce the phonon field

$$\hat{\phi}(\mathbf{r}) = \mathrm{i}\sum_{\mathbf{k}}\sqrt{\frac{v_s k}{2V}}\left(\hat{b}_{\mathbf{k}}e^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} - h.c.\right), \quad \hat{\phi}(\mathbf{r}) = v_s\sqrt{\rho}\nabla\cdot\mathbf{u}(\mathbf{r}), \ \rho: \text{ mass density}$$

- Replace the impurity potential with the phonon potential

$$V_{imp}(\mathbf{r}) \to V_{ph}(\mathbf{r}) = g\hat{\phi}(\mathbf{r}), \quad g: \text{ electron} - \text{phonon coupling}$$

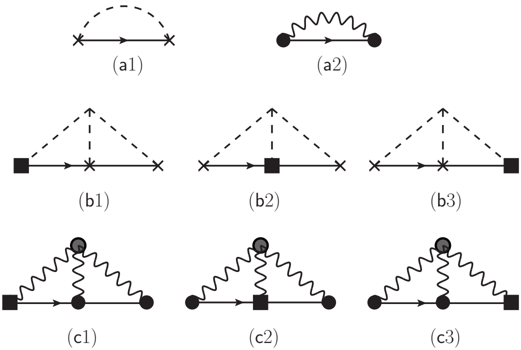
- Average over phonon configurations taking anharmonic cubic terms

$$H_{an} = \frac{\Lambda}{3!}\int \mathrm{d}\mathbf{r}\ \hat{\phi}^3(\mathbf{r})$$

where $\Lambda = -\gamma/v_s\sqrt{\rho}$ is related to the Grüneisen parameter $\gamma \sim 2-3$

Diagrams

- Dashed line = impurity average
- Wavy line = phonon propogator
- $\times$ = impurity potential
- $\bullet$ = phonon potential
- filled square = spin-orbit coupling
- gray dot = three-phonon term



High-temperature regime

Debye temperatures in metals: $T_D = 165$ K for Au, $T_D = 240$K for Pt and Ta
For $T > T_D$,

- phonon dynamics becomes irrelevant, and phonon potential behaves almost as a static one as for the impurity potential
- phonon averages can be done semiclassically with the equipartition theorem

$$\langle \hat{\phi}(\mathbf{r}_1)\hat{\phi}(\mathbf{r}_2)\rangle = kT\delta(\mathbf{r}_1 - \mathbf{r}_2)$$

$$\langle \hat{\phi}(\mathbf{r}_1)\hat{\phi}(\mathbf{r}_2)\hat{\phi}(\mathbf{r}_3)\rangle = -\Lambda(kT)^2\delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_1 - \mathbf{r}_3)$$

- The Keldysh technique in the high-T regime confirms this result

$$\frac{\Lambda g^3}{4}\int_4 [D^R_{14}D^K_{24}D^K_{34} + D^K_{14}D^R_{24}D^K_{34} + D^K_{14}D^K_{24}D^R_{34}] \sim -3\Lambda g^3(k_B T)^2$$

by using

$$D^K(\mathbf{k},\omega) = -\mathrm{i}\frac{\hbar\omega_{\mathbf{k}}}{2}\coth\left(\frac{\beta\hbar\omega_{\mathbf{k}}}{2}\right)2\pi[\delta(\omega - \omega_{\mathbf{k}}) + \delta(\omega + \omega_{\mathbf{k}})] \to \sim T$$

Naive derivation (Gorini et al. PRL **115**, 076602 (2015))

Correspondence impurity-phonons

$$n_i v_0^2 \quad \to \quad g^2 kT = \frac{1}{2\pi N_0 \tau_{e-ph}}$$

$$n_i v_0^3 \quad \to \quad -3\Lambda g^3 (kT)^2 = \frac{1}{2\pi N_0 \tau_{e-ph}}(-3kTg\Lambda)$$

