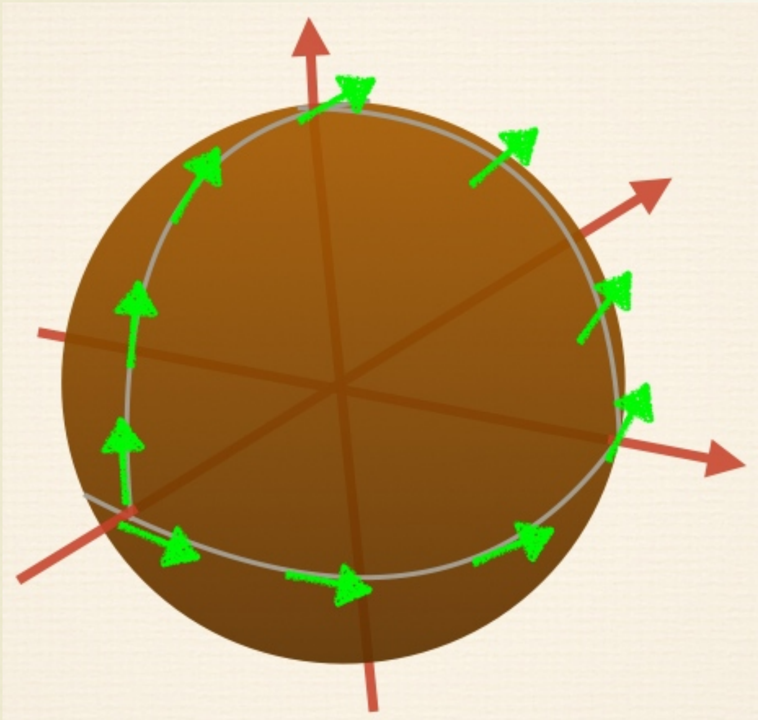
# Effective forces in ferromagnetic conductors
# - spinmotive force, anomalous Hall effect, and magnetoresistance -

Yuta Yamane
from Jairo Sinova gr.

# **introduction:** gauge fields



you restrict your Hilbert space
(e.g., 3D space to the surface of a 2D sphere)

↓

you feel curvature in your new subspace

↓

you describe the curvature
by gauge fields

**U(1) gauge symmetry**

electromagnetic potential $(A_0, \boldsymbol{A})$

**non-relativistic limit:**
**from 4×4 to 2×2 Hamiltonian**

Zeeman coupling $A_0^{\mathrm{Z}} = -\mu \boldsymbol{\sigma} \cdot \boldsymbol{B}$

μ: electron's magnetic moment

spin-orbit coupling $\boldsymbol{A}^{\mathrm{SO}} = \frac{m_{\mathrm{e}} \eta_{\mathrm{so}}}{\hbar} \boldsymbol{\sigma} \times \boldsymbol{E}$

$m_e$: electron's mass
$\eta_{so}$: spin-orbit coupling parameter
**σ**: Pauli's matrices

**exchange coupling in ferromagnets:**
**from 2×2 to two 1×1 Hamiltonians**
**for the majority and minority spins**

$$A_0^{\mathrm{B}} = \sigma_z \frac{\hbar}{2e} \cos\theta \partial_t \varphi, \quad \boldsymbol{A}^{\mathrm{B}} = -\sigma_z \frac{\hbar}{2e} \cos\theta \nabla\varphi$$

