Optimal persistent currents for interacting bosons on a ring with gauge field

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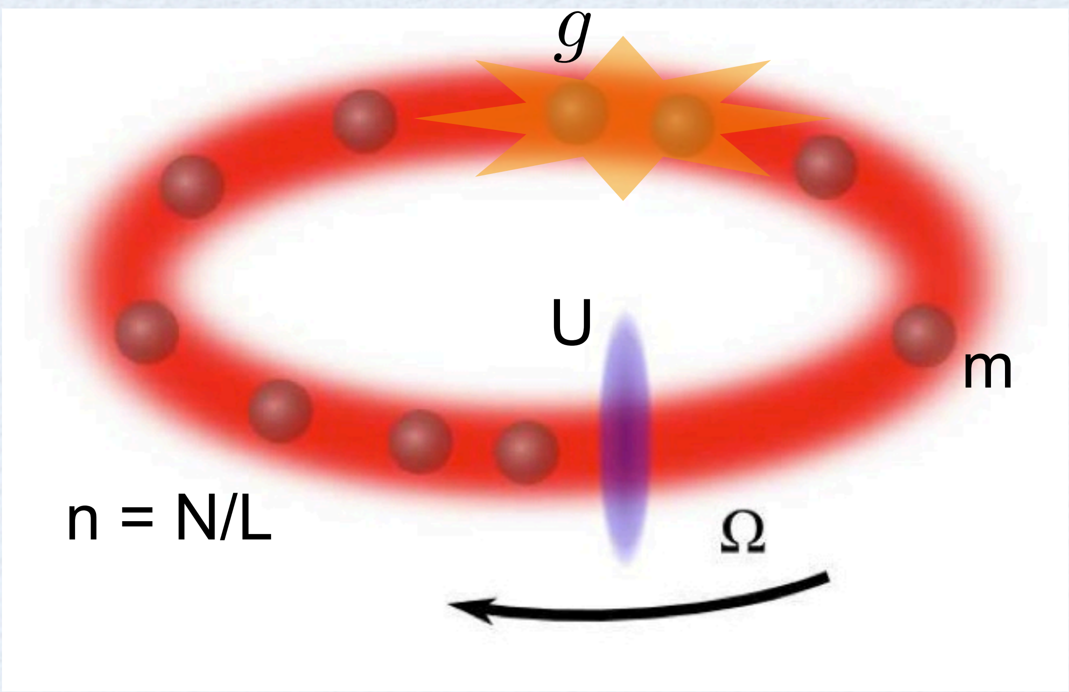
KOMET Seminar - 27.05.2014

M.Cominotti, D. Rossini, M. Rizzi, F. Hekking, A. Minguzzi, arXiv:1310.0382

The system Hamiltonian

Problem



- 1D mesoscopic ring (PBC's, length L)
- ultracold bosons (T=0)
- contact interactions (g)
- rotation or gauge field (Ω)
- localized barrier (U)
- density (n) of particles with mass (m)

$$\mathcal{H} = \sum_{j=1}^{N} \left[\frac{\hbar^2}{2M} \left(-i \frac{\partial}{\partial x_j} - \frac{2\pi\Omega}{L} \right)^2 + U\,\delta(x_j) + g \sum_{l<j}^{N} \delta(x_l - x_j) \right]$$

TARGET: Persistent current $I(\Omega) = -\frac{1}{2\pi\hbar} \frac{\partial E(\Omega)}{\partial \Omega}$ in all physical regimes

Bloch, PRB 2, 109 (1970)

Reasons of interest

Motivation

Ahranov-Bohm effect + macroscopic many-body quantum coherence

→ observed in bulk superconductors and even normal metallic rings

B. S. Deaver and W. M. Fairbank, PRL 7, 43 (1961)
N. Byers and C. N. Yang, PRL 7, 46 (1961)
L. Onsager, PRL 7, 50 (1961)

L. P. Levy, et al., PRL 64, 2074 (1990)
D. Mailly, et al., PRL 70, 2020 (1993)
H. Bluhm et al., PRL 102, 136802 (2009)
A. C. Bleszynski-Jayich, et al., Science 326, 272 (2009)

Cold atoms as testbed for fundamental questions

[long coherence times, tunability, internal structure, easy detection, …]

→ low-dimensional superfluidity / interactions vs. quantum fluctuations / …

Cherny et al., Front. Phys. 7, 54 (2012)
Fisher et al., MET 345, 331 (1997)

Büchler et al., PRL 87, 100403 (2001)
A. Ramanathan et al., PRL 106, 130401 (2011)
Moulder et al., PRA 86, 013629 (2012)

Possible macroscopic superposition of current states (flux qubits)

→ quantum computation / atomic SQUID gyroscopes / atomtronics

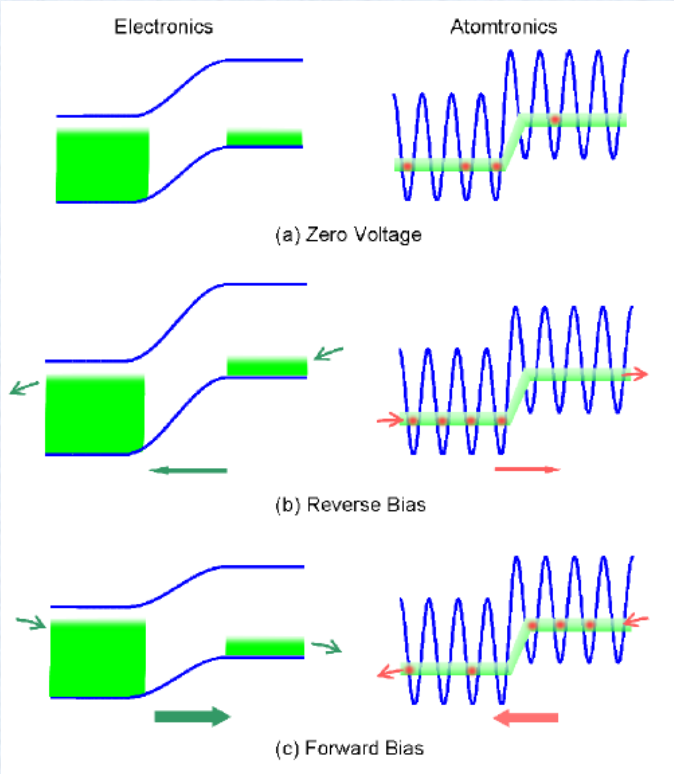
D. W. Hallwood, et al., PRA 82, 063623 (2010)
D. Solenov, D. Mozyrsky, PRA 82, 061601 (2010)

A. Nunnenkamp, et al, PRA 84, 053604 (2011)
C. Schenke et al., PRA 85, 053627 (2012)

‘Atomtronics’

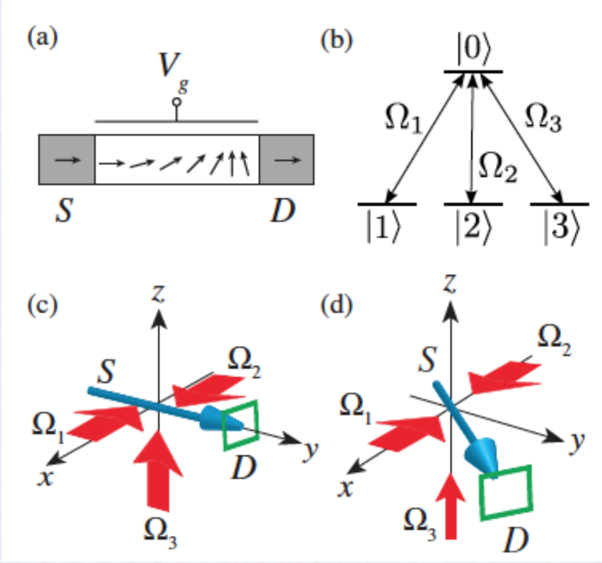
Motivation

P-N Junction diode in optical lattices



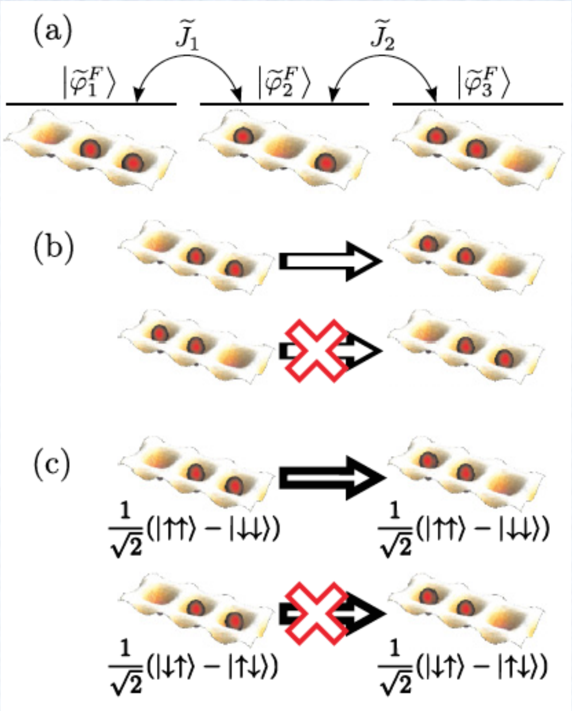
B. Seaman, et al., PRA 75, 023615 (2007)

Atomic Spin-Field Effect Transistor



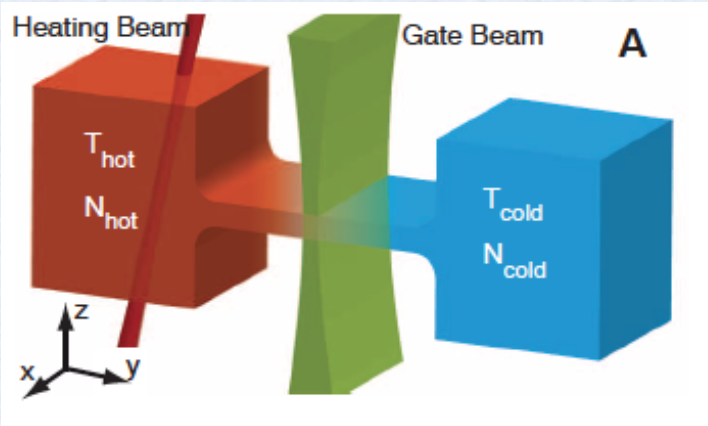
Vaishnav, et al., PRL 101, 265302 (2008)

STIRAP Diode for holes



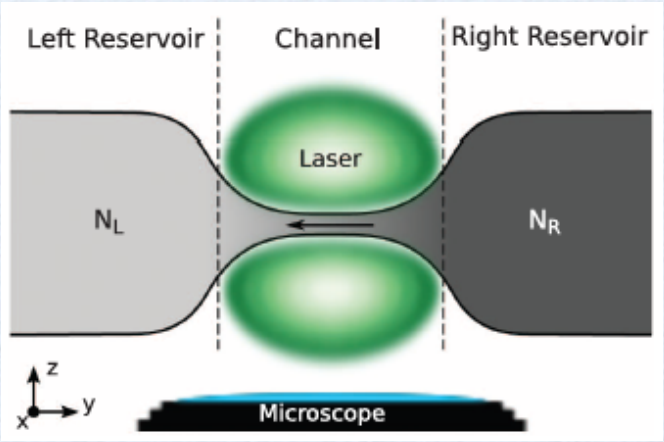
Benseny, et al., PRA 82, 013604 (2010)

Thermoelectric Heat Engine



Brantut, et al., Science 342, 713 (2013)

Mesoscopic Ohmic Conduction

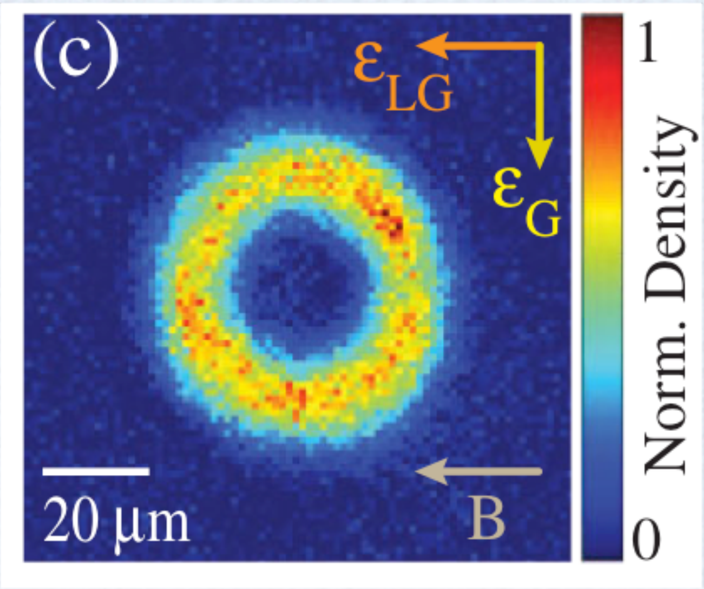


Brantut, et al., Science 337, 1069 (2012)

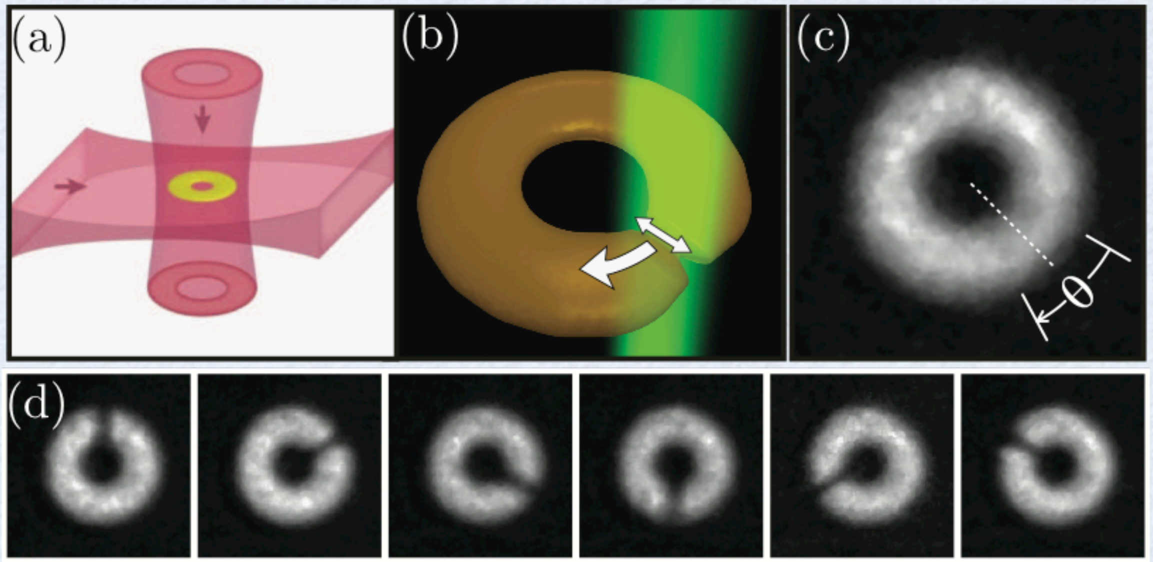
… and many more proposals & experiments in the last few years !

