# Disordered Weyl Semimetals

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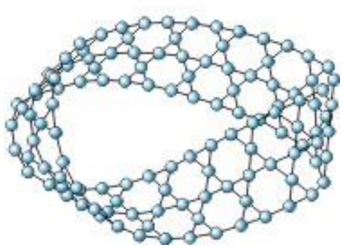
Virtual Institute: New States of Matter and their Excitations



Mainz, 25.07.2014

# Outline

1. Introduction
   - What is a Weyl semimetal?
   - Topological protection of Weyl node
   - Nice features of Weyl metals
   - Labtour: Dirac semimetals

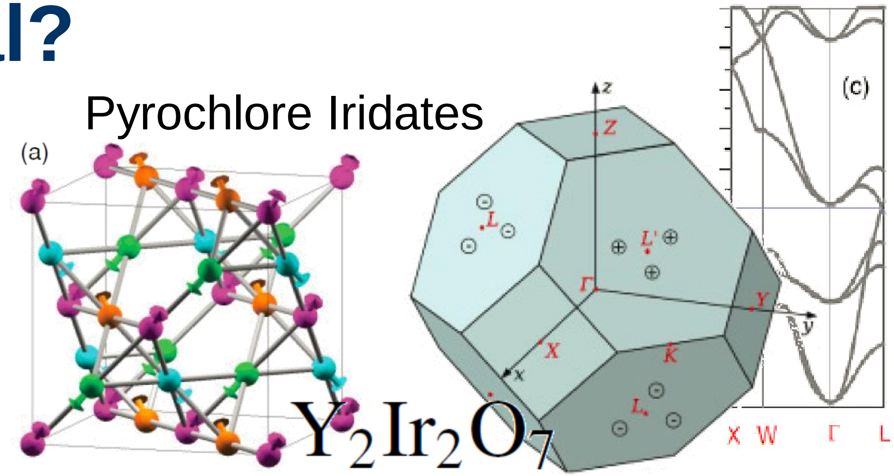
2. Disordered Weyl semimetals
   - Irrelevance of weak disorder
   - Numerical method
   - Results
3. Open questions
4. Summary

# What is a Weyl Semimetal?

High energy physics:
Massless Dirac Particles

Weyl-Equation: $\sigma^\mu \partial_\mu \psi = 0$

Pyrochlore Iridates

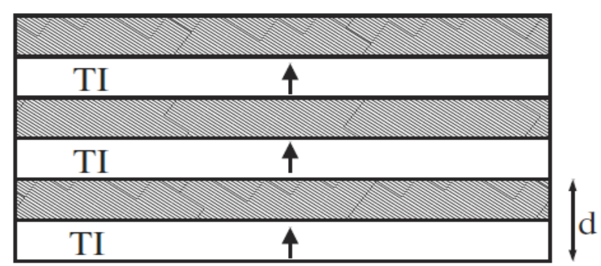



[Wan et al.,PRB2011]

“LSDA+U+SO” – pseudospin σ

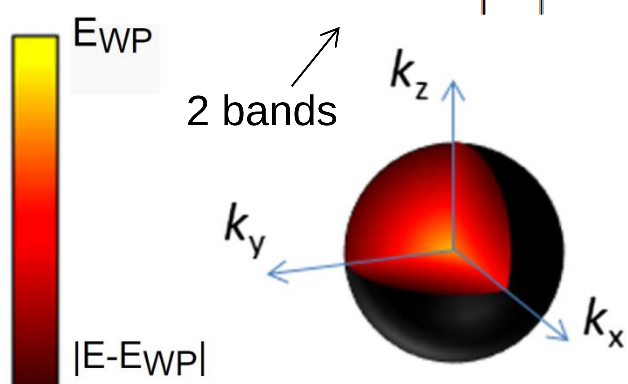
$$H_0\left(\mathbf{k}\right) = \pm \hbar v \boldsymbol{\sigma} \cdot \mathbf{k}$$

- [Burkov et al, PRL2011]:
  TI/OI mulitlayer
- [Bulmash et al, PRB2014]:
  Strained $Hg_{1-x-y}Cd_xMn_yTe$
- [Xu et al, PRL2014]:
  Cold atoms
- …




$$\varepsilon = \pm \hbar v |k|$$

2 bands




[Borisenko et al, PRL2014]

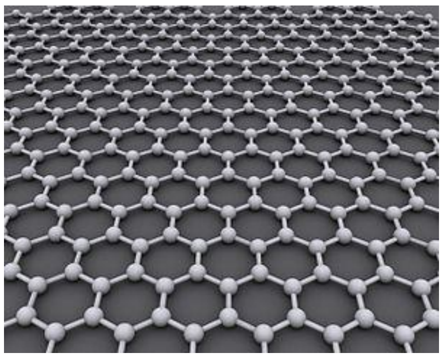
Weyl metal:

$$H = H_0(\mathbf{k}) + \mu$$
$$\varepsilon = \mu \pm \hbar v |k|$$

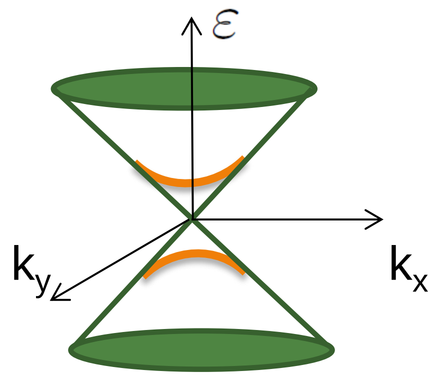
# Topological Protection of Weyl Node

$$H_0(\mathbf{k}) = \pm\hbar v\left(\sigma_x k_x + \sigma_y k_y + \sigma_z k_z\right) \longrightarrow$$ "3d – graphene" ?