Using the correspondence with impurity

- Skew-scattering from impurities

$$\sigma^{sH}_{ss,imp} = \frac{\lambda^2 k_F^2}{4}\frac{en}{m}2\pi N_0 v_0 \tau_{imp}$$

  scales as conductivity $\sim \tau_{imp}$

- Skew-scattering from phonons

$$\sigma^{sH}_{ss,ph} = -\frac{\lambda^2 k_F^2}{4}\frac{en}{m}\frac{\hbar\Lambda}{g} \sim \sigma^{sH}_{ss,imp}\frac{\gamma}{\epsilon_F \tau_{imp}} \sim 0.1\sigma^{sH}_{ss,imp}$$

  is T-independent, while $\sigma \sim \tau_{e-ph} \sim T^{-1}$

Temperature dependence of the Spin Hall Angle

New point of view

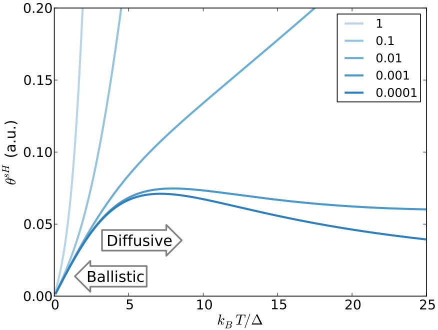
- $\sigma \sim T^{-1}$ is T-dependent via e-ph scattering
- σ_{ss}^{sH} is T-independent at high T
- Combine with Rashba (Δ) spin splitting

$$\theta^{sH} = \frac{1}{\sigma} \frac{\sigma_{int}^{sH} + \sigma_{ext}^{sH}}{1 + \tau_{EY}/\tau_{DP}}$$

Diffusive $\Delta \ll T$

$$\sigma_{int}^{sH} \sim T^{-2}, \tau_{EY} \sim T^{-1}, \tau_{DP} \sim T$$

From weak (darker) to stronger (lighter) $\sigma_{ext}^{sH}/(e/8\pi)$



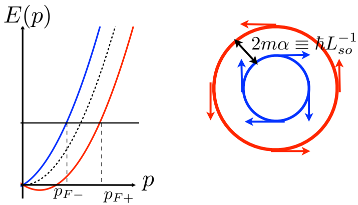
Warnings and future perspectives

- higher anharmonic terms may give a T-behavior of σ_{ss}^{sH} opposite to σ
- intermediate temparature regime $T \leq T_D$ needs to be studied
- non-parabolic terms may affect the T-behavior of side-jump (Gorini 2015)

- E. I. Rashba, Sov. Phys. Solid State 2, 1109 (1960); Bychkov and Rashba, JETP Lett. **39**, 78(1984); J. Phys. C: Solid State Phys. **17**, 6093 (1984).

$$H = \frac{p^2}{2m} + \alpha(p_y\sigma^x - p_x\sigma^y) + V(\mathbf{r})$$

- Intrinsic SHE $j_y^z = \sigma^{sH} E_x = (e/8\pi) E_x$, Sinova et al. PRL **92**, 126603 (2004)



Disorder introduces spin relaxation

$$\tau_{DP} = \frac{L_{so}^2}{D} = (2m\alpha)^2 D = 2m^2\alpha^2 v_F^2 \tau$$

Dyakonov and Perel, SOv. Phys.-Solid State **13**, 3023 (1971)

However, "subtle is the Lord" and there can be no SHE in *static* and *uniform* conditions

$$\partial_t s^y + \nabla \cdot \mathbf{j}^y = -2m\alpha j_y^z$$

Dimitrova, PRB **71**, 245327 (2005).
although ⇒SHE still possible at *edges*, in *transient regime* (Mishchenko et al. PRL **93**, 226602 (2004); Raimondi et al. PRB **74**, 035340 (2006)), with random SOC (Moca et al. PRB **77**, 193302 (2008); Dugaev et al. PRB **82**, 121310 (2010); Dyrdal et al. Acta Phys. Pol. A **127**, 499 (2015))

Question

Can we taylor the SOC so to have SHE in static and uniform conditions?