Cold atoms in ring traps

Motivation



A. Ramanathan et al., PRL 106, 130401 (2011)



Wright et al., PRL 110, 025302 (2013)

- ✓ Mesoscopic size (Ø ~ tens µm)
- ✓ Long coherence times (~ 40 s observed)
- ✤ Still in 3D regime…
  (vortex => phase slip decay)

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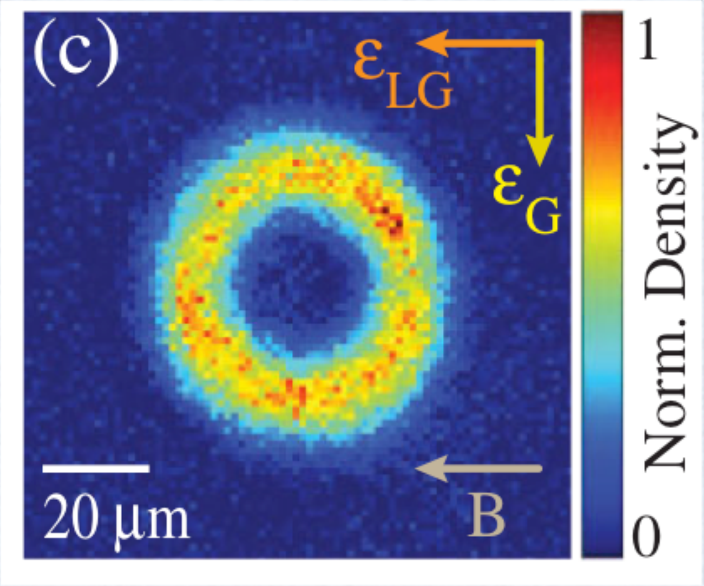

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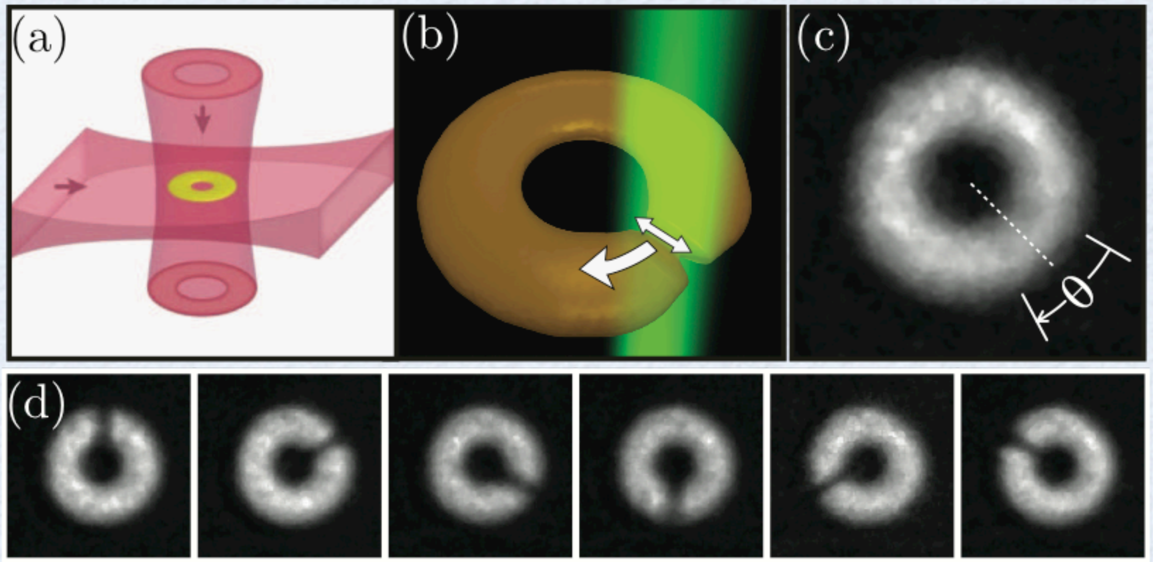

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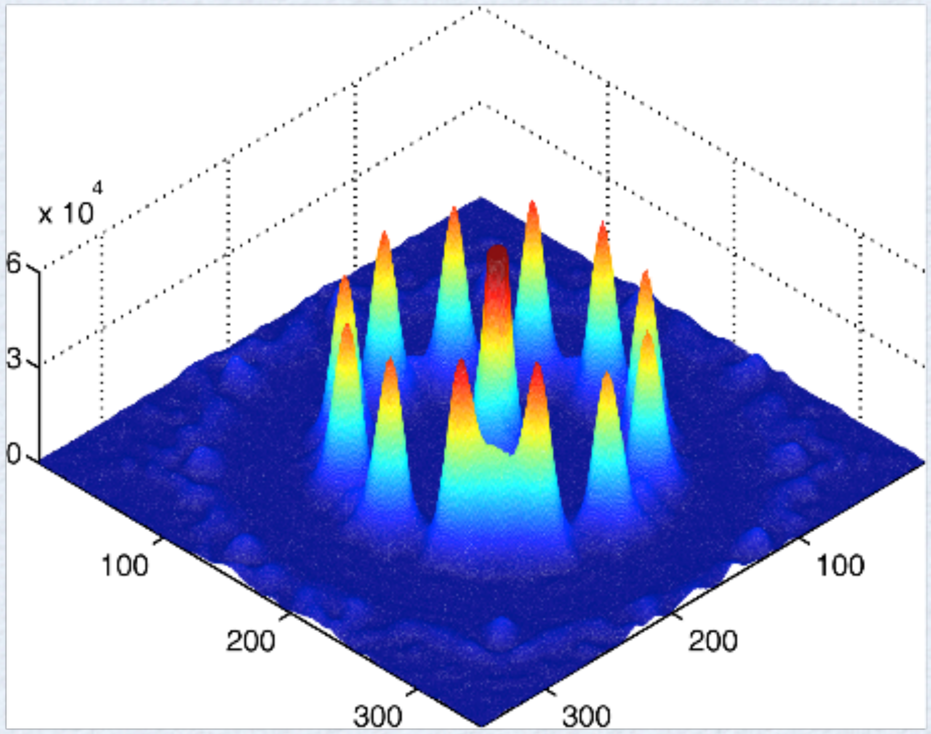
- ✓ Proposals for 1D confinement around !



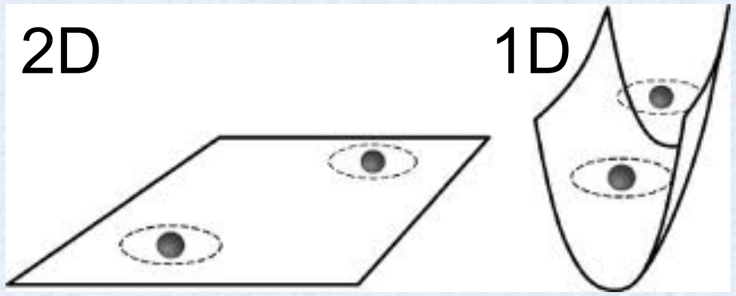
Amico et al., arXiv:1304.4615 (2013)

Richness & oddness of 1D

Motivation

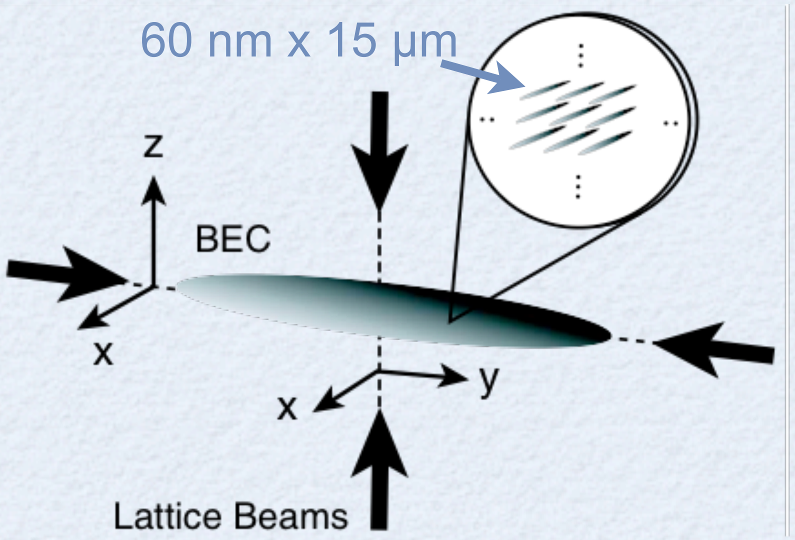
Strong transverse confinement

$$\hbar\omega_\perp \gg k_B T,\ \mu$$



Effectively reduced dimensionality

$$\Psi_B(\vec{r}_1,\ldots,\vec{r}_N) = \psi_B^{1D}(x_1,\ldots,x_N)\prod_{i=1}^{N}\phi_0(\vec{r}_i^{\perp})$$



Greiner et al., PRL 87, 160405 (2001)

Moritz et al., PRL 91, 250402 (2003)

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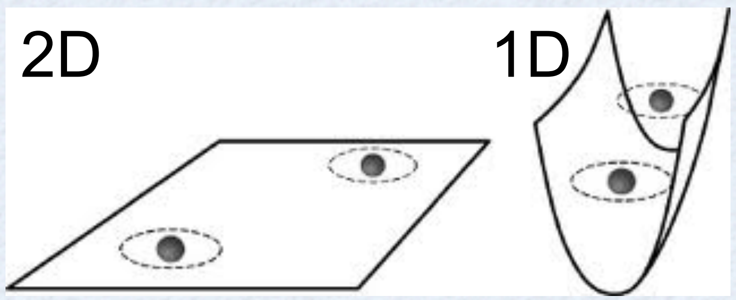

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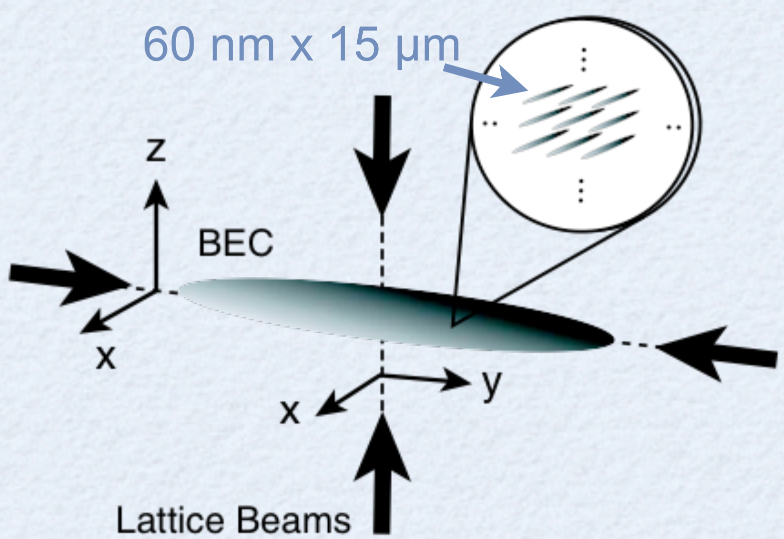


Greiner et al., PRL 87, 160405 (2001)

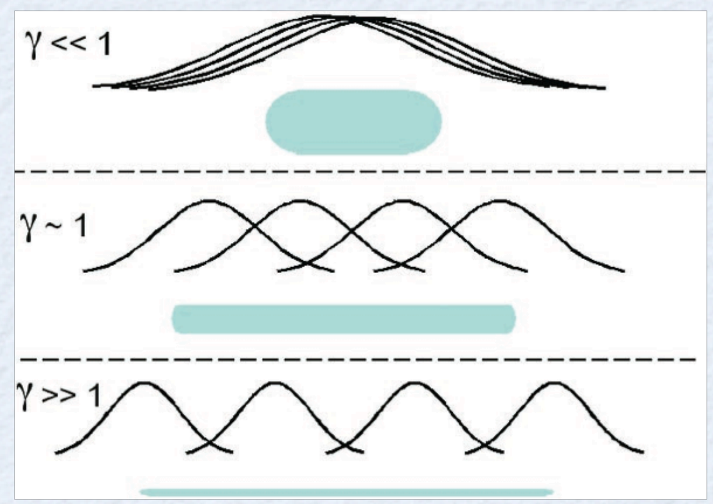
Moritz et al., PRL 91, 250402 (2003)

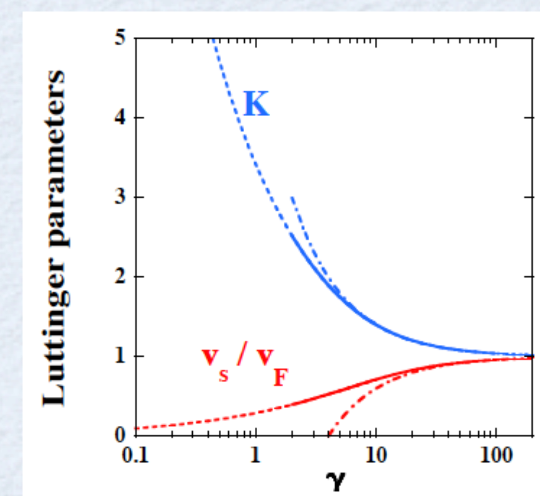
✓ Lots of analytically treatable regimes

- Interaction growth with diluteness !

$$\gamma = \frac{E_{int}}{E_{kin}} = \frac{gn}{\hbar^2 n^2/m} = \frac{gm}{\hbar^2 n} \qquad n = \frac{N}{L}$$

- "Fermionization" of hard-core bosons





- Quantum fluctuations: extremely relevant !
  => no long-range order … "only" Luttinger K

$$\langle \hat{\Psi}^\dagger(x)\hat{\Psi}(x')\rangle \simeq \frac{1}{|x-x'|^{1/2K}}$$

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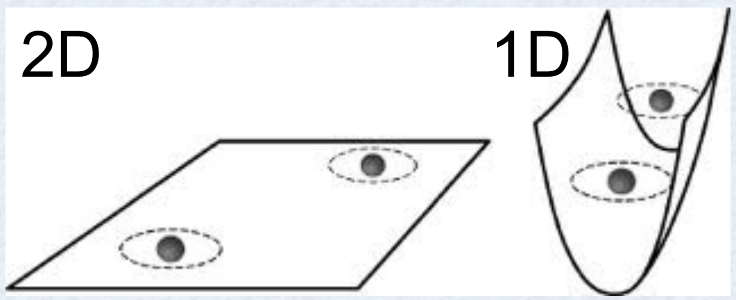