1) 'Counting' argument



$$H_G = \hbar v\left(\sigma_x k_x + \sigma_y k_y\right) + \sigma_z m$$

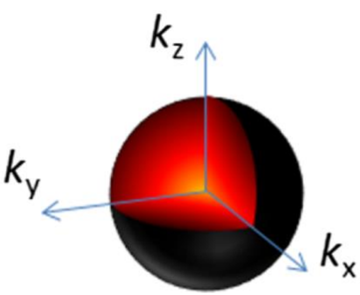



$$\varepsilon = \pm\sqrt{(\hbar v)^2\left(k_x^2 + k_y^2\right) + m^2}$$

Inversion symmetry breaking (e.g. by substrate)

2) Chern flux – relation to quantum Hall effect

*r-space - analogy*




$$\mathbf{A}_n(\mathbf{k}) = -i\left\langle u_n(\mathbf{k})\middle|\nabla_k\middle|u_n(\mathbf{k})\right\rangle$$ (vector potential)

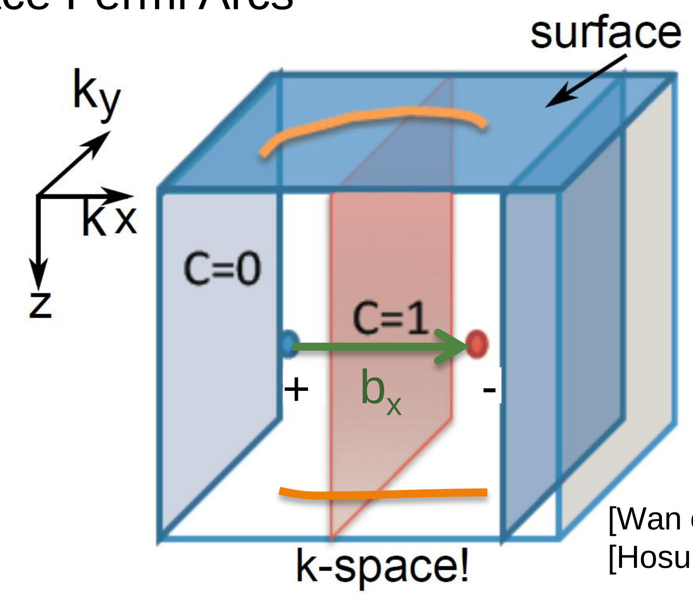
$$\mathbf{B}_n(\mathbf{k}) = \nabla_k \times \mathbf{A}_n(\mathbf{k})$$ (B-field)

$$\nabla_{\mathbf{k}} \cdot \mathbf{B}_n(\mathbf{k}) \propto \pm\delta(\mathbf{k})$$ (magn. monopole)

Weyl node cannot be gapped by *translational invariant* perturbation. Come in pairs.

# Nice Features of Weyl Metals

1.) Surface Fermi Arcs



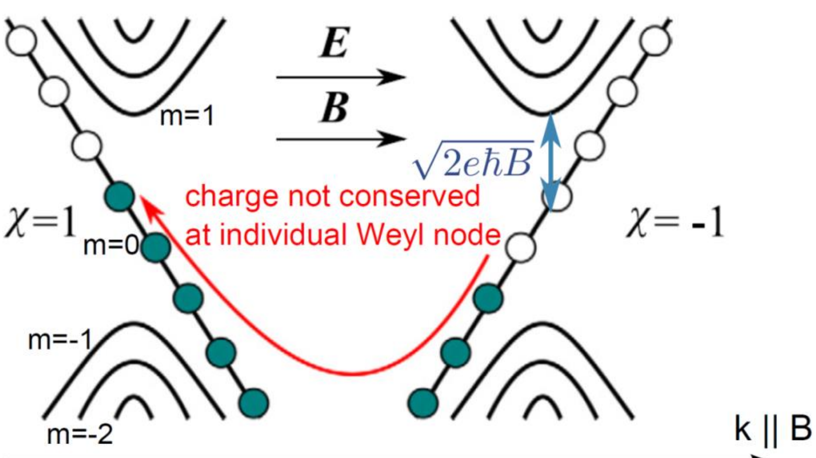

[Wan et al, PRB2011]
[Hosur et al, 2013]

QH–stack of "height" $b_x$ :
Anomalous Hall effect

$$\sigma_{yz} = -\frac{e^2}{2h\pi} b_x$$

(Field theory: Hosur et al)

2.) Chiral Anomaly (with B-field)




[Nielsen Ninomiya, PhysLett 1983]

$$\partial_\mu j^\mu = -\chi \frac{e^3}{h^2} \mathbf{E} \cdot \mathbf{B}$$

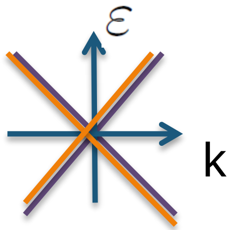
Magnetoconductivity:

$$\sigma_{MR} = \frac{\tau_I e^3 v B}{h^2}$$

($\tau_I$ internode scattering)

# Labtour: Dirac Semimetals

i) TRS and inversion symmetry (I) usually present in real materials
ii) Weyl node has non-degenerate band away from nodal point




$$|\mathbf{k}, n\rangle \stackrel{TRS}{\rightarrow} |-\mathbf{k}, TRS(n)\rangle \stackrel{I}{\rightarrow} |\mathbf{k}, \underbrace{I(TRS(n))}_{\neq n}\rangle$$

TRS+I: Double Weyl node = *Dirac* node, not topological!