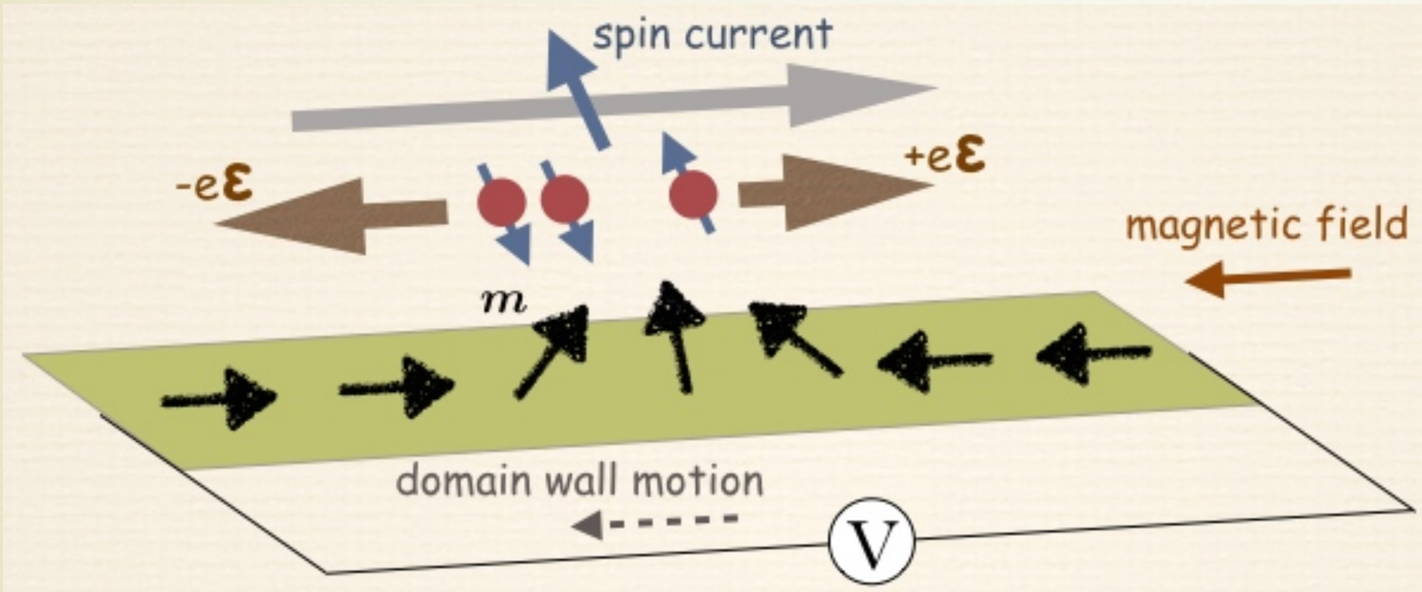
(Volovik, J. Phys. C 1987)

θ and φ are polar angles of the local magnetization direction **m**:
$\boldsymbol{m} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$

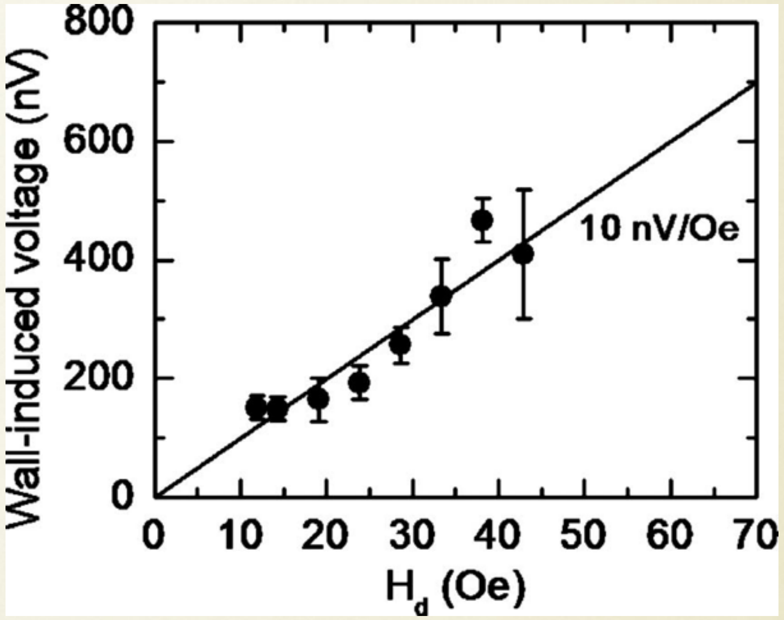
# introduction: "electric" and "magnetic" fields in FM

$$\sigma_z \boldsymbol{\mathcal{E}} = -\nabla A_0^{\mathrm{B}} - \partial \boldsymbol{A}^{\mathrm{B}} = \sigma_z \frac{\hbar}{2e} \boldsymbol{m} \times \partial_t \boldsymbol{m} \cdot \nabla \boldsymbol{m}$$