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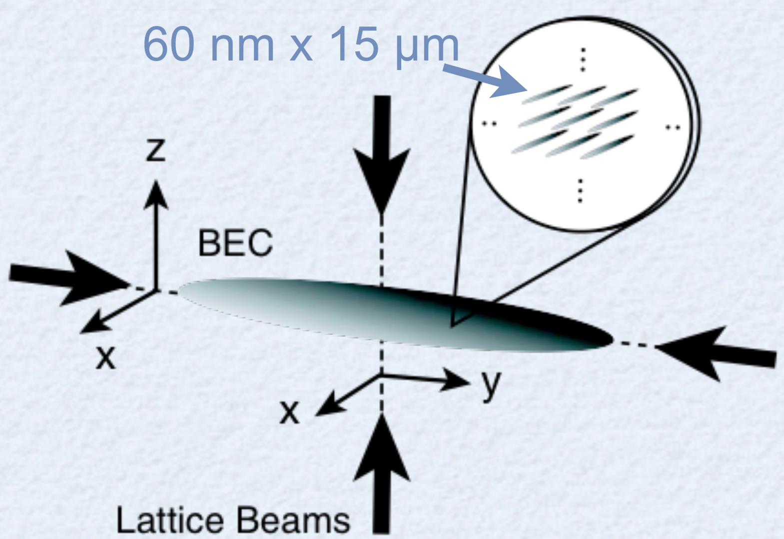

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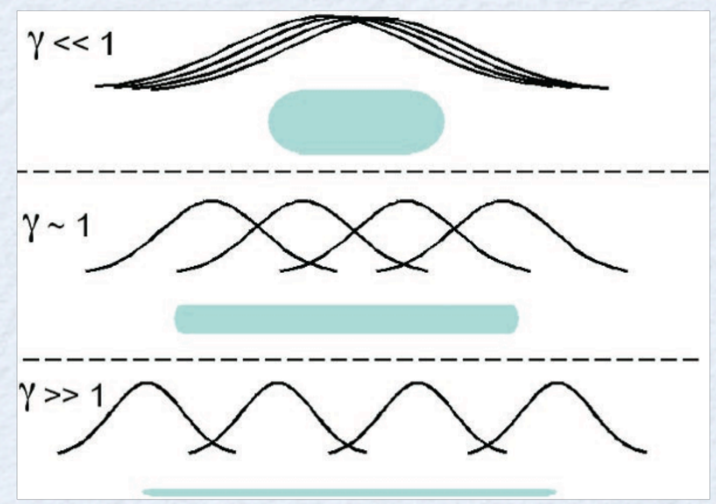

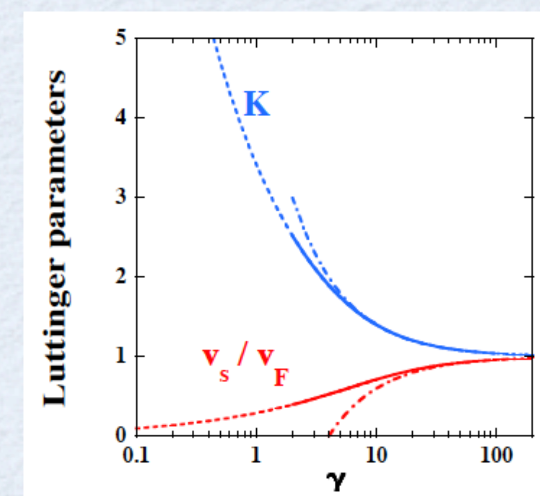

- Quantum fluctuations: extremely relevant !
  => no long-range order … "only" Luttinger K

$$\langle \hat{\Psi}^\dagger(x)\hat{\Psi}(x')\rangle \simeq \frac{1}{|x-x'|^{1/2K}}$$

✓ Reduced amount of entanglement
=> efficient numerics by DMRG !

The system Hamiltonian

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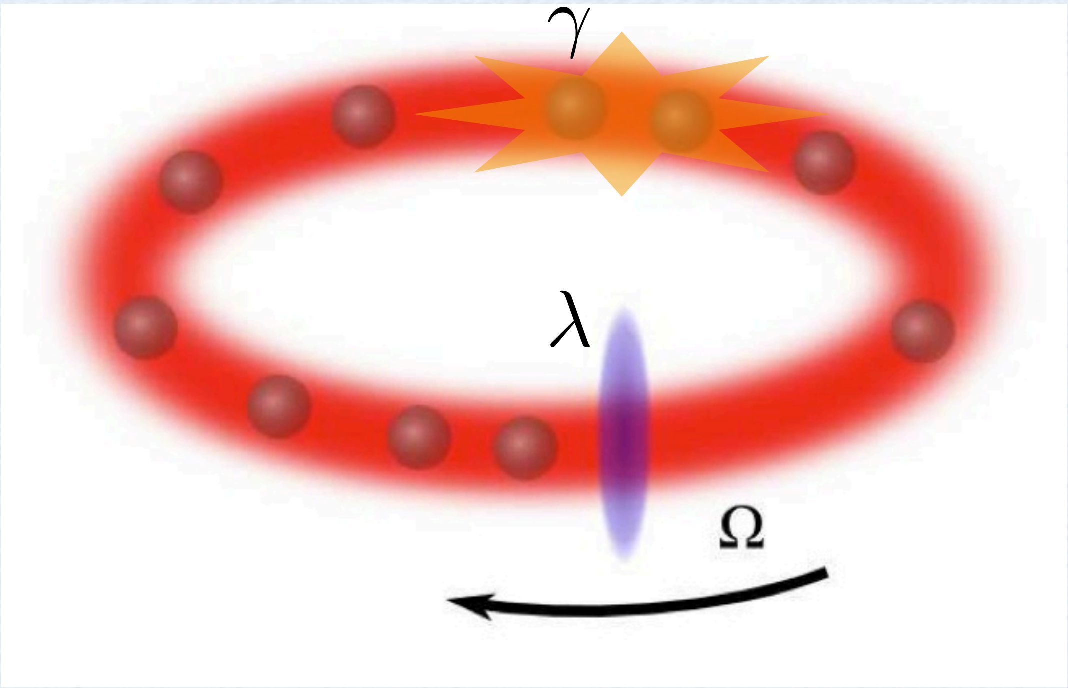

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- ultracold bosons (T=0)
- contact interactions (g)
- rotation or gauge field (Ω)
- localized barrier (U)
- density (n) of particles with mass (m)

$$\gamma = \frac{gm}{\hbar^2 n}$$

$$\lambda = \frac{mUL}{\pi\hbar^2}$$

$$\mathcal{H} = \frac{\hbar^2}{2M(L/2\pi)^2} \sum_{j=1}^{N} \left[\left(-i\frac{\partial}{\partial\theta_j} - \Omega \right)^2 + \lambda\,\delta(\theta_j) + \frac{N\gamma}{\pi} \sum_{l<j}^{N} \delta(\theta_l - \theta_j) \right]$$

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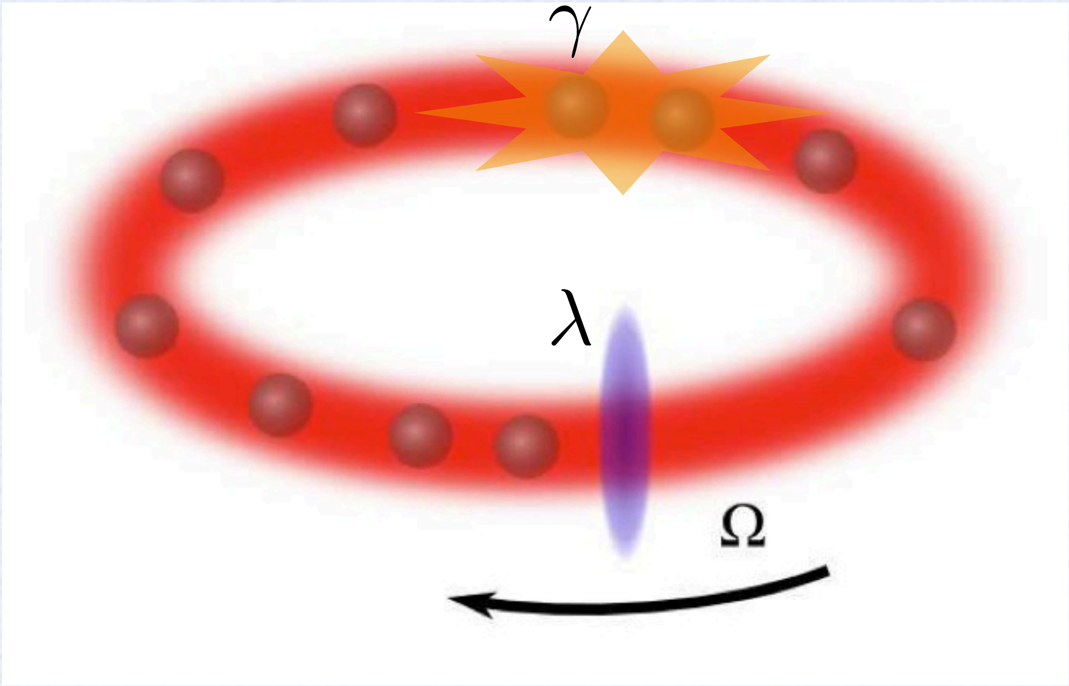

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TARGET: Persistent current $I(\Omega) = -\frac{1}{2\pi\hbar} \frac{\partial E(\Omega)}{\partial \Omega}$ in all regimes of γ & λ

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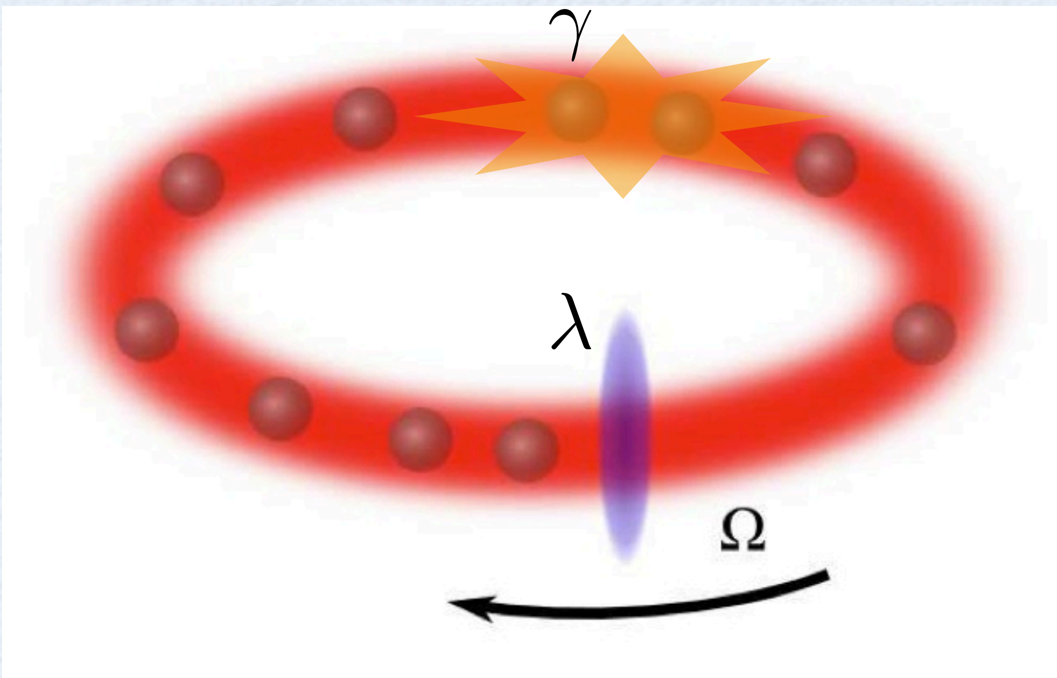

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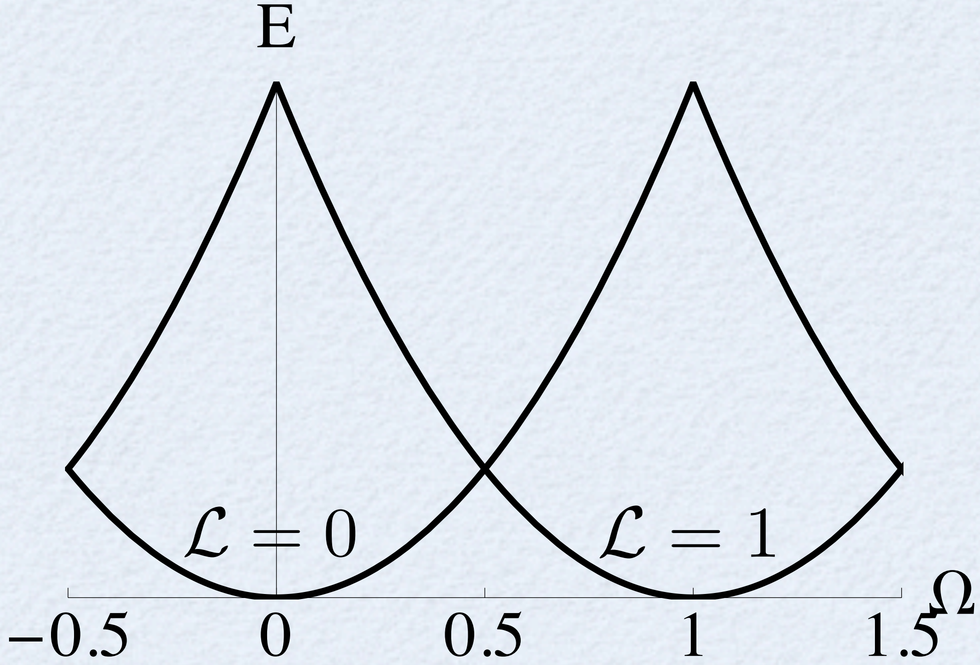
Presence of a barrier/defect