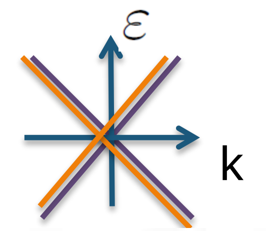
<u>Conclusions:</u>

a) Weyl nodes require either broken I or TRS.

b) Further point group symmetries (i.e. $C_4$) can protect Dirac node.

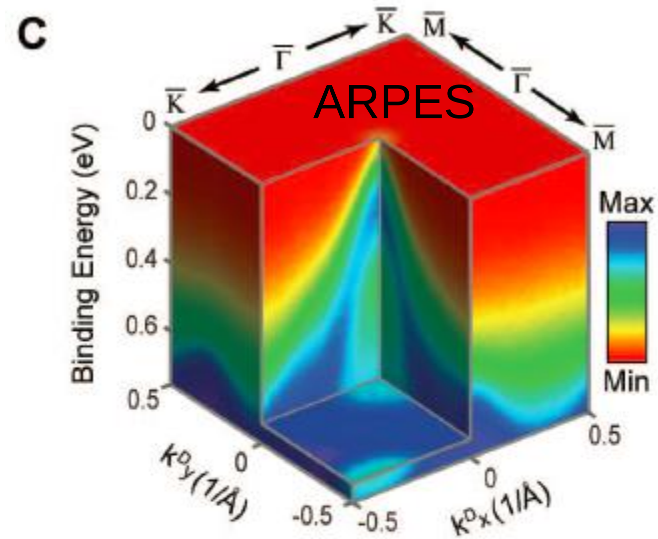
# Labtour: Dirac Semimetals

REPORTS [Science 2014]




## Discovery of a Three-Dimensional Topological Dirac Semimetal, $Na_3Bi$


Z. K. Liu,[1*] B. Zhou,[2,3*] Y. Zhang,[3] Z. J. Wang,[4] H. M. Weng,[4,5] D. Prabhakaran,[2] S.-K. Mo,[3] Z. X. Shen,[1] Z. Fang,[4,5] X. Dai,[4,5] Z. Hussain,[3] Y. L. Chen[2,6]†



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ARTICLE

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## Observation of a three-dimensional topological Dirac semimetal phase in high-mobility $Cd_3As_2$


Madhab Neupane[1,*], Su-Yang Xu[1,*], Raman Sankar[2,*], Nasser Alidoust[1], Guang Bian[1], Chang Liu[1], Ilya Belopolski[1], Tay-Rong Chang[3], Horng-Tay Jeng[3,4], Hsin Lin[5], Arun Bansil[6], Fangcheng Chou[2] & M. Zahid Hasan[1,7]



PRL **113,** 027603 (2014) PHYSICAL REVIEW LETTERS 11 JULY 2014


**break TRS or inversion: Dirac → Weyl**

## Experimental Realization of a Three-Dimensional Dirac Semimetal


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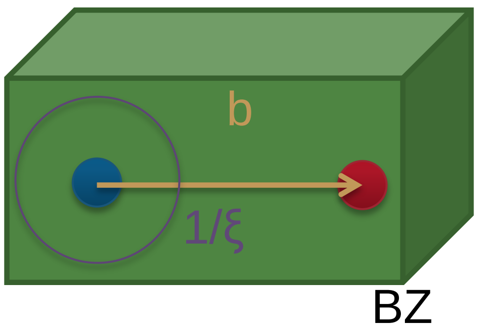
(Received 3 February 2014; published 8 July 2014)

# 2. Disordered Weyl Semimetals

Reminder:
Weyl node cannot be gapped by *translational invariant* perturbation and comes in pairs.

Question: What happens in a disordered system?

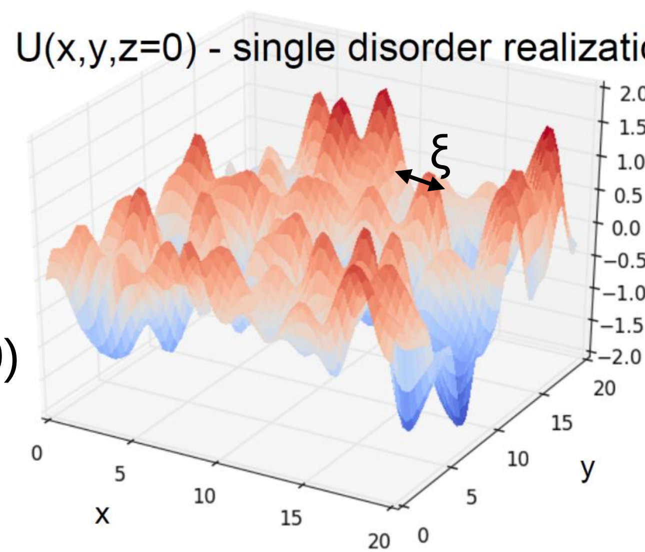
- If b >> 1/ ξ : single Weyl node




- Potential disorder U(r), strength K
- Gaussian distributed, zero mean
- Finite correlation length ξ

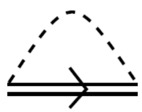
$$\langle U(\mathbf{r})U(\mathbf{r}')\rangle_{dis} = \frac{K(\hbar v)^2}{\sqrt{2\pi}^3 \xi^2} e^{-\frac{|r-r'|^2}{2\xi^2}}$$