→ spinmotive force



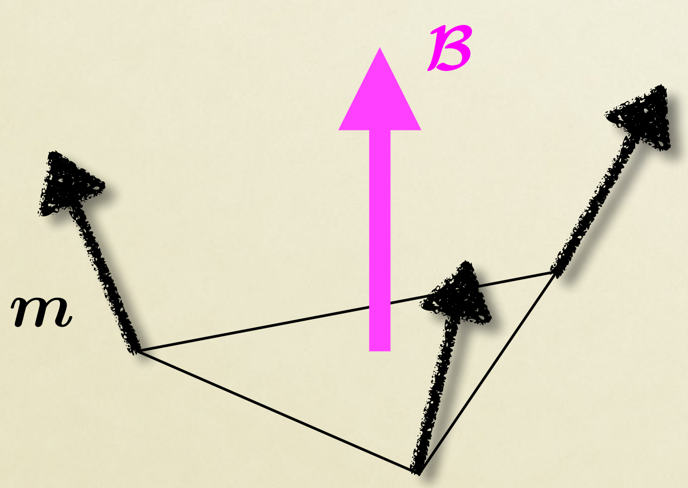

(Barnes and Maekawa, PRL 2007)

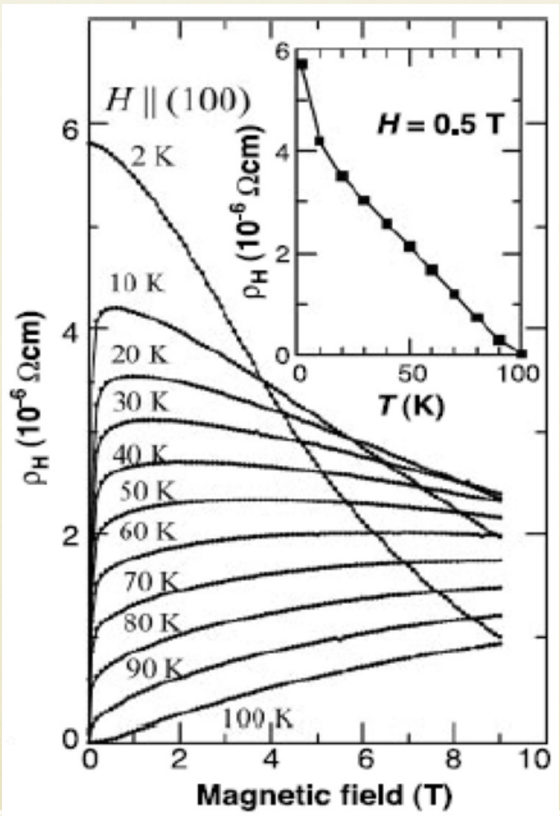



(Yang et al., PRL 2009)

$$\sigma_z \boldsymbol{\mathcal{B}}_i = (\nabla \times \boldsymbol{A}^{\mathrm{B}})_i = -\sigma_z \frac{\hbar}{4e} \epsilon_{ijk} \boldsymbol{m} \cdot \partial_j \boldsymbol{m} \times \partial_k \boldsymbol{m}$$

→ anomalous Hall effect




(Taguchi et al., Science 2001)

# questions

what about

- contribution from the **non-adiabatic** spin dynamics ?

- combination effects of **spin-orbit** and exchange couplings ?

# questions

what about

- contribution from the **non-adiabatic** spin dynamics ?

- combination effects of **spin-orbit** and exchange couplings ?

# nonadiabatic effects -Newtonian equation approach-

(Yamane et al., PRB 2013)

## starting Hamiltonian

$$\mathcal{H} = \frac{\boldsymbol{p}^2}{2m_\mathrm{e}} + J_\mathrm{ex}\boldsymbol{\sigma}\cdot\boldsymbol{m}$$

$J_{ex}$: exchange coupling energy
**σ**: Pauli's matrices
**m**: unit vector of the magnetization direction

## Heisenberg's equation of motion

$$m_\mathrm{e}\ddot{\boldsymbol{r}} = \frac{1}{(i\hbar)^2}\left[\left[\boldsymbol{r},\mathcal{H}\right],\mathcal{H}\right] = -J_\mathrm{ex}\boldsymbol{\sigma}\cdot\nabla\boldsymbol{m}$$