Problem



$\lambda = 0$ Rotational Invariance

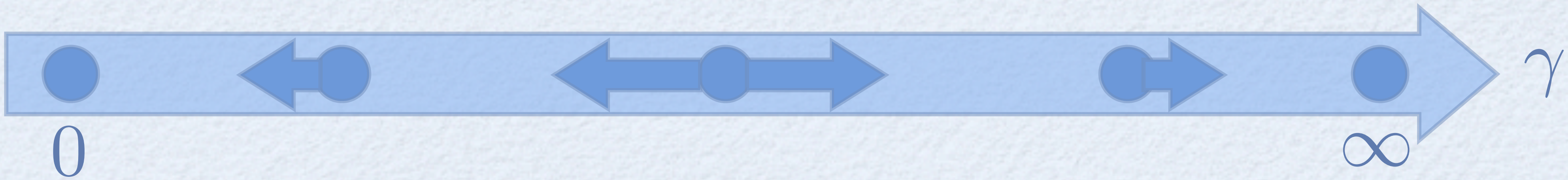
⇓

Periodicity in angular momenta

⇓

Current is a perfect sawtooth $\forall\gamma$

Loss, PRL 69, 343 (1992); Mueller et al., EPL 22, 193 (1993)



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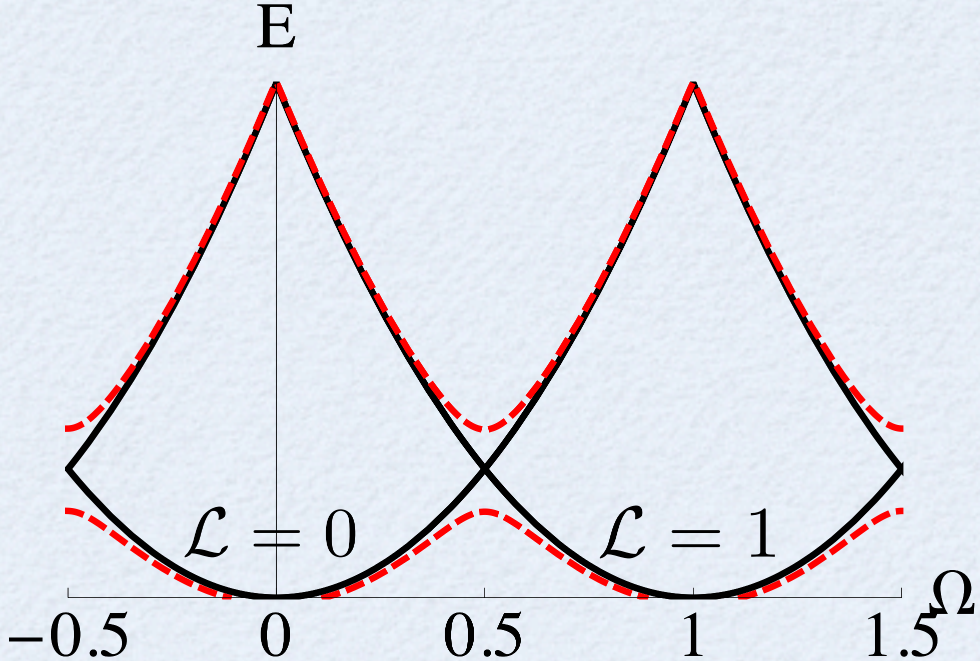
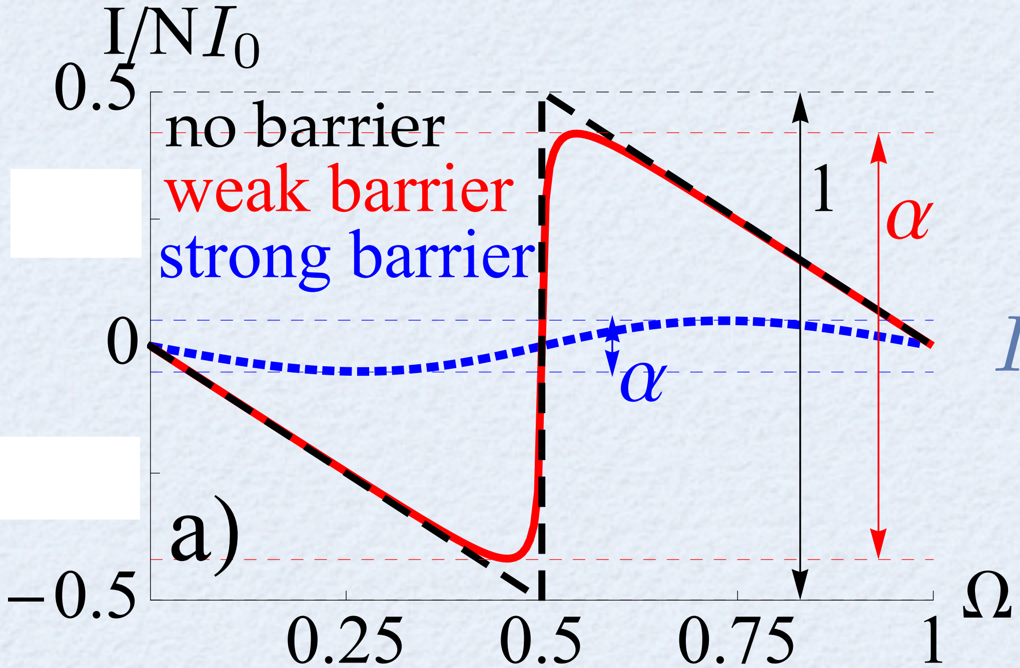




$I_0 = \frac{2\pi\hbar\Omega}{mL^2}$

$\lambda = 0$ Rotational Invariance

⇓

Periodicity in angular momenta

⇓

Current is a perfect sawtooth $\forall\gamma$

Loss, PRL 69, 343 (1992); Mueller et al., EPL 22, 193 (1993)

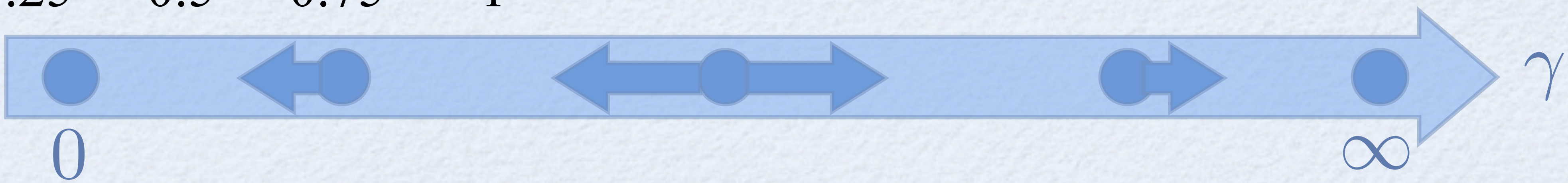
$\lambda > 0$ Symmetry Breaking

⇓

Degeneracy lifted (flux qubit !?)

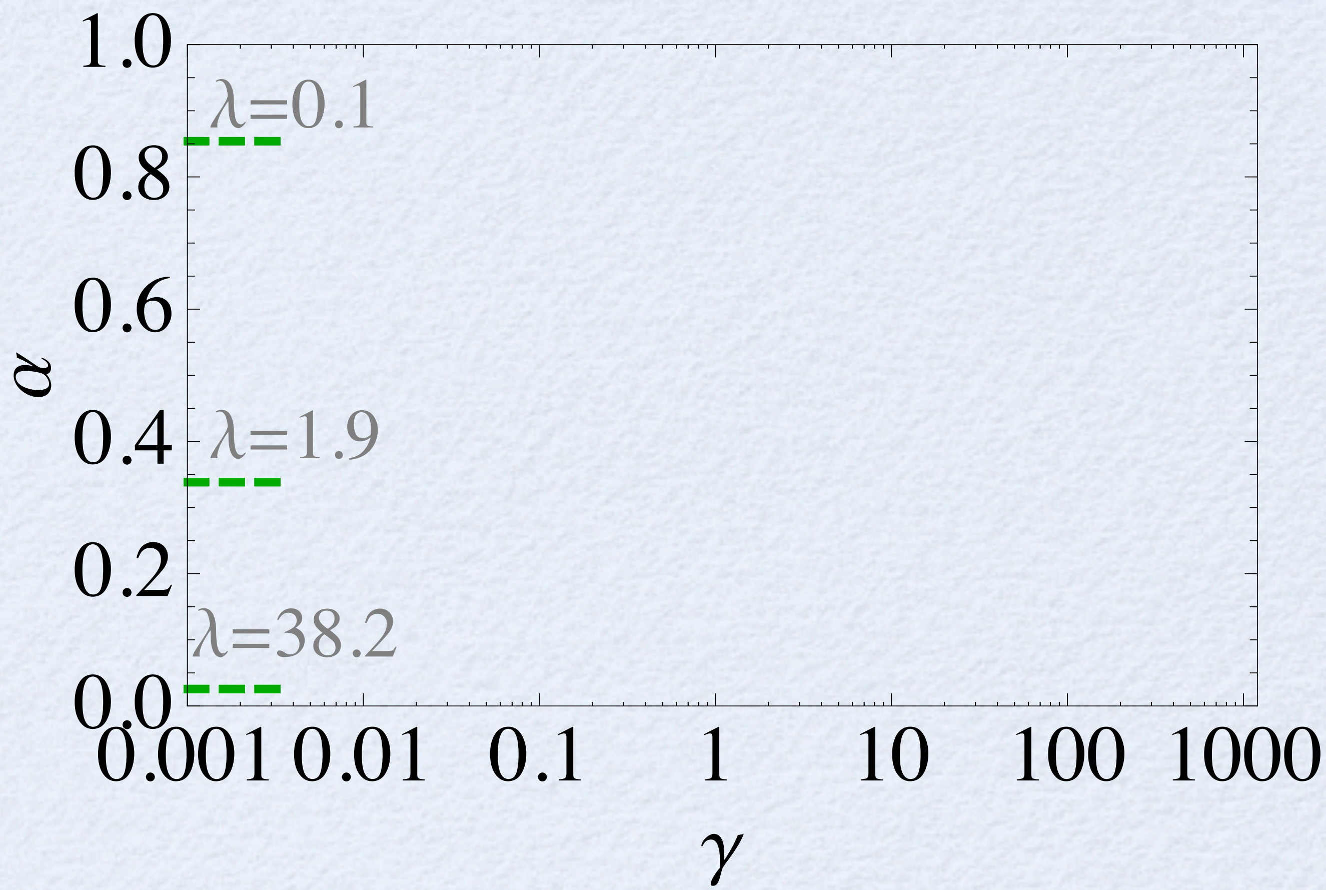
⇓

Current α depends on γ & λ



Non-Interacting regime

Problem



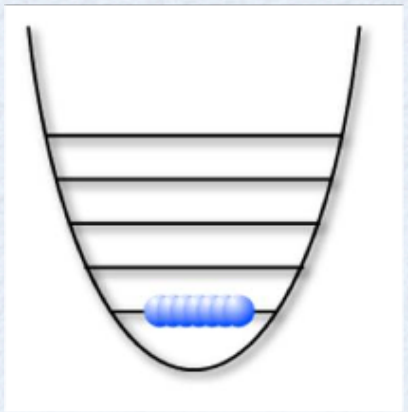
✓ eigenfunctions are plane waves
+ twisted boundary conditions
+ cusp at barrier position

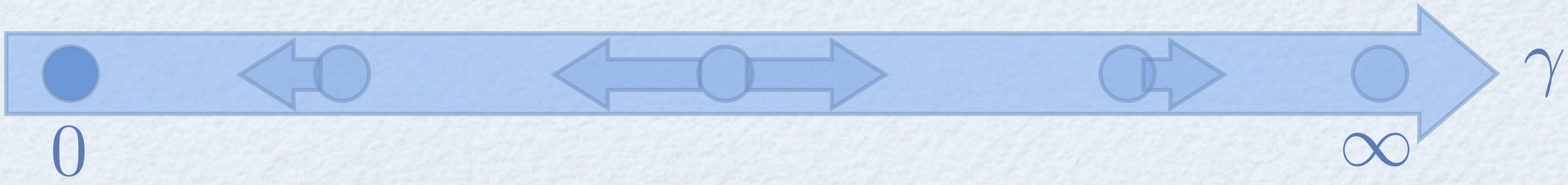
$$k_n = \pm\lambda \frac{\pi}{L} \frac{\sin(k_n L)}{\cos(2\pi\Omega) \mp \cos(k_n L)}$$

$$\varepsilon_n = \hbar^2 k_n^2 / 2m$$

✓ ideal bosons scenario: condensation in the ground

$$E = N\varepsilon_0$$





Hard-core regime

Problem

- ✓ eigenfunctions are plane waves
  + twisted boundary conditions
  + cusp at barrier position