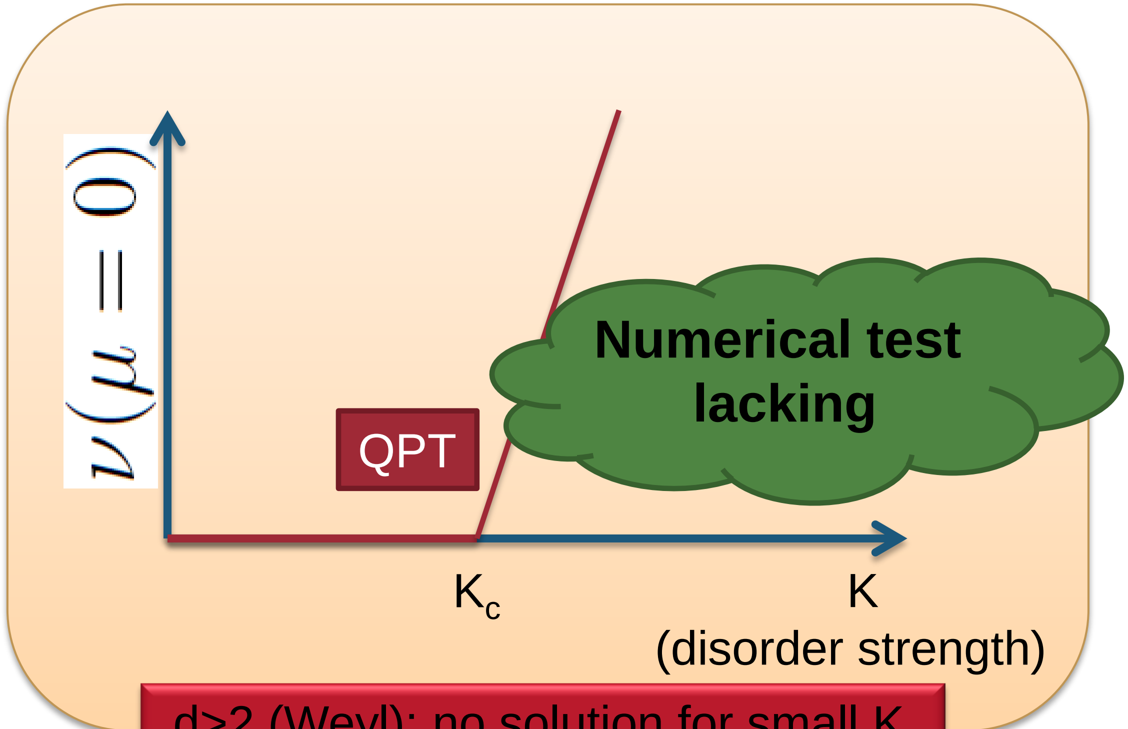
U(x,y,z=0) - single disorder realization




- From k-space to density of states (DOS)
- Clean system: $\nu(\varepsilon) \propto \varepsilon^2$
- Focus on semimetal case (μ=0, stoichiometric, T=0)
- Question': Effect of disorder on $\nu(\mu = 0) = 0$

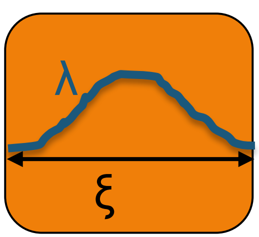
# Weak Disorder Irrelevant

- 1/N expansion
  [Fradkin, 1986]
- SCBA
  [Ominato et al, 2014]
  $\Sigma^{\mathrm{SCBA}} =$ 
- RG
  [Syzranov et al, 2014]
  [Burkov et al, 2011]




d>2 (Weyl): no solution for small K

Poor man's derivation ( $\hbar v \equiv 1$ , d-dim) – fit wavepacket:



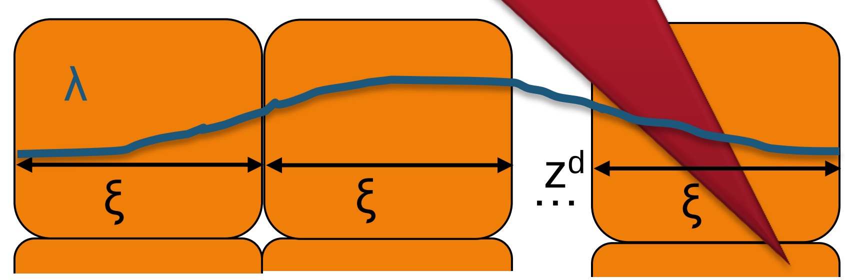

$\bar{U} = \sqrt{K}/\xi$

$\bar{U} \overset{!}{>} 1/\xi$

⇕

$K > 1$

What if K<1?
try z>1 puddles
in each direction



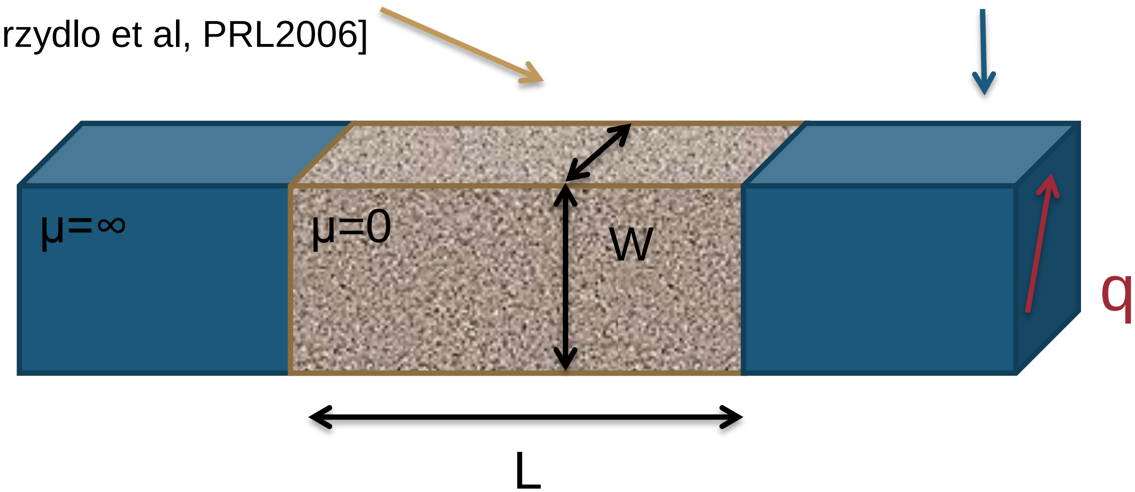

$\bar{U}/\sqrt{z^d} > 1/z\xi$ ⟺ $\sqrt{K} > z^{d/2-1}$

# Numerical Method

Idea: Study quantum transport instead of DOS

1st step: Clean Weyl Node between highly doped leads *(avoid Fermion doubling)*

[Tworzydlo et al, PRL2006]



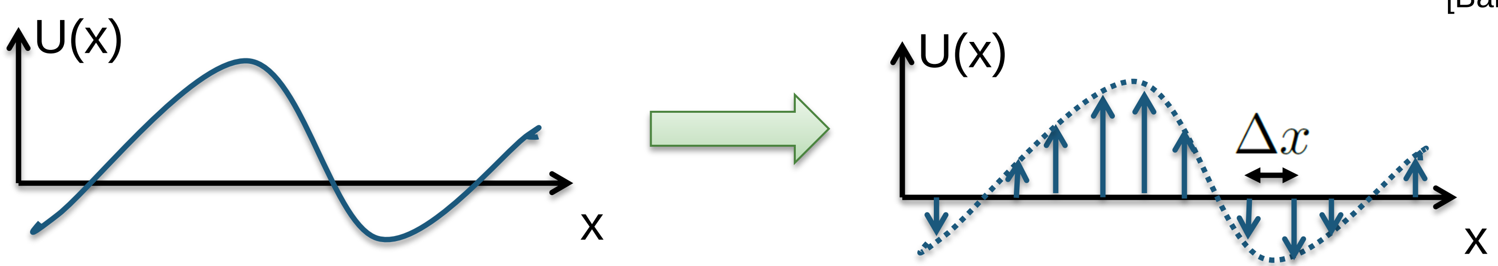

S-matrix for lead modes q

Landauer Formula:
Conductance G
Fano Factor F (noise)

$$G^0 = \left(\frac{W}{L}\right)^2 \times 0.110$$
$$F^0 = 0.574$$

[Baireuther et al, PRB2014]

2nd step: Add disorder – iterate step 1




free propagation

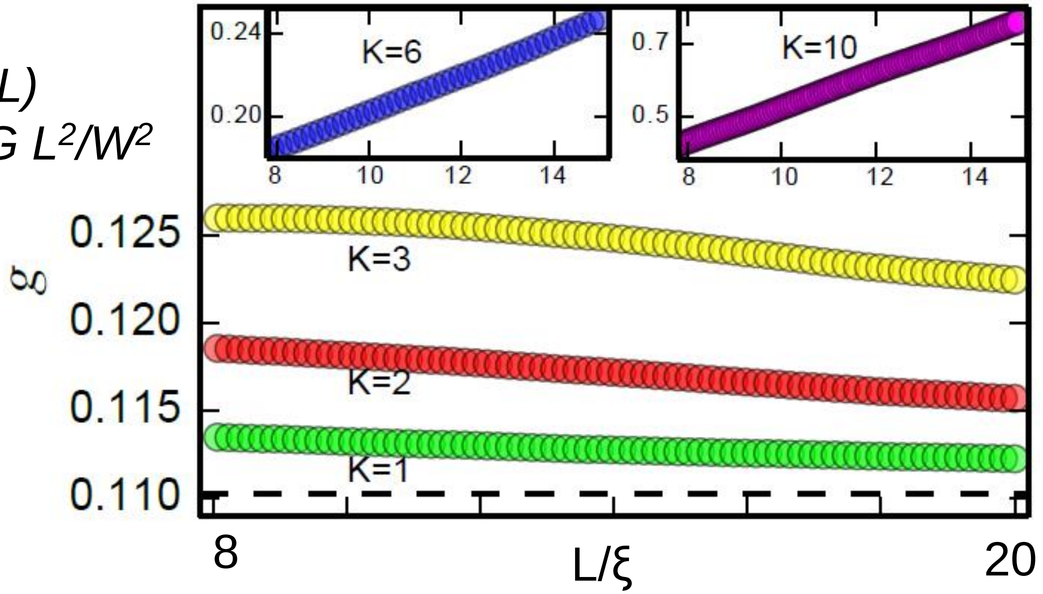
$$S = S_N^{imp} \otimes S_{\Delta x}^0 \otimes S_{N-1}^{imp} \otimes S_{\Delta x}^0 \otimes \ldots \otimes S_1^{imp} \otimes S_{\Delta x}^0$$

scattering (Born, q–mixing)

[Bardarson et al, PRL 2007]