## electron spin dynamics

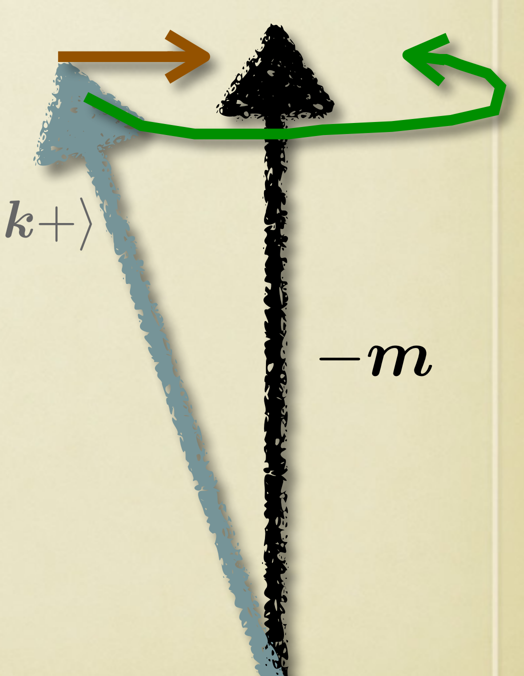
$$(\partial_t + \boldsymbol{v}^\pm_{\boldsymbol{k}}\cdot\nabla)\langle\boldsymbol{k}\pm|\boldsymbol{\sigma}|\boldsymbol{k}\pm\rangle = -\frac{\hbar}{2J_\mathrm{ex}}\langle\boldsymbol{k}\pm|\boldsymbol{\sigma}|\boldsymbol{k}\pm\rangle\times\boldsymbol{m} - \frac{\langle\boldsymbol{k}\pm|\boldsymbol{\sigma}|\boldsymbol{k}\pm\rangle\pm\boldsymbol{m}}{\tau_\mathrm{sf}}$$

Larmor precession (adiabatic) relaxation (non-adiabatic)

$|\boldsymbol{k}\pm\rangle$: one-electron state with momentum $\hbar\boldsymbol{k}$ and majority (+) or minority (-) spin

$\tau_{sf}$: relaxation time for the electron spin flip

$\boldsymbol{v}^\pm_{\boldsymbol{k}} = \langle\boldsymbol{k}\pm|\dot{\boldsymbol{r}}|\boldsymbol{k}\pm\rangle$




→

$$\langle\boldsymbol{k}\pm|\boldsymbol{\sigma}|\boldsymbol{k}\pm\rangle = \mp\boldsymbol{m} \pm \frac{\hbar}{2J_\mathrm{ex}}\left[\boldsymbol{m}\times(\partial_t + \boldsymbol{v}^\pm_{\boldsymbol{k}}\cdot\nabla)\boldsymbol{m} + \beta(\partial_t + \boldsymbol{v}^\pm_{\boldsymbol{k}}\cdot\nabla)\boldsymbol{m}\right]$$

adiabatic non-adiabatic

(Zhang and Li, PRL 2004)

with $\beta = \dfrac{\hbar}{2J_\mathrm{ex}}\dfrac{1}{\tau_\mathrm{sf}}$

# nonadiabatic effects -Newtonian equation approach-

(Yamane et al., PRB 2013)

**equations of motion for the majority and minority electrons**

$$\langle \boldsymbol{k} \pm | m_{\rm e} \ddot{\boldsymbol{r}} | \boldsymbol{k} \pm \rangle = -J_{\rm ex} \langle \boldsymbol{k} \pm | \boldsymbol{\sigma} | \boldsymbol{k} \pm \rangle \cdot \nabla \boldsymbol{m}$$

$$= \pm \left[ -e \left( \boldsymbol{\mathcal{E}} + \boldsymbol{\mathcal{E}}^{\rm NA} + \boldsymbol{v}_{\boldsymbol{k}}^{\pm} \times \boldsymbol{\mathcal{B}} \right) \mp \frac{\hbar \beta}{2} (\boldsymbol{v}_{\boldsymbol{k}}^{\pm} \cdot \nabla) \boldsymbol{m} \cdot \nabla \boldsymbol{m} \right]$$

spinmotive force

$$\boldsymbol{\mathcal{E}} = \frac{\hbar}{2e} \boldsymbol{m} \times \partial_t \boldsymbol{m} \cdot \nabla \boldsymbol{m}$$

frictional force

anomalous Hall effect

$$\mathcal{B}_i = -\frac{\hbar}{4e} \epsilon_{ijk} \boldsymbol{m} \cdot \partial_j \boldsymbol{m} \times \partial_k \boldsymbol{m}$$

additional electric field

(Tserkovnyak and Mecklenburg, PRB 2008)

$$\boldsymbol{\mathcal{E}}^{\rm NA} = \frac{\hbar \beta}{2e} \partial_t \boldsymbol{m} \cdot \nabla \boldsymbol{m}$$

for an important application of this term,
see, e.g., Yamane et al., Sci. Rep. 2014

# questions

what about

- contribution from the **non-adiabatic** spin dynamics ?