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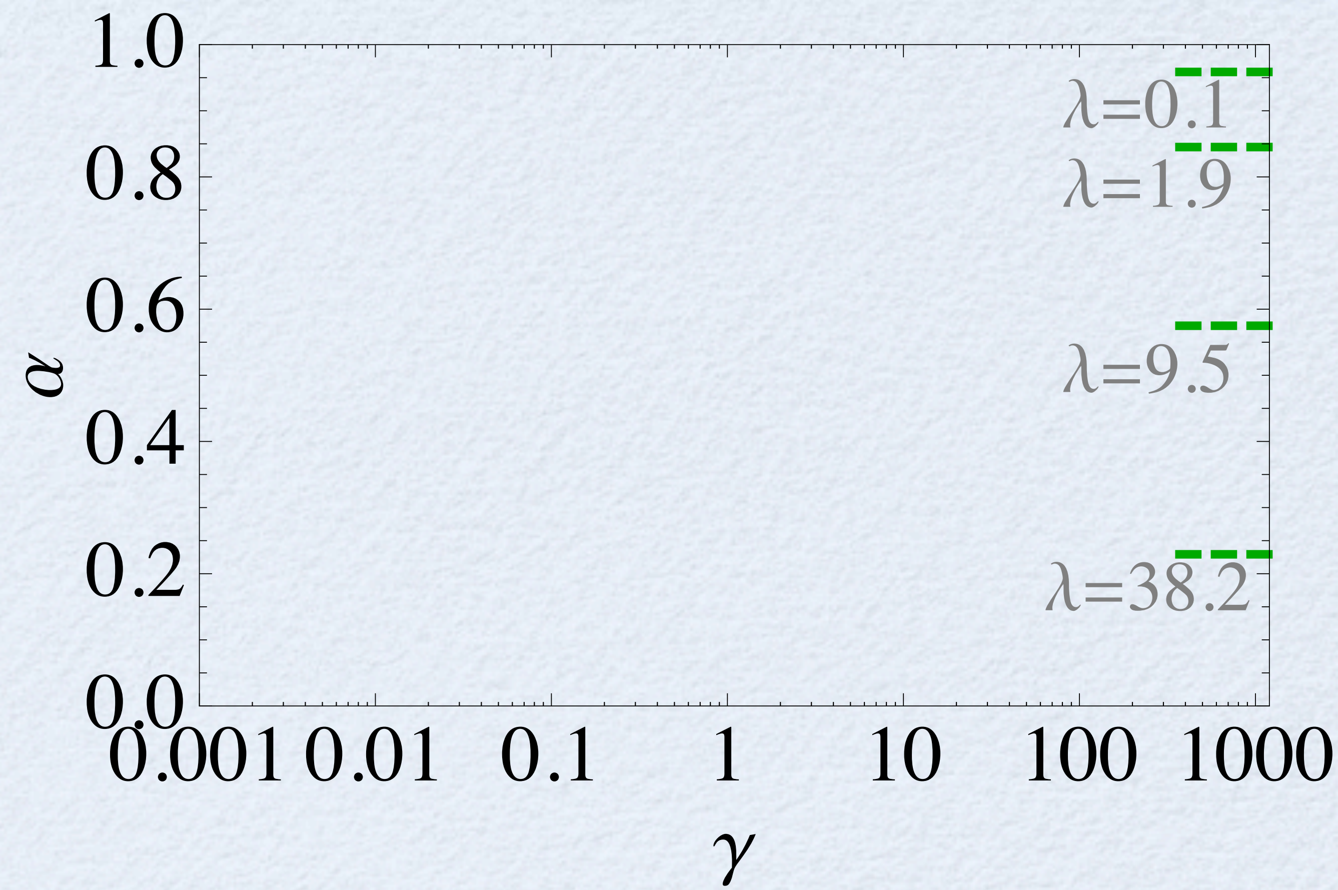
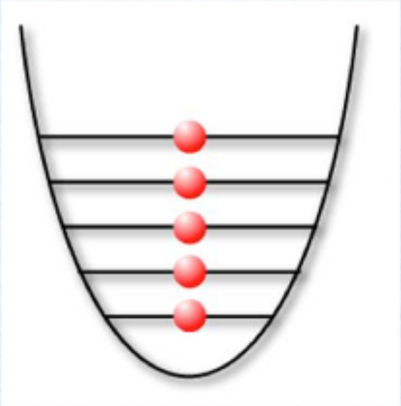
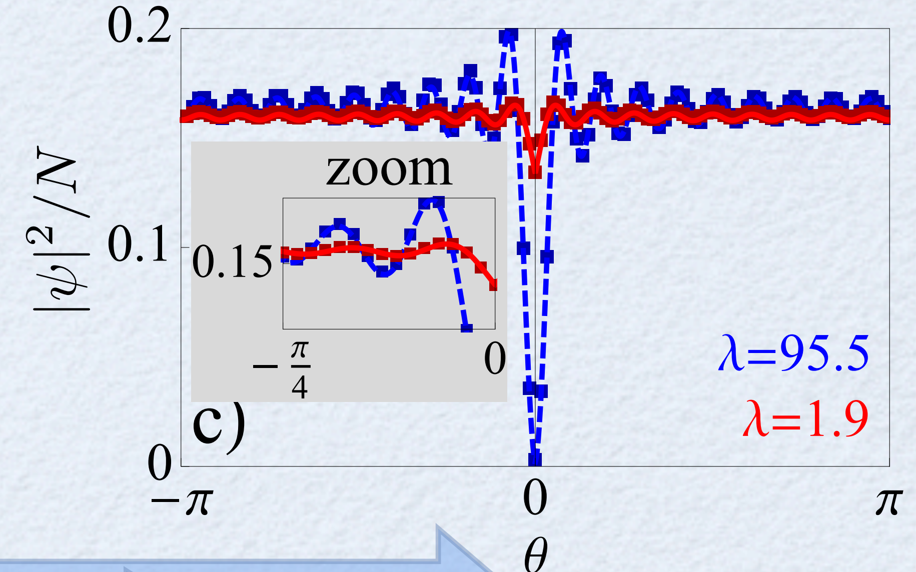
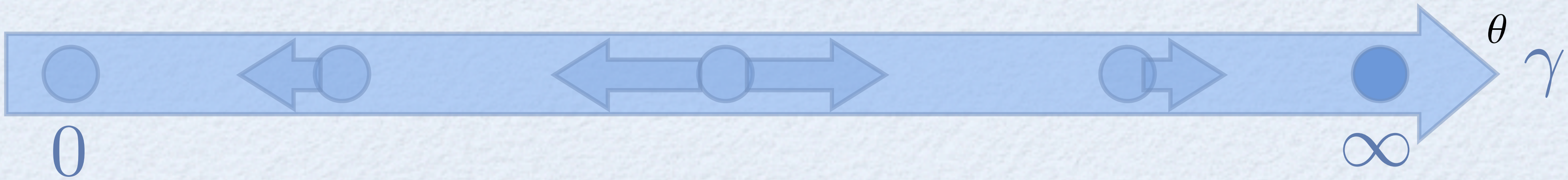


- ✓ ideal "fermionized" scenario
  => Friedel oscillations

$$E = \sum_{n=0}^{N-1}\varepsilon_n$$







Weakly interacting regime

Problem

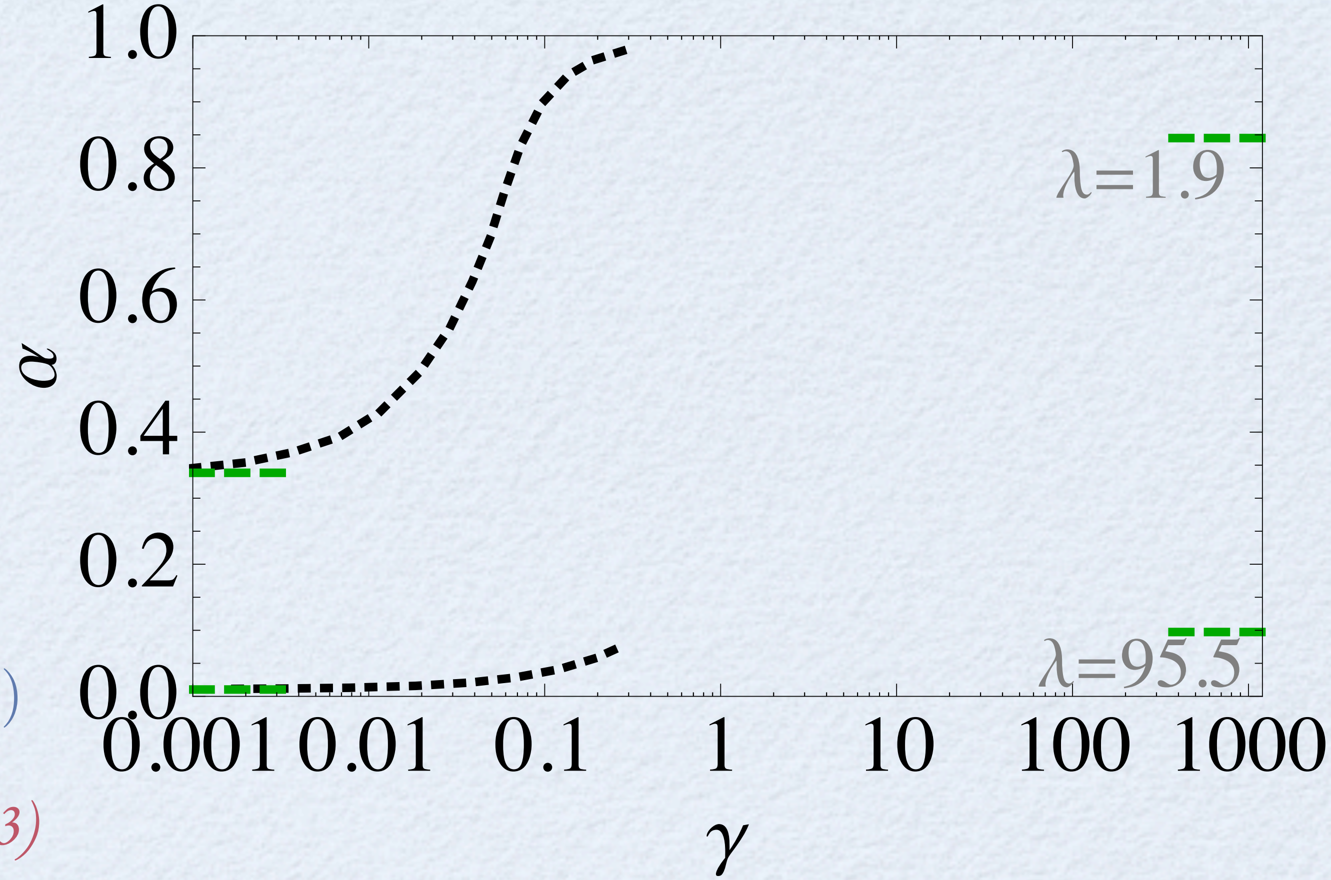
✓ mean-field (~ classical) approach with order parameter

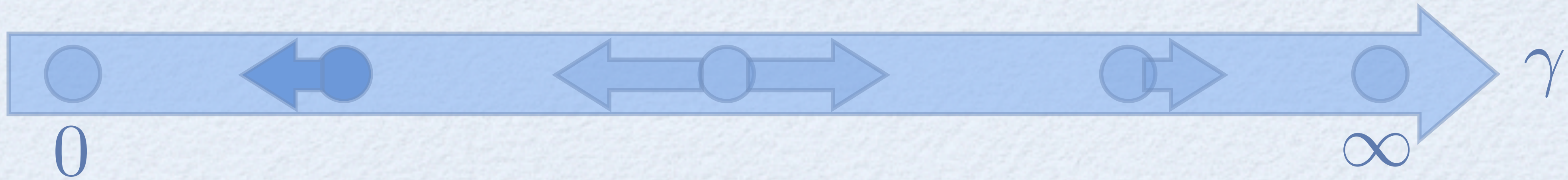
$$\langle\hat{\psi}(\theta)\rangle = |\Psi(\theta)|\, e^{i\phi(\theta)}$$

◉ non-linear Schrödinger equation (Gross-Pitaevski equation)

$$\left[\left(-i\frac{\partial}{\partial\theta} - \Omega\right)^2 + \lambda\delta(\theta) + \tilde{g}|\Psi(\theta)|^2\right]\Psi(\theta) = \mu\Psi(\theta)$$

Pitaevskii & Stringari, Bose-Einstein Cond., Oxford (2003)





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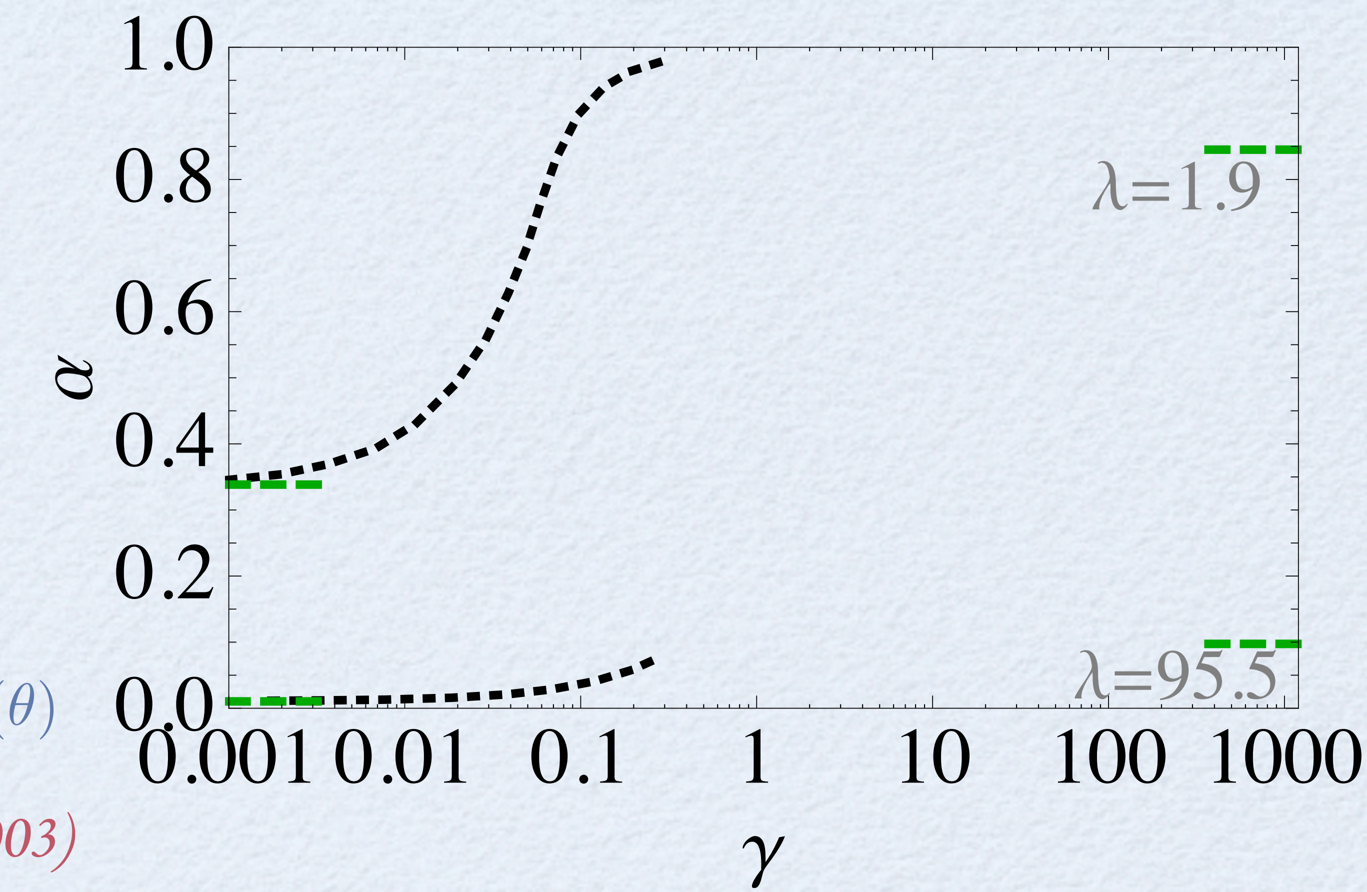
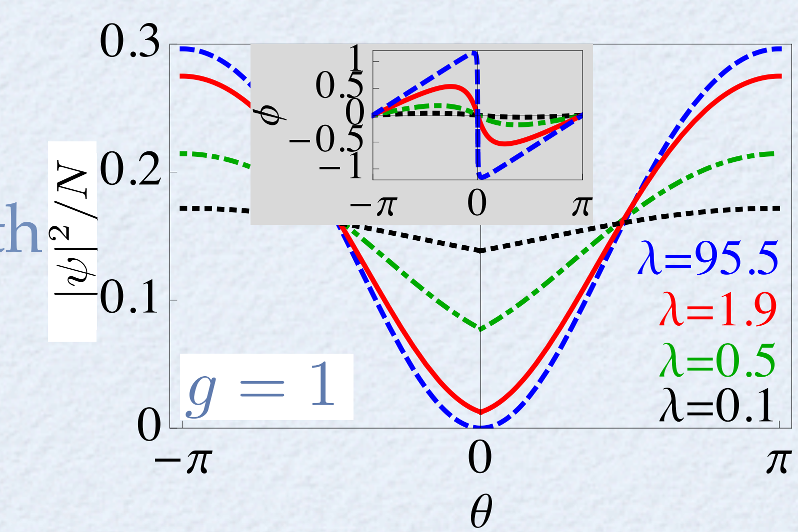
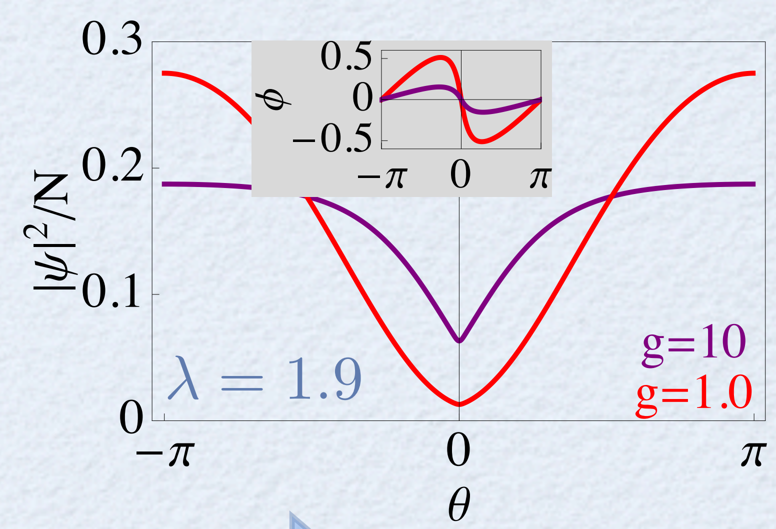
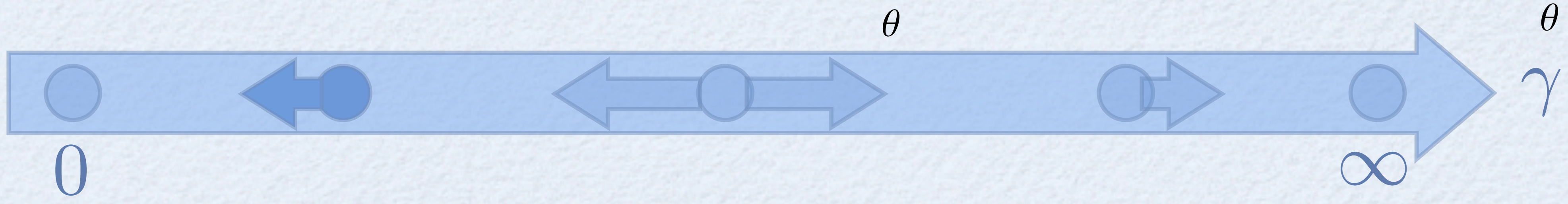