# Results - Conductance

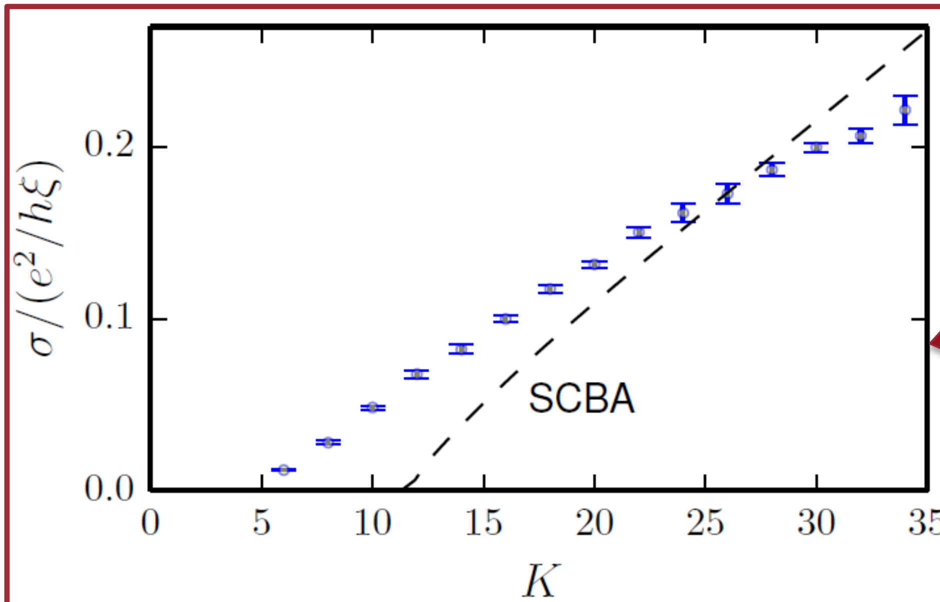
*g(L)*
*=G L²/W²*




Strong disorder: diffusive
$G = \sigma W^2/L \rightarrow g = \sigma L$

Weak disorder: pseudoballistic

$\sigma = G L/W^2$
$= g / L$

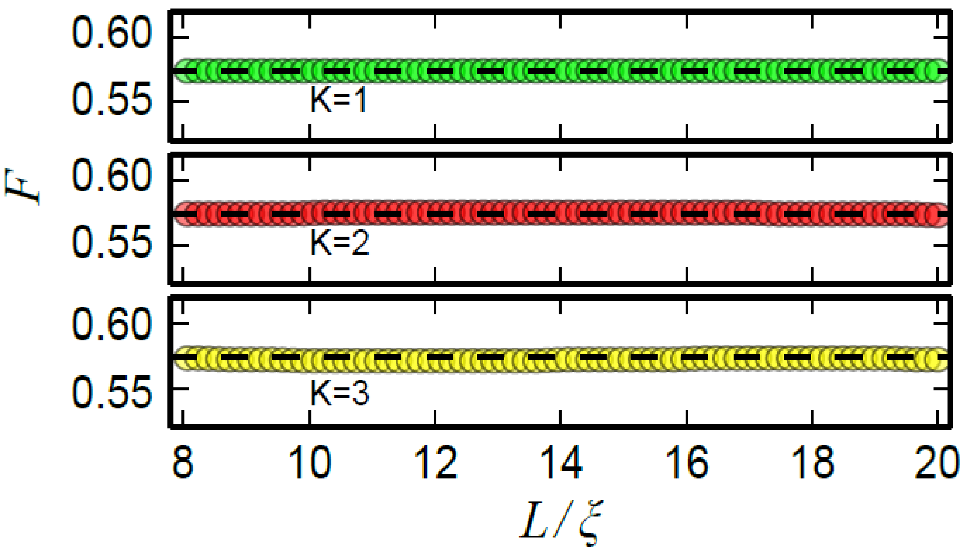



critical disorder confirmed

[Sbierski et al, PRL2014]