- combination effects of **spin-orbit** and exchange couplings ?

# effects of spin-orbit coupling

## starting Hamiltonian

$$\mathcal{H} = \frac{1}{2m_e}\left(\boldsymbol{p} + \frac{em_e\eta_{so}}{\hbar}\boldsymbol{\sigma}\times\boldsymbol{E}\right)^2 + J_{ex}\boldsymbol{\sigma}\cdot\boldsymbol{m}$$

$\eta_{so}$: spin-orbit coupling parameter

$J_{ex} \gg e\eta_{so}|k_F||\boldsymbol{E}|$

## Heisenberg's equation of motion

$$\dot{\boldsymbol{r}} = \frac{1}{i\hbar}[\boldsymbol{r},\mathcal{H}] = \frac{\boldsymbol{p}}{m_e} + \frac{e\eta_{so}}{\hbar}\boldsymbol{\sigma}\times\boldsymbol{E}$$

anomalous velocity

$$m_e\ddot{\boldsymbol{r}} = \frac{1}{i\hbar}[m_e\dot{\boldsymbol{r}},\mathcal{H}] + \frac{\partial(m_e\dot{\boldsymbol{r}})}{\partial t}$$

$$= -J_{ex}\boldsymbol{\sigma}\cdot\nabla\boldsymbol{m} + \frac{em_e\eta_{so}J_{ex}}{i\hbar^2}[\boldsymbol{\sigma}\times\boldsymbol{E},\boldsymbol{\sigma}\cdot\boldsymbol{m}] + \frac{em_e\eta_{so}}{\hbar}\boldsymbol{\sigma}\times\partial_t\boldsymbol{E} - e\dot{\boldsymbol{r}}\times\left[\nabla\times\frac{em_e\eta_{so}}{\hbar}(\boldsymbol{\sigma}\times\boldsymbol{E})\right]$$

## electron spin dynamics

$$(\partial_t + \boldsymbol{v}_{\boldsymbol{k}}^{\pm}\cdot\nabla)\langle\boldsymbol{k}\pm|\boldsymbol{\sigma}|\boldsymbol{k}\pm\rangle = -\frac{2J_{ex}}{\hbar}\langle\boldsymbol{k}\pm|\boldsymbol{\sigma}|\boldsymbol{k}\pm\rangle\times\left(\boldsymbol{m} + \frac{em_e\eta_{so}}{\hbar J_{ex}}\boldsymbol{v}_{\boldsymbol{k}}^{\pm}\times\boldsymbol{E}\right) - \frac{\langle\boldsymbol{k}\pm|\boldsymbol{\sigma}|\boldsymbol{k}\pm\rangle \pm \boldsymbol{m}}{\tau^{sf}}$$

→

$$\langle\boldsymbol{k}\pm|\boldsymbol{\sigma}|\boldsymbol{k}\pm\rangle = \mp\boldsymbol{m} \pm \frac{\hbar}{2J_{ex}}\left[\boldsymbol{m}\times(\partial_t + \boldsymbol{v}_{\boldsymbol{k}}^{\pm}\cdot\nabla)\boldsymbol{m} + \beta(\partial_t + \boldsymbol{v}_{\boldsymbol{k}}^{\pm}\cdot\nabla)\boldsymbol{m}\right]$$
$$\pm\frac{em_e\eta_{so}}{\hbar J_{ex}}\left[\boldsymbol{m}\times\left\{\boldsymbol{m}\times\left(\boldsymbol{v}_{\boldsymbol{k}}^{\pm}\times\boldsymbol{E}\right)\right\} + \beta\boldsymbol{m}\times\left(\boldsymbol{v}_{\boldsymbol{k}}^{\pm}\times\boldsymbol{E}\right)\right]$$

(Kim et al., PRB 2012)

# effects of spin-orbit coupling

## equations of motion for the majority and minority electrons

additional electric field

$$\boldsymbol{\mathcal{E}}^{\mathrm{SO}} = \frac{m_{\mathrm{e}}\eta_{\mathrm{so}}}{\hbar}\partial_t(\boldsymbol{m}\times\boldsymbol{E})$$

applications of this term:
Kim et al., PRL 2012
Yamane et al., PRB 2013

anomalous Hall effect

$$\boldsymbol{\mathcal{B}}^{\mathrm{SO}} = -\frac{m_{\mathrm{e}}\eta_{\mathrm{so}}}{\hbar}\nabla\times(\boldsymbol{m}\times\boldsymbol{E})$$