- ✓ soliton get pinned at defect... width depends on healing length

Kanamoto et al., PRL 100, 060401 (2008)







Strongly interacting regime

Problem

✓ 1D breakdown of Fermi Liquid ==> Luttinger liquid description !

$$\omega(k) \simeq \hbar v_s |k|$$

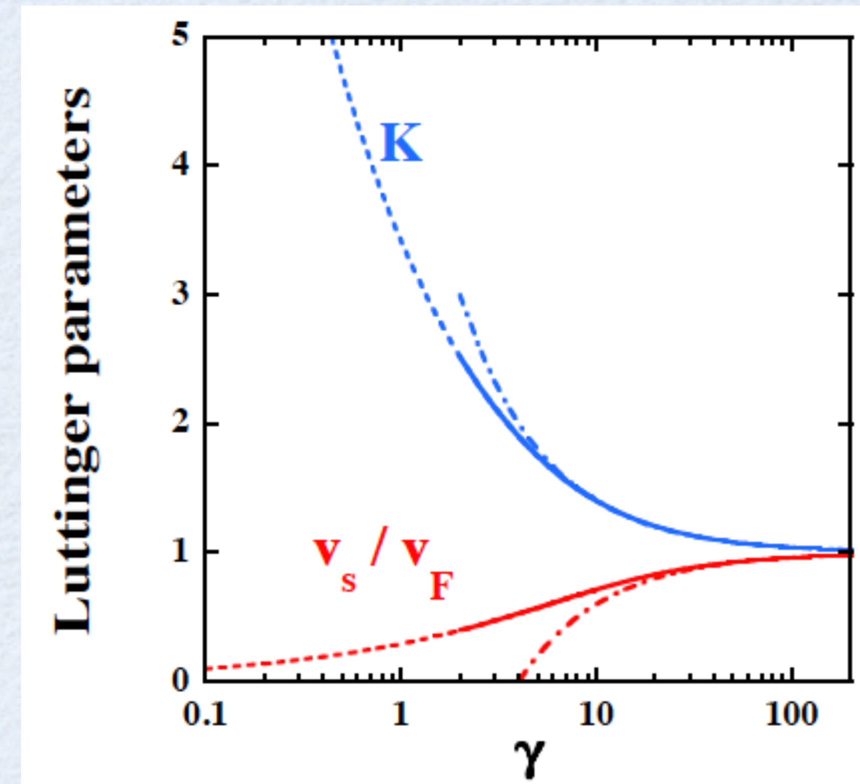
$$n_0 = N/L$$

$$\psi(x) = \sqrt{\rho(x)} e^{i\phi(x)}$$

$$\rho(x) \simeq (n_0 + \partial_x \theta(x)/\pi) \sum_{l \in \mathbb{Z}} e^{2il(\theta(x) + \pi n_0 x)}$$

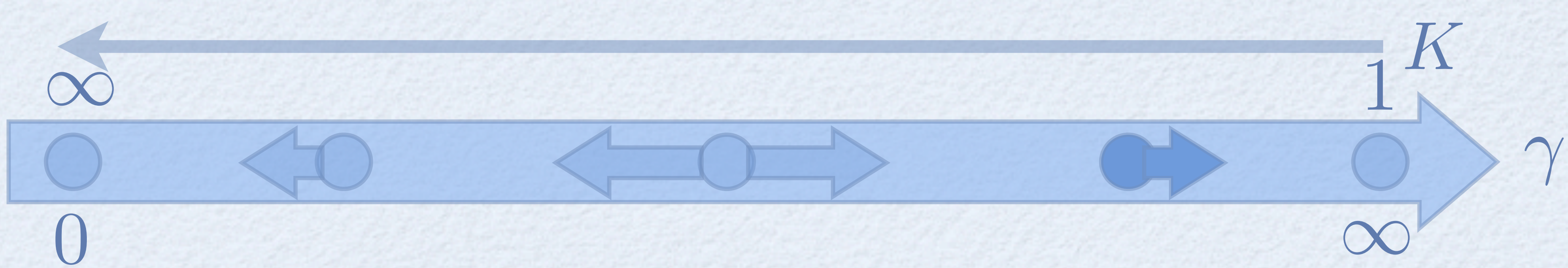
$$[\partial_x \theta(x), \phi(x')] = i\pi\delta(x - x')$$

Cazalilla, J. Phys. B: At. Mol. Opt. Phys. 37, S1 (2004)



✓ presence of gauge field ~ shift in the phase field

$$\mathcal{H}_{LL} = \frac{\hbar v_s}{2\pi} \int_0^L \mathrm{d}x \left[K \left(\partial_x \phi(x) - \frac{2\pi}{L}\Omega \right)^2 + \frac{1}{K} (\partial_x \theta(x))^2 \right]$$



Strongly interacting regime

Problem

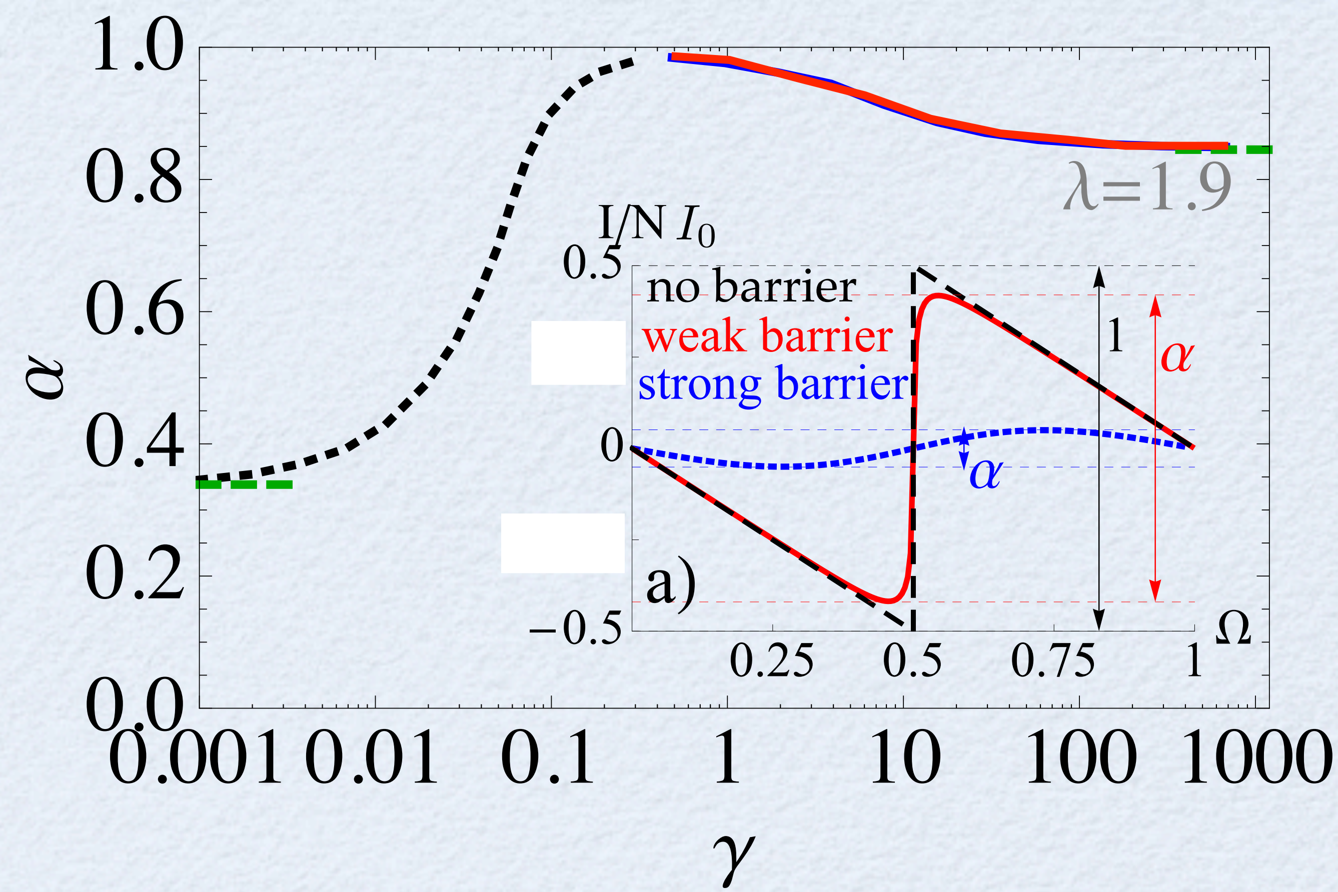
- ✓ effective field theory (no mean-field!)
  => Luttinger liquid

$$\psi(x) = \sqrt{\rho(x)} e^{i\phi(x)}$$

$$\rho(x) \simeq (n_0 + \partial_x \theta(x)/\pi) \sum_{l \in \mathbb{Z}} e^{2il(\theta(x) + \pi n_0 x)}$$

$$[\partial_x \theta(x), \phi(x')] = i\pi\delta(x - x')$$

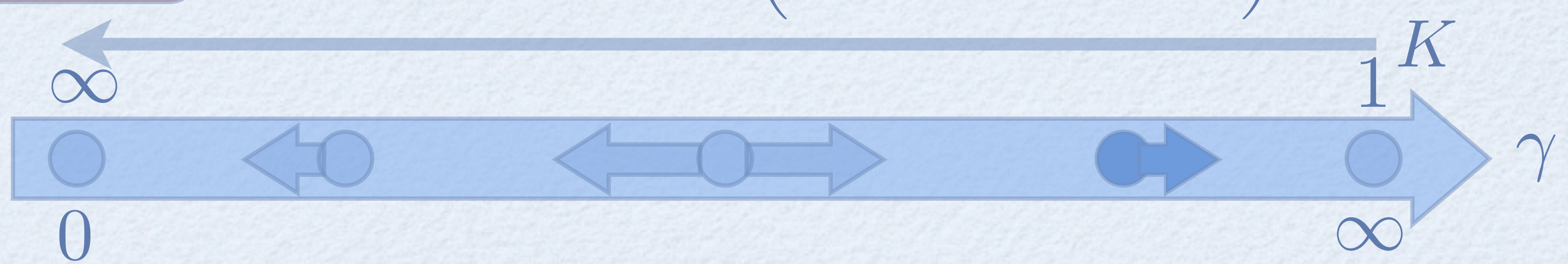
Cazalilla, J. Phys. B: At. Mol. Opt. Phys. 37, S1 (2004)



- ✓ weak barrier ~ backscattering term

$$\mathcal{H}_{LL} = \frac{\hbar v_s}{2\pi} \int_0^L \mathrm{d}x \left[K \left(\partial_x \phi(x) - \frac{2\pi}{L}\Omega \right)^2 + \frac{1}{K} (\partial_x \theta(x))^2 \right] + 2U_0 n_0 \cos[2\theta(0)]$$

$$\lambda_{\text{eff}} = \lambda (d/L)^K \qquad \delta E(0.5 + \delta\Omega) \propto \left(\delta\Omega^2 - \sqrt{\delta\Omega^2 + \lambda_{\text{eff}}^2} \right)$$