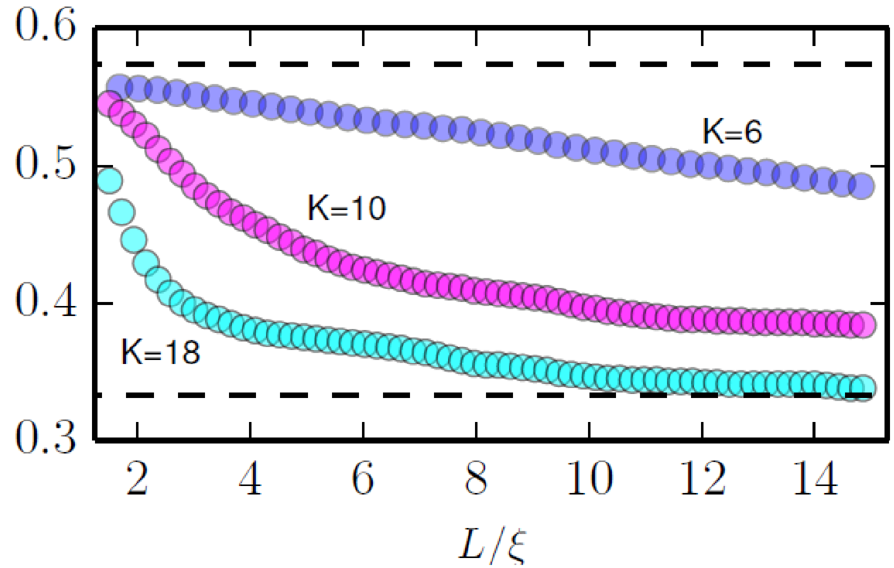
# Results – Fano Factor

Weak disorder - pseudoballistic




$F = F^0 = 0.574$

Strong disorder - diffusive



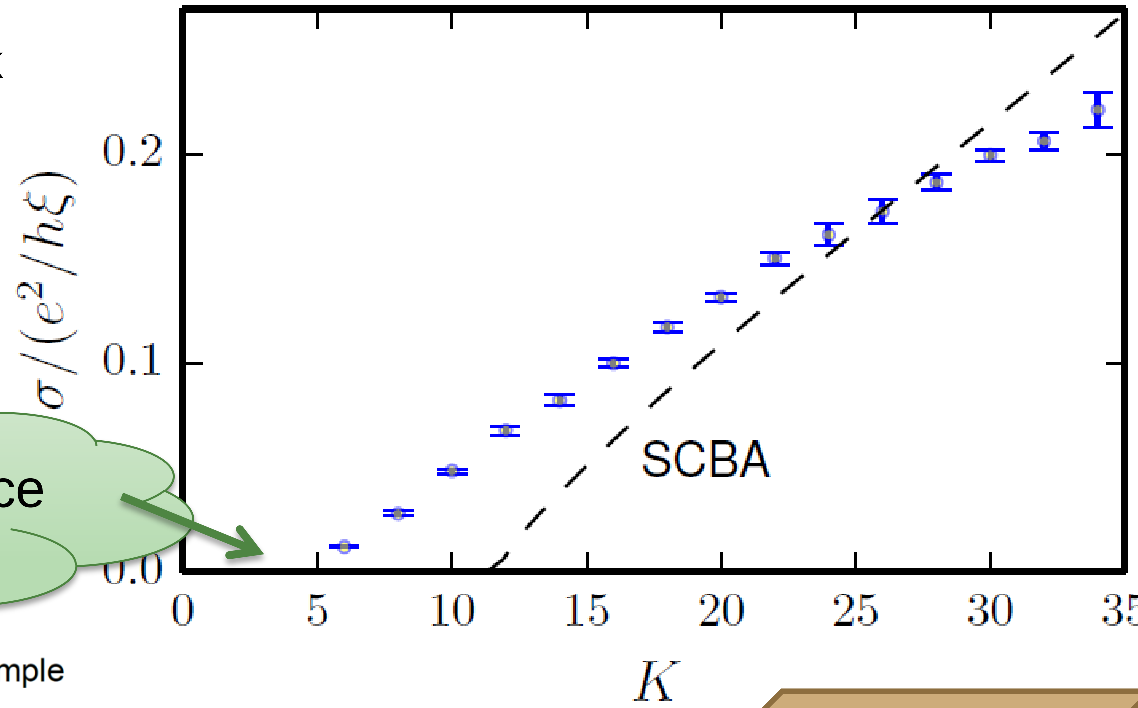

F=1/3

Fano factor excellent indicator to discriminate the two regimes.

[Sbierski et al, PRL2014]

# Back to Topology…

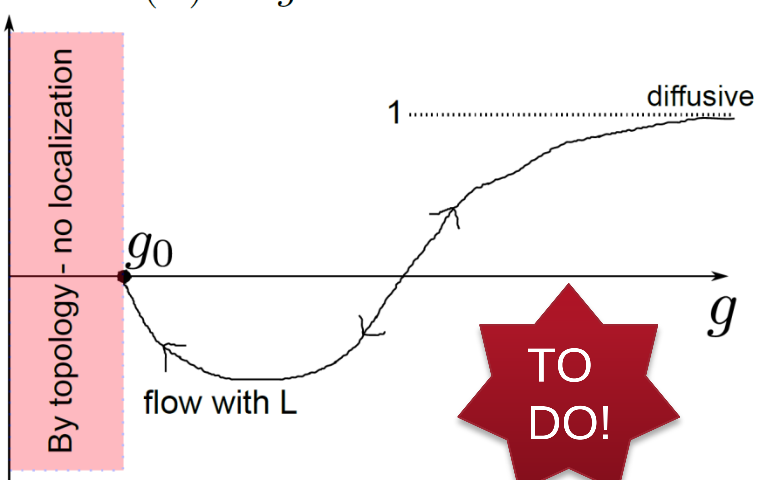
Critical disorder strength H ~ k
[Syzranov et al, 2014]