(Chudnovsky, PRL 2007)

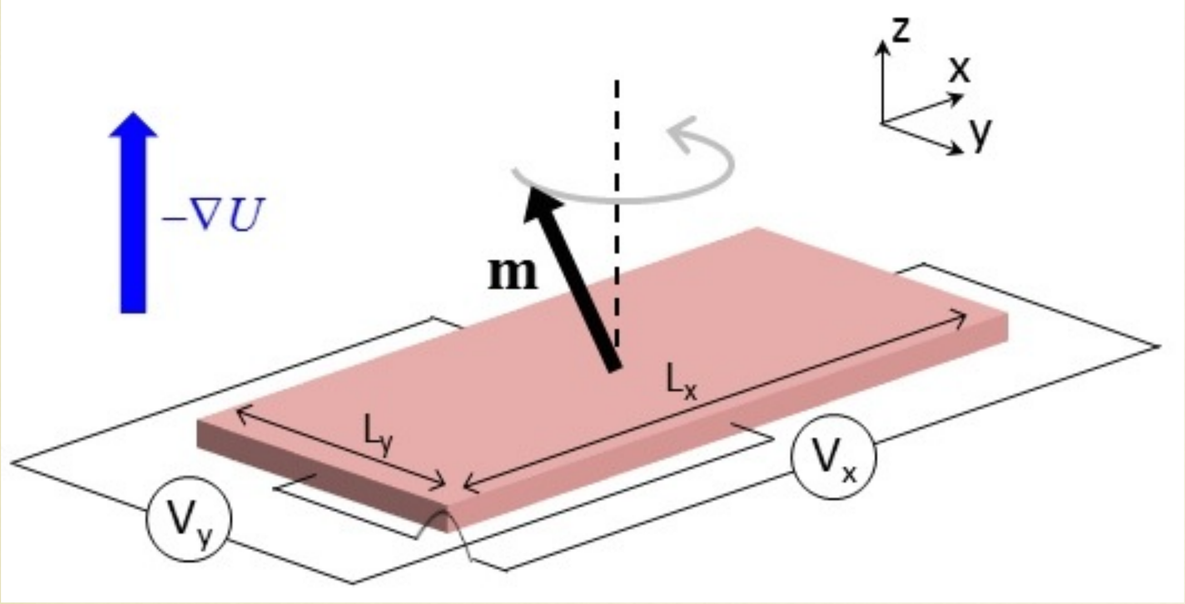
$$\langle \boldsymbol{k}\pm|m_{\mathrm{e}}\ddot{\boldsymbol{r}}|\boldsymbol{k}\pm\rangle = \pm\left[-e\left\{\boldsymbol{\mathcal{E}}+\boldsymbol{\mathcal{E}}^{\mathrm{NA}}+\boldsymbol{\mathcal{E}}^{\mathrm{SO}}+\boldsymbol{v}_{\boldsymbol{k}}^{\pm}\times\left(\boldsymbol{\mathcal{B}}+\boldsymbol{\mathcal{B}}^{\mathrm{SO}}\right)\right\}\right]$$
$$\mp\left[\frac{\hbar\beta}{2}(\boldsymbol{v}_{\boldsymbol{k}}^{\pm}\cdot\nabla)\boldsymbol{m}\cdot\nabla\boldsymbol{m}+\frac{e m_{\mathrm{e}}\eta_{\mathrm{so}}}{\hbar}\left\{(\boldsymbol{v}_{\boldsymbol{k}}^{\pm}\cdot\nabla)\boldsymbol{m}\times\boldsymbol{E}+\boldsymbol{v}_{\boldsymbol{k}}^{\pm}\cdot(\nabla\boldsymbol{m}\times\boldsymbol{E})\right\}\right]$$