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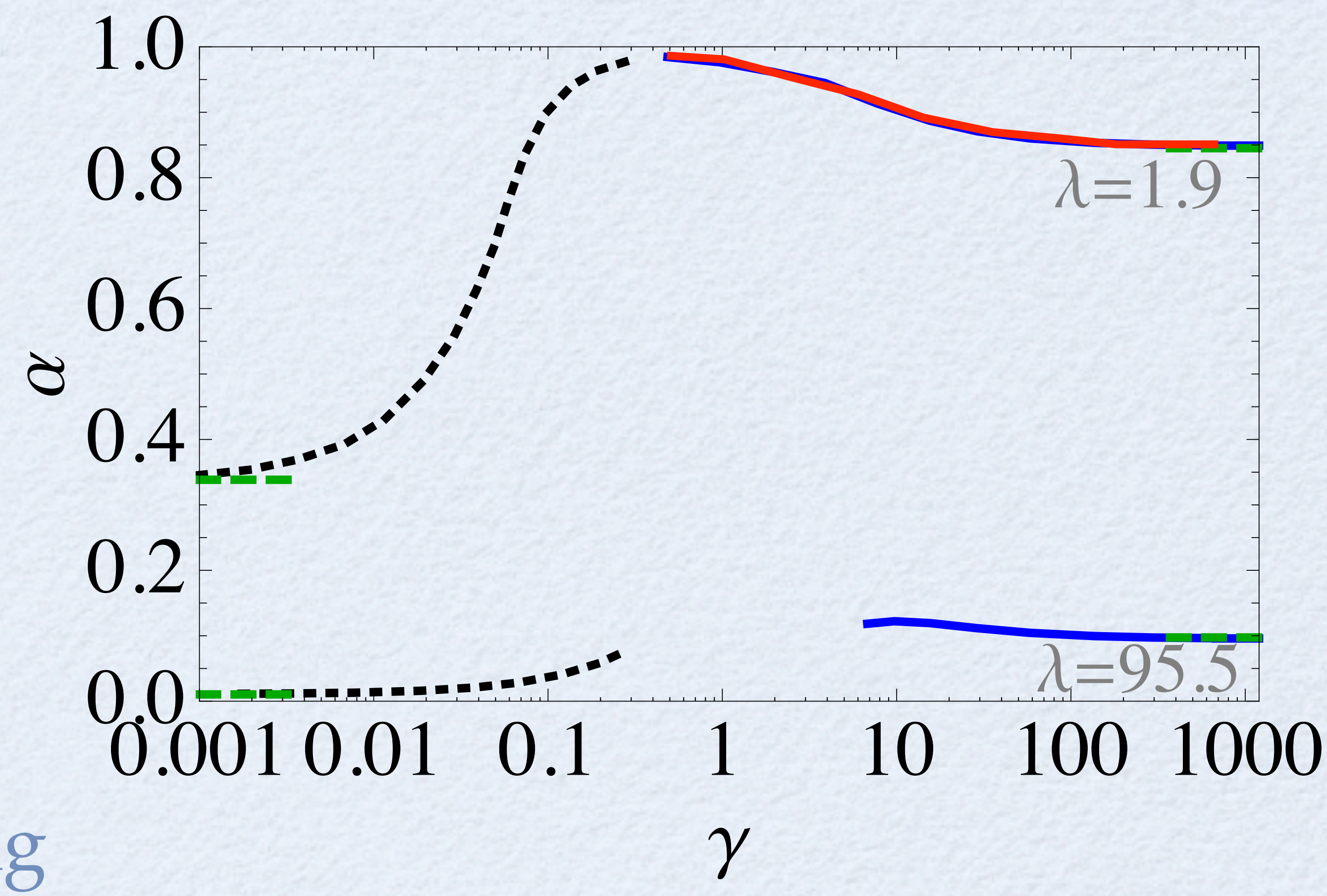
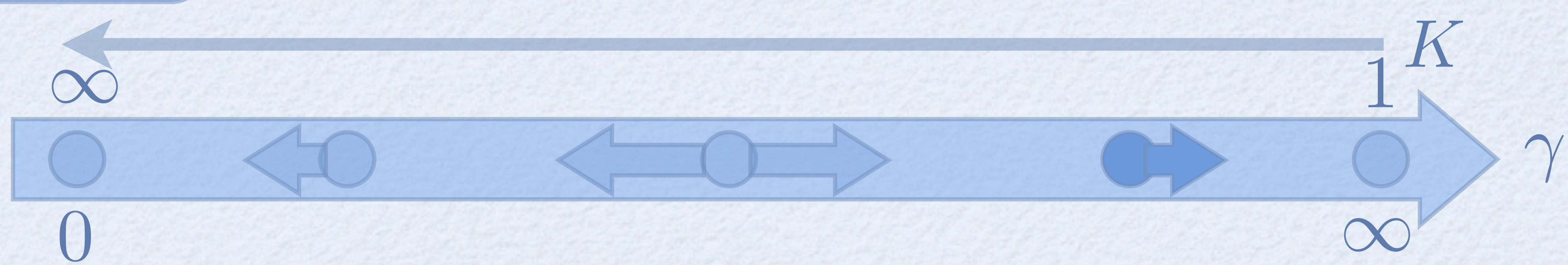


✓ strong barrier ~ weak link tunnelling

$$\mathcal{H}_{LL} = \frac{\hbar v_s}{2\pi}\int_0^L \mathrm{d}x \left[K\left(\partial_x\phi(x) - \frac{2\pi}{L}\Omega\right)^2 + \frac{1}{K}(\partial_x\theta(x))^2\right] - 2\,t\,n_0\cos[\phi(L)-\phi(0)+2\pi\Omega]$$

$$t_{\mathrm{eff}} = t(d/L)^{1/K} \qquad t = f(K)\,\lambda^{-K} \qquad E(\Omega) = -2\,t_{eff}\,n_0\cos(2\pi\Omega)$$

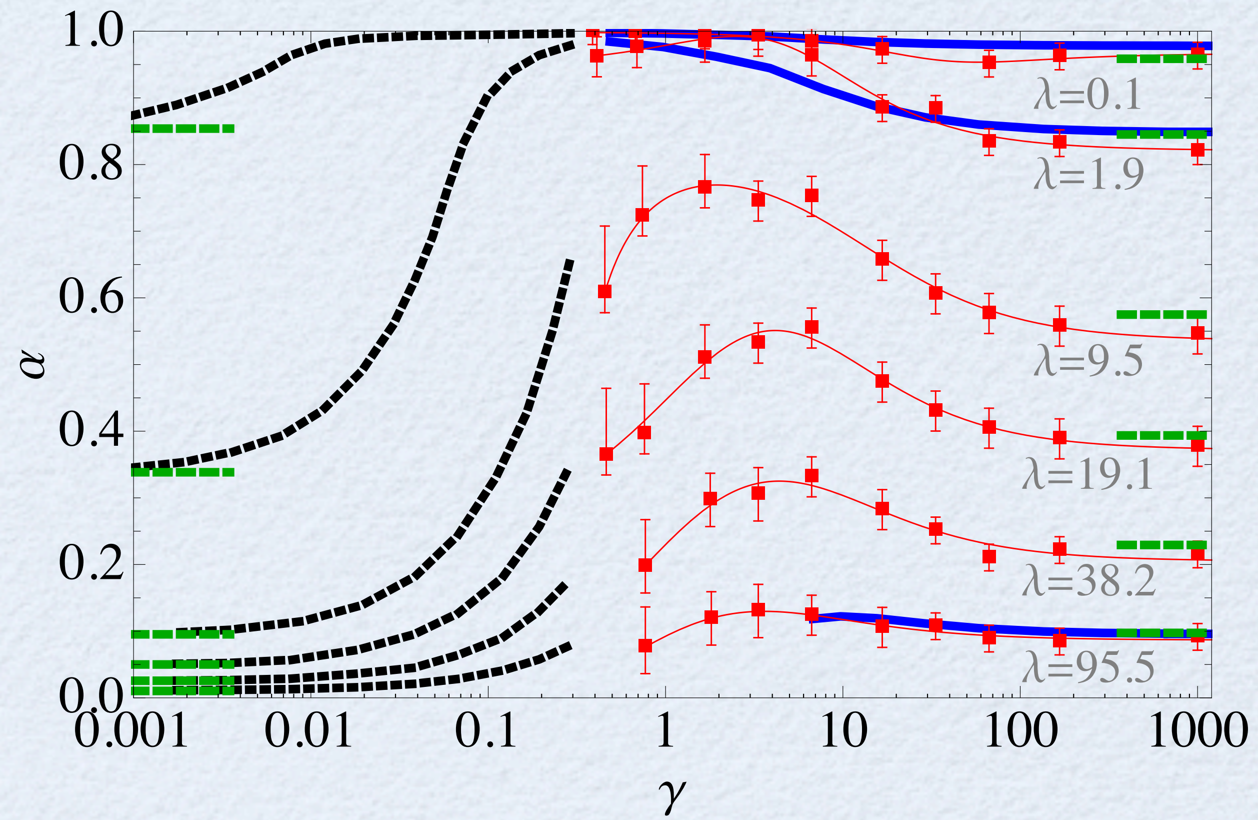


Scanning through diverse regimes: MPS

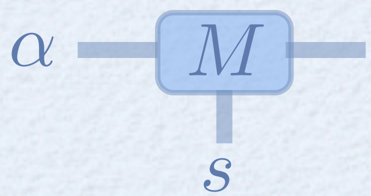
Problem

- lattice discretization (@ low filling): Bose-Hubbard + Peierls phase

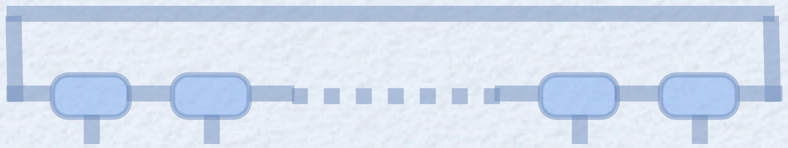
$$\mathcal{H}_{\rm lat} = -t_{\rm BH} \sum_{j=1}^{N_s} \left(e^{-\frac{2\pi i \Omega}{N_s}} b_j^\dagger b_{j+1} + {\rm H.c.} \right) + \frac{U_{\rm BH}}{2} \sum_{j=1}^{N_s} n_j (n_j - 1) + \sum_j (\lambda_{\rm BH}\, \delta_{j,1} n_j - \mu\, n_j)$$



✓ matrix product state variational ansatz (MPS/DMRG)



$M^s_{\alpha,\beta}$



$\psi_{s_1 \dots s_L} = {\rm Tr}\left[M^{s_1}_{[1]} \dots M^{s_L}_{[L]} \right]$

- not so trivial & stable with PBC's … but some tricks help :)

Verstraete, Porras & Cirac, PRL. 93, 227205 (2004)
Schollwock, Ann. Phys. 326, 96 (2011)
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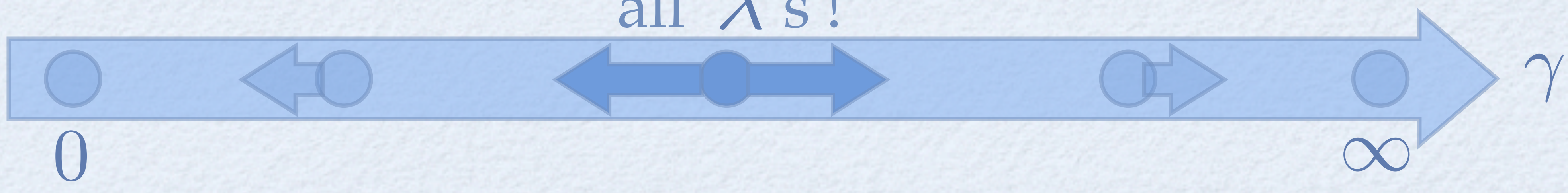
all λ's !



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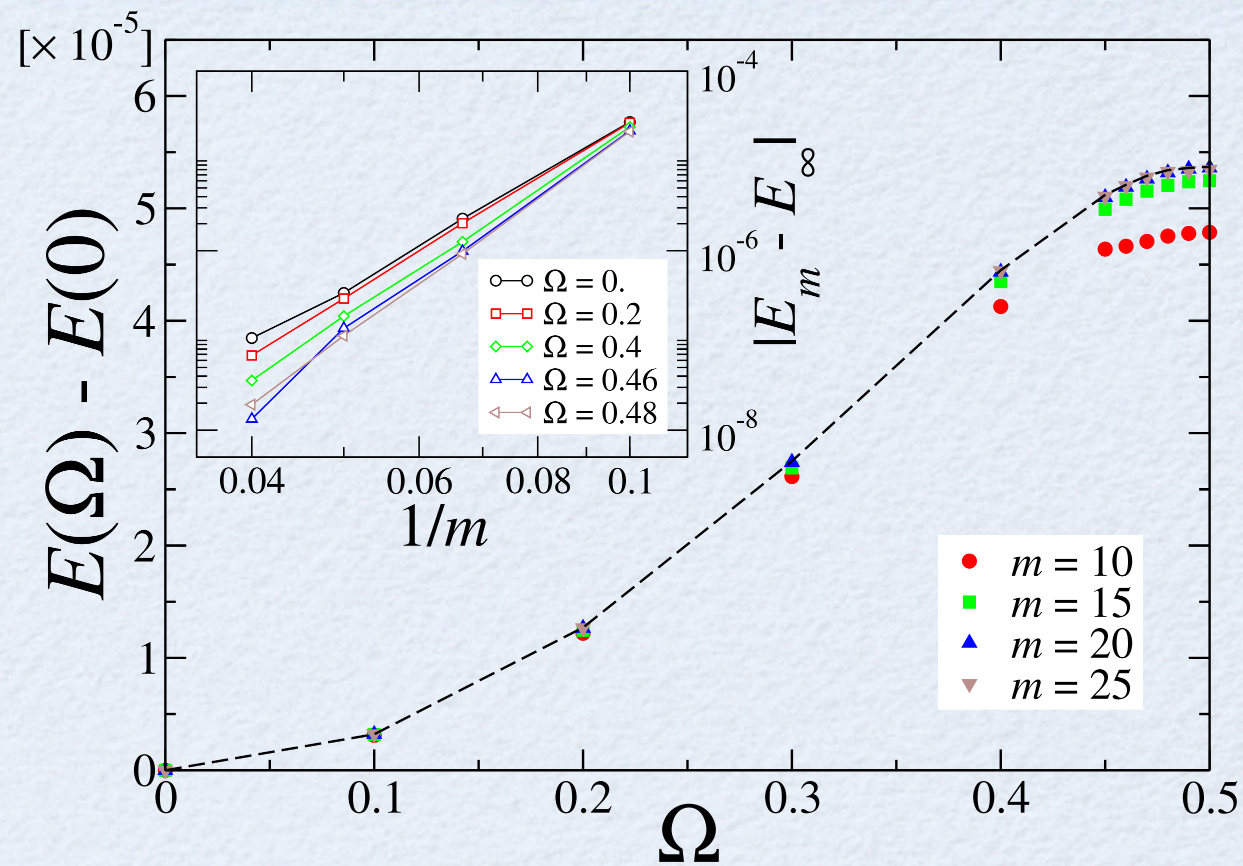