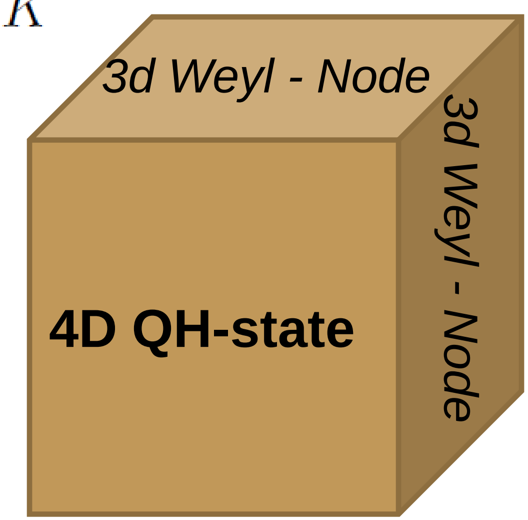



finite *cube* conductance

*g=G L²/W²*

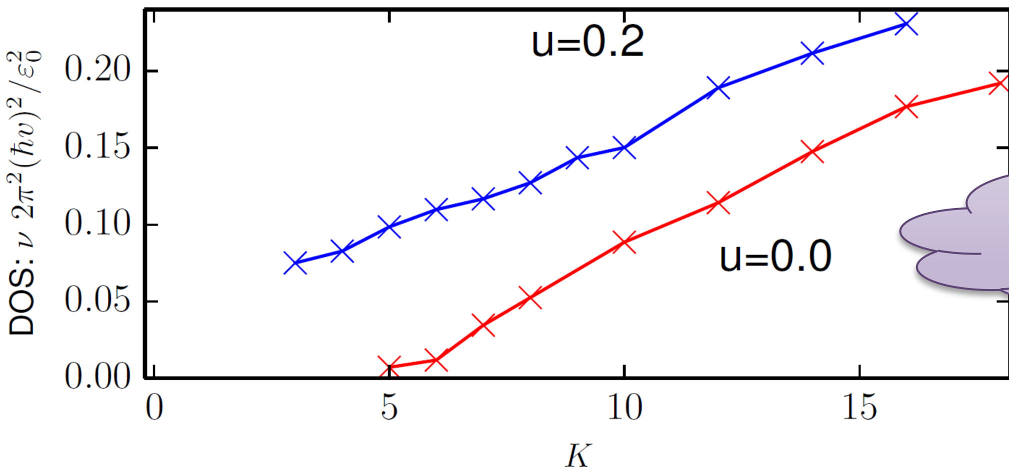
$$\beta(g) = \frac{d\,ln(g)}{d\,ln(L)} = \frac{L}{g}\frac{dg}{dL}$$ : cubic sample



# Open Questions

1. Compute DOS from Scattering matrix

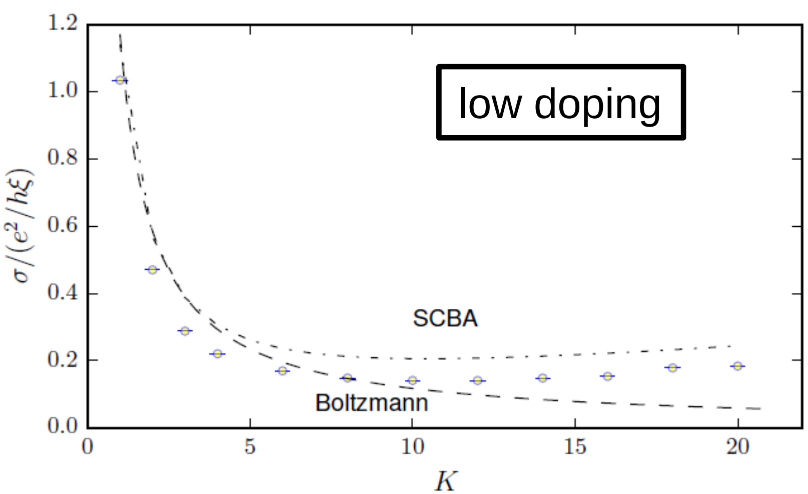


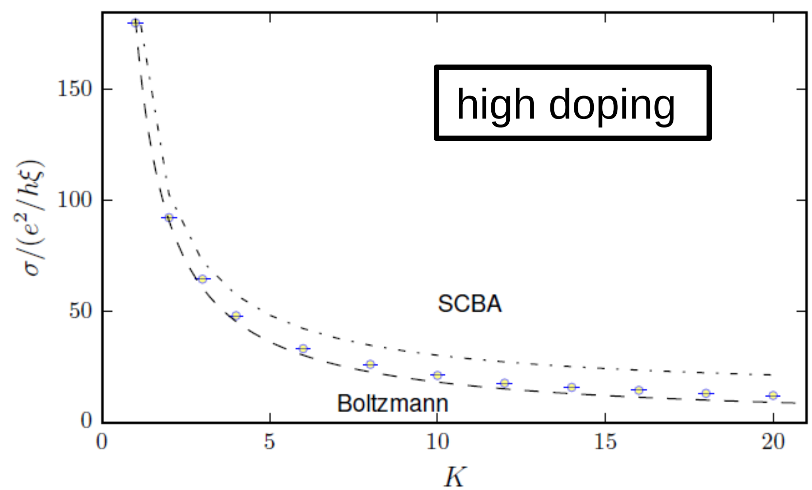

$$\nu = \frac{1}{2\pi i} tr \left\{ \mathbf{S}^\dagger (V) \frac{d\mathbf{S}(V)}{dV} \right\}$$

**Fresh from CPU!**

2. Quantitative understanding [SCBA, (f)RG,…]

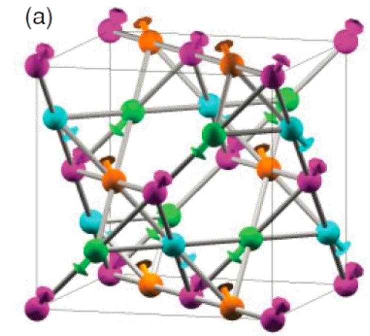
SCBA: $k_F$ l >> 1
(high doping, low disorder)



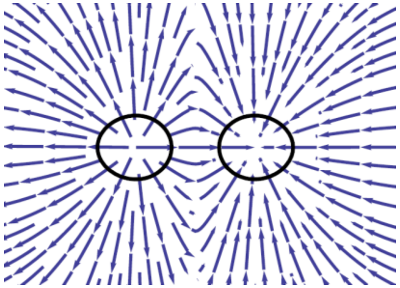




3. One parameter scaling?

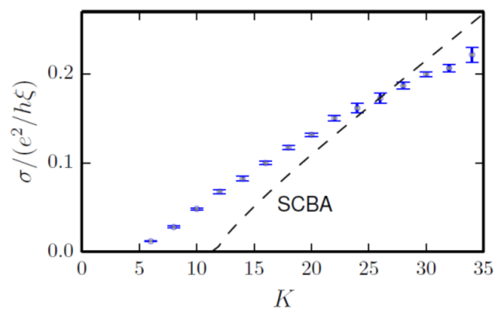
# Summary




- Weyl (Semi-)metals: High energy meets solid state



- Topological protection, predicted effects, Dirac Semimetals
- Disordered Weyl semimetals and the critical disorder strength (analytics and numerics)




**THE END.**