- With a space-dependent α, the Dimitrova constraint no longer implies the vanishing of the spin current
- The standard vanishing occurs due to an exact compensation between two terms:
  $\Rightarrow$ *diffusion* contribution from non-abelian SU(2) covariant derivative
  $\Rightarrow$ *drift* contribution from Lorentz-like force due to $SU(2)$ *magnetic field*

$$j_y^z = \sigma^{sH} E_x - D(-\varepsilon^{zxy} 2m\alpha s^y)$$

- It may be possible to unbalance such compensation so to have a finite spin current

The inhomogeneous Rashba model (Götz et al. EPL, **112**, 17004 (2015))

General idea

Single-interface model

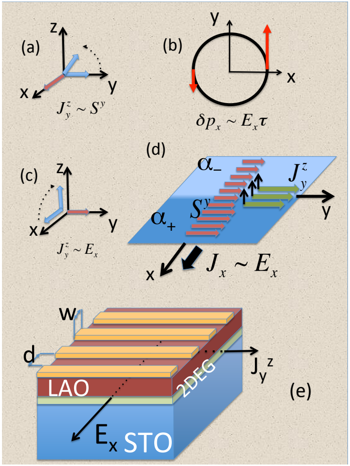
$$\alpha \to \alpha(x) = \theta(x)\alpha_+ + \theta(-x)\alpha_-$$

Interpolating solution

$$\begin{aligned} S^y(x) &= \theta(x)\left(S_{0,+} + \delta s_+ e^{-x/L_+}\right) \\ &+ \theta(-x)\left(S_{0,-} + \delta s_- e^{x/L_+}\right), \end{aligned}$$

Main message

Non-zero spin current exponentially localized at the interface



Strong Rashba coupling in LAO/STO Nitta et al. PRL **78**, 1335 (1997); Caviglia et al. PRL **104**, 126803 (2010); Hurand et al. Sci. Rep. **5**, 12751 (2015)

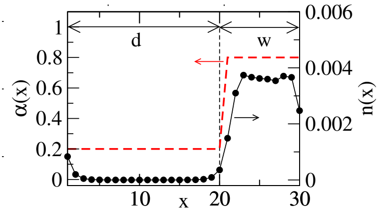
Why the lattice? No need for a small expansion parameter (Nomura et al. PRB **72**, 165316 (2005))

$$H = \sum_{ij\sigma} t_{ij} c^{\dagger}_{i\sigma} c_{j\sigma} + \sum_{i\sigma} (V_i - \mu) c^{\dagger}_{i\sigma} c_{i\sigma} + H^{RSO},$$

Disorder distrubution $-V_0 \leq V_i \leq V_0$

$$H^{RSO} = -i \sum_{i\sigma\sigma'} \alpha_{i,i+x} \left[c^{\dagger}_{i\sigma} \tau^{y}_{\sigma\sigma'} c_{i+x,\sigma'} - c.c. \right] + i \sum_{i\sigma\sigma'} \alpha_{i,i+y} \left[c^{\dagger}_{i\sigma} \tau^{x}_{\sigma\sigma'} c_{i+y,\sigma'} - c.c. \right]$$

Rashba SOC on a lattice



System size: 3060×3060 sites.
$V_0 = 0$
$\mu = -4.3\,t$

The Stripes modulation

$$\alpha_{i,i+x} = \frac{1}{2} \left[a_0 + a_1 + (a_0 - a_1)\,\mathrm{sgn}(\sin \frac{2\pi i_x}{2L}) \right]$$
$$\alpha_{i,i+y} = \alpha_{i,i+x},$$

The generalized Dimitrova relation

$$\dot{S}_i^y + [\mathrm{div}\,\mathbf{J}^y]_i + \alpha_{i,i+y} J^z_{i,i+y} + \alpha_{i-y,i} J^z_{i-y,i} = 0\,.$$

For a homogeneous RSOC, where $[\mathrm{div}\,\mathbf{J}^y]_i = 0$, this implies that the total z-spin current has to vanish under stationary conditions. On the contrary, when α varies in space, a cancellation occurs between $\mathrm{div}\,\mathbf{J}^y$ and the last two terms, so that the stationarity condition $\dot{S} = 0$ does not imply the vanishing of J^z.

$$-\sum_i \dot{S}_i^y = \sum_i \left\{ \alpha_{i,i+y} J^z_{i,i+y} + \alpha_{i-y,i} J^z_{i-y,i} \right\}.$$