frictional forces

# effects of spin-orbit coupling: spinmotive force

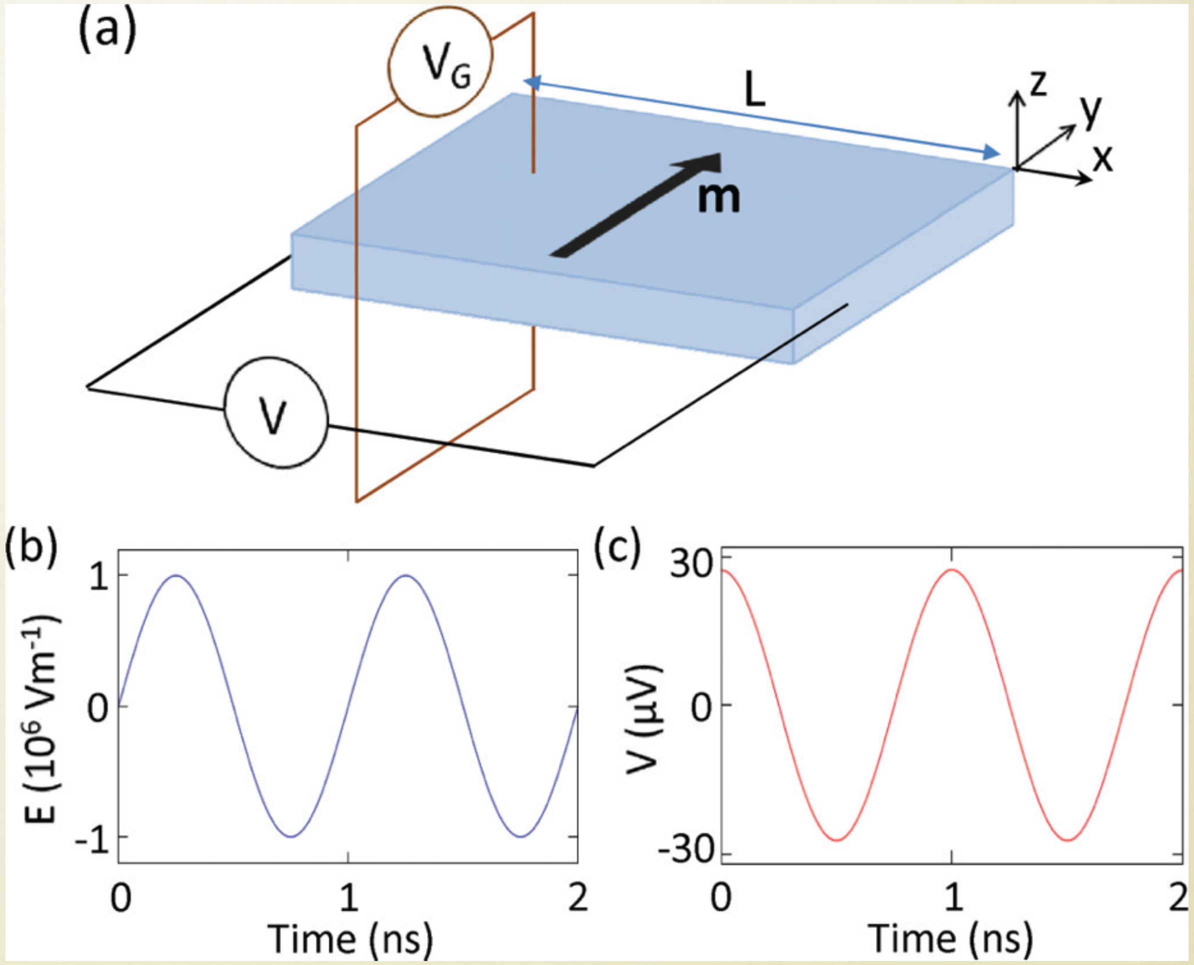
$$\boldsymbol{\mathcal{E}}^{\rm SO} = \frac{m_{\rm e}\eta_{\rm so}}{\hbar}\partial_t(\boldsymbol{m}\times\boldsymbol{E})$$

with $\dfrac{\partial \boldsymbol{m}}{\partial t} \neq 0$ and $\dfrac{\partial \boldsymbol{E}}{\partial t} = 0$




(Kim et al., PRL 2012)

with $\dfrac{\partial \boldsymbol{m}}{\partial t} = 0$ and $\dfrac{\partial \boldsymbol{E}}{\partial t} \neq 0$




(Yamane et al., PRB 2013)

# conclusion

we have investigated the Newtonian equation for the conduction electron
in ferromagnets, taking into account
the nonadiabatic effects and the spin-orbit coupling.

This semi-classical formalism allows us to address
spinmotive force, anomalous Hall effect, and domain wall resistances.

one of the future works may be extension of this formalism to
antiferromagnetic systems.