The Kubo formula

$$\sigma_{ij}^{sH} \equiv \frac{2}{N} \sum_{\substack{E_n < E_F \\ E_m > E_F}} \frac{Im\langle n|j^z_{i,i+y}|m\rangle\langle m|j^{ch}_{j,j+x}|n\rangle}{(E_n - E_m)^2 + \eta^2}\,.$$

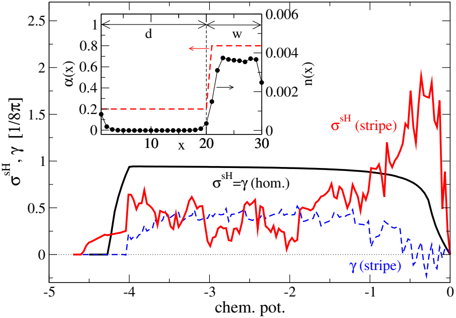
Here, $\eta \to 0$ is a small regularization term which acts as an inverse electric-field turn-on time

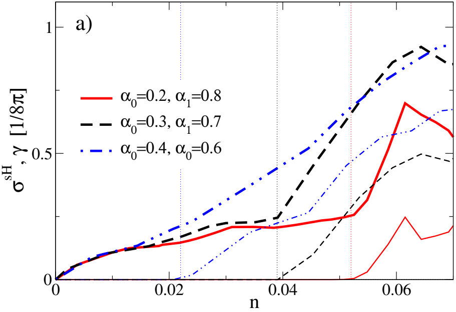
The stationarity "detector"

$$\gamma = 2\sum_{ij} \alpha_{i,i+y} \sigma_{ij}^{sH}$$

$\gamma = 0$ quantifies the "stationarity" of the solution

Results for the lattice model: no disorder



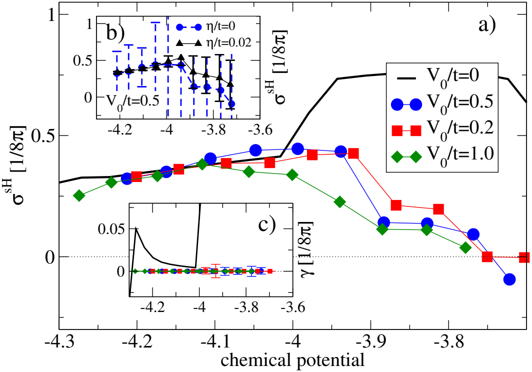


For $a_0 = 0.2\,t$ and $a_1 = 0.8\,t$, for a non-negligible range of chemical potential near the bottom of the band, a substantial σ^{sH} (red solid curve) is present while $\gamma = 0$ (blue dashed curve) $\Rightarrow$ SHE in stationary conditions.

Relevance of states that are extended along y, while they are nearly localized along x due to the modulation of α.

This occurs for increasingly large density ranges by increasing the inhomogeneity of α

Results for the lattice model: with disorder



- Clean case: $\sigma^{sH} \neq 0$, $\gamma = 0$ for bottom band energies, where electron states are localized along x but extendend along y
- Disorder case: σ^{sH} robust and almost insensitive in value
- Strong fluctuations due to finite size effects
- Disorder guarantees even more "stationarity" behavior with respect to the clean case even when $\eta = 0$ (electric field turn-on time)

Take-home message

A system with modulated RSOC can sustain a finite SHE in stationary conditions. the response of the charge current J_x^{ch} to the electric field along the modulation direction is strongly suppressed which can lead to large spin Hall angles eJ_y^z / J_x^{ch} for the striped system.

Conclusions and Perspectives

- Theory of spin current swapping and conditions to observe it. Future: interplay of extrinsic and intrinsic SOC.
- Temperature dependence of the spin Hall angle taking into account electron-phonon phonon-phonon scattering. Future: determine the full crossover behavior from low to high (room) temperature.
- SHE in modulated systems to achieve a strong response. Future: explore also spin current swapping.

Acknowledgements and References

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Relevant papers

- PRB **92**, 035201 (2015);
- PRL **115**, 076602 (2015);
- EPL **112**, 17004 (2015);
- PRB **82**, 195316 (2010);
- Ann. Phys. (Berlin) **524**, 153 (2012);
- PRL **109**, 246604 (2012);
- PRL **112**, 096601 (2014);
- PRB **90**, 245302 (2014).

Thanks for your attention!