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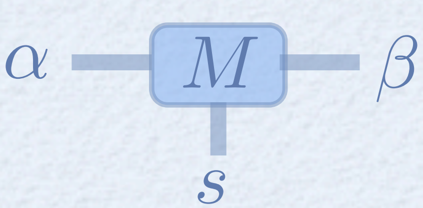

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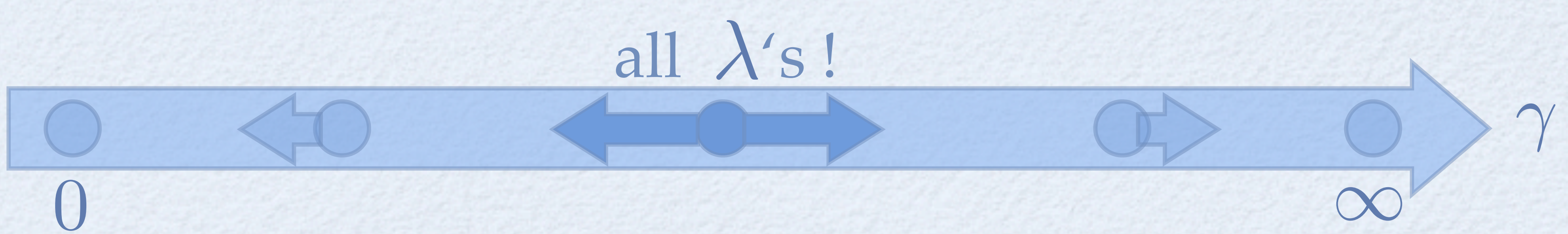
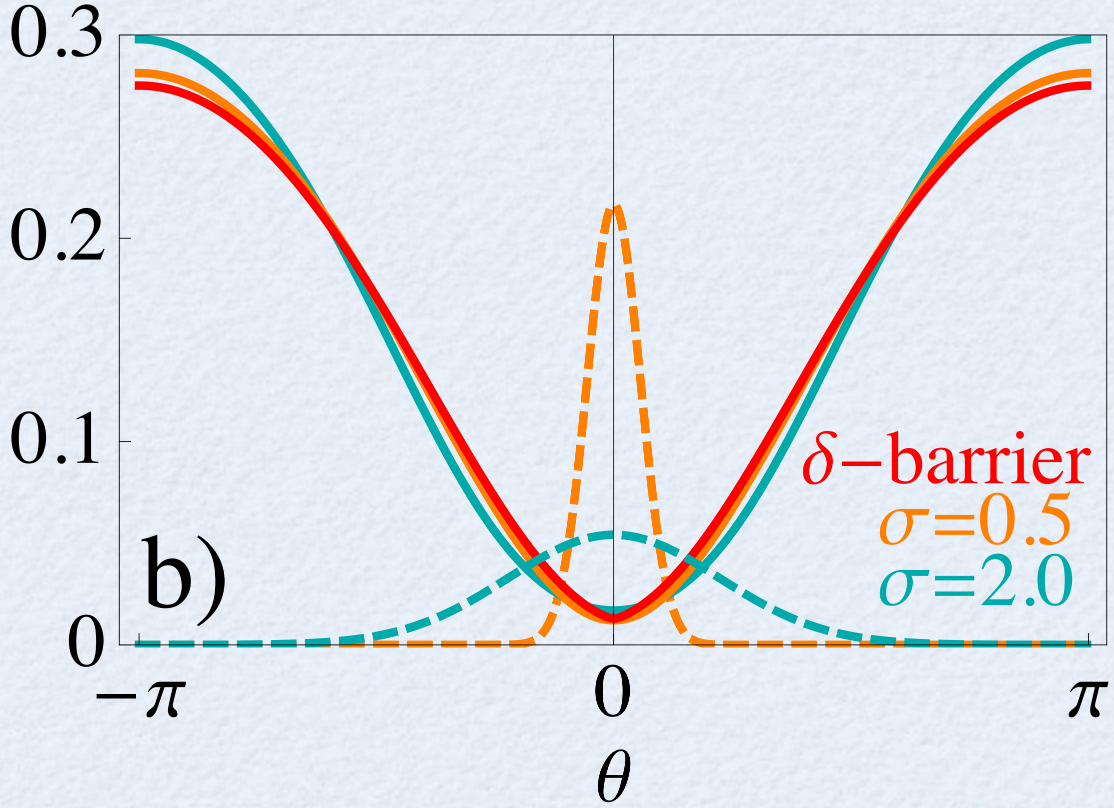
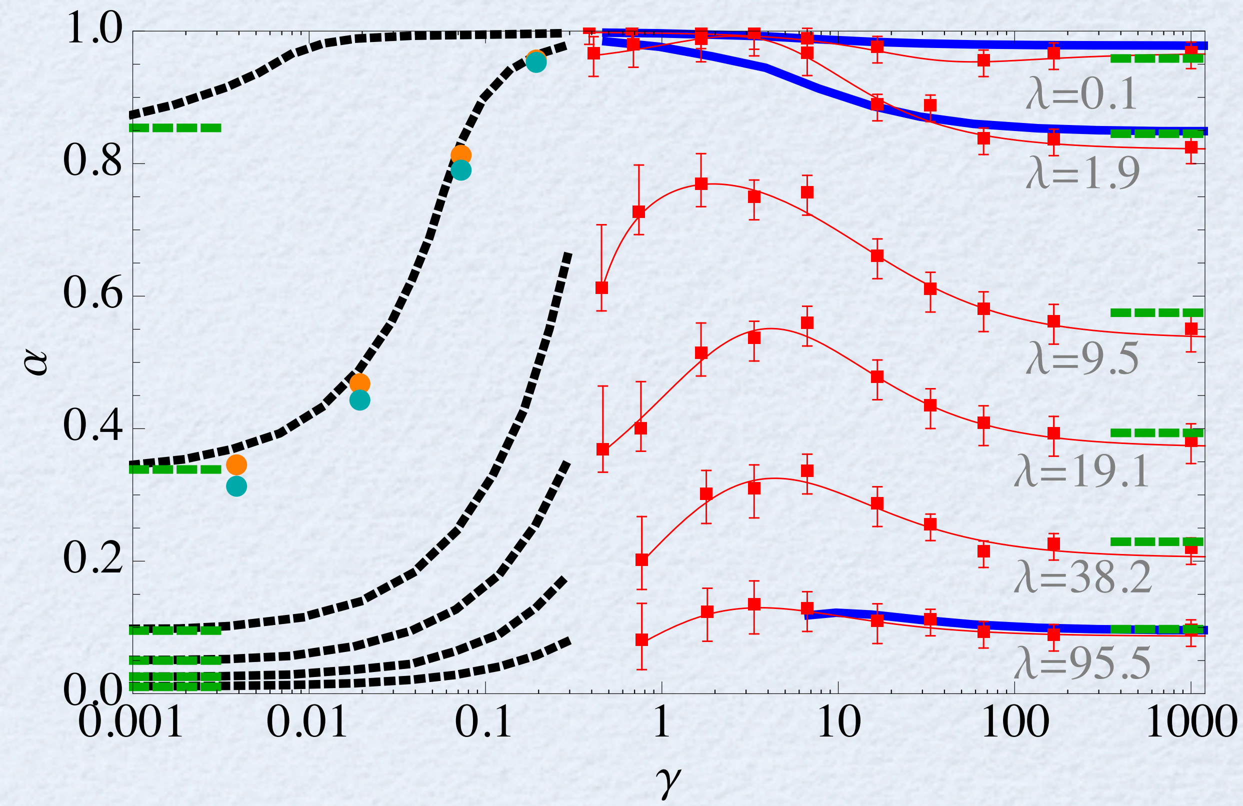


Vicinity to experiments

Problem





✓ gaussian barriers (closer to experiments) only weakly affect results !

Vicinity to experiments

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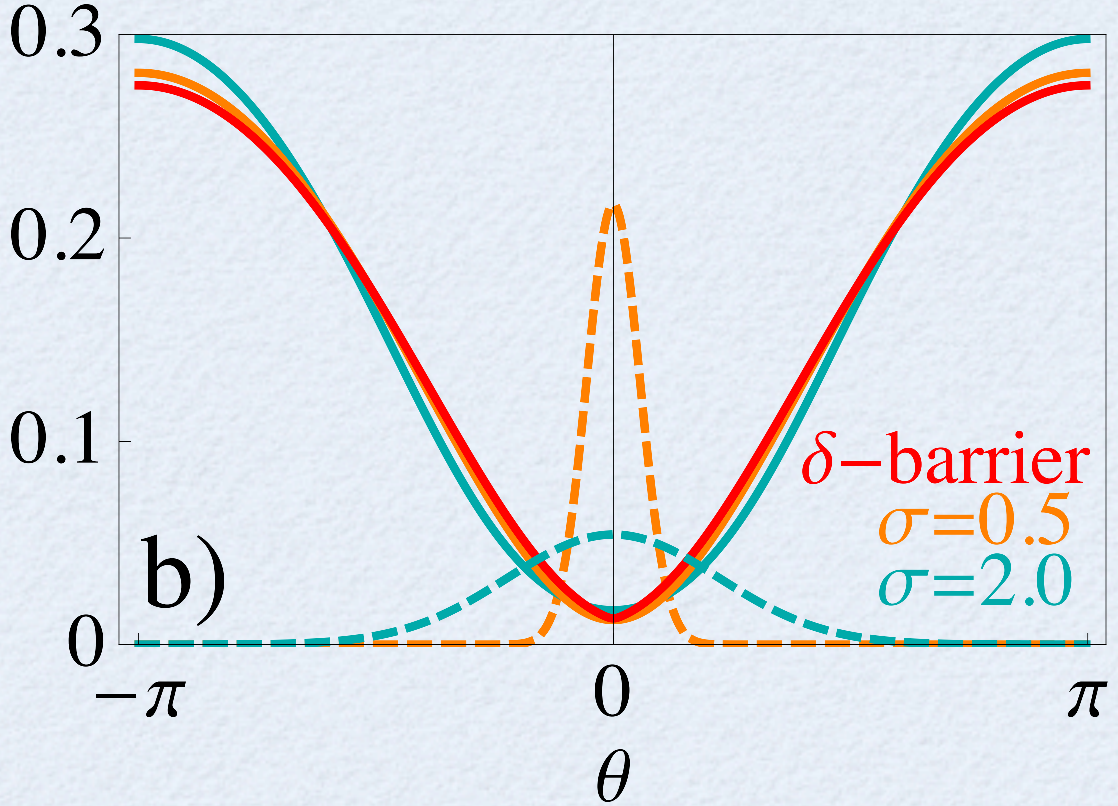

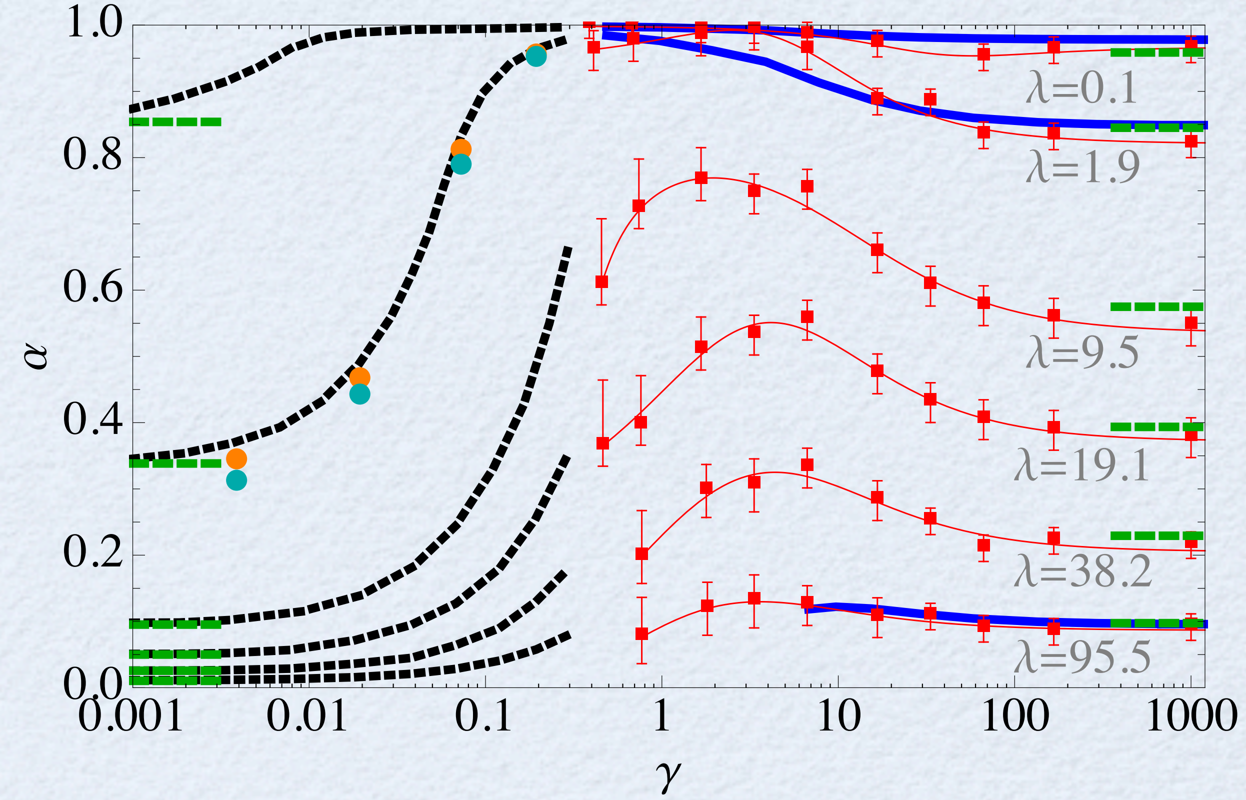

✔ gaussian barriers (closer to experiments) only weakly affect results !

✔ further smearing by thermal fluctuations above $K_{\rm B}T \simeq NE_0 = \dfrac{\pi\hbar^2 n_0}{MR}$

$n_0 \simeq 0.15 \quad R \simeq 5\mu m \quad {}^{87}\mathrm{Rb} \qquad K_{\rm B}T \simeq 550\mathrm{Hz} \simeq 25\mathrm{nK}$

not dramatic but should be taken into account in further studies

Take-Home message

Conclusion

✳ MF regime, i.e. low $\gamma = (gm)/(\hbar^2 n)$:
interaction ↗ ==> barrier effect ↘
(shorter density-density healing length …)

✳ LL regime, i.e. large $\gamma = (gm)/(\hbar^2 n)$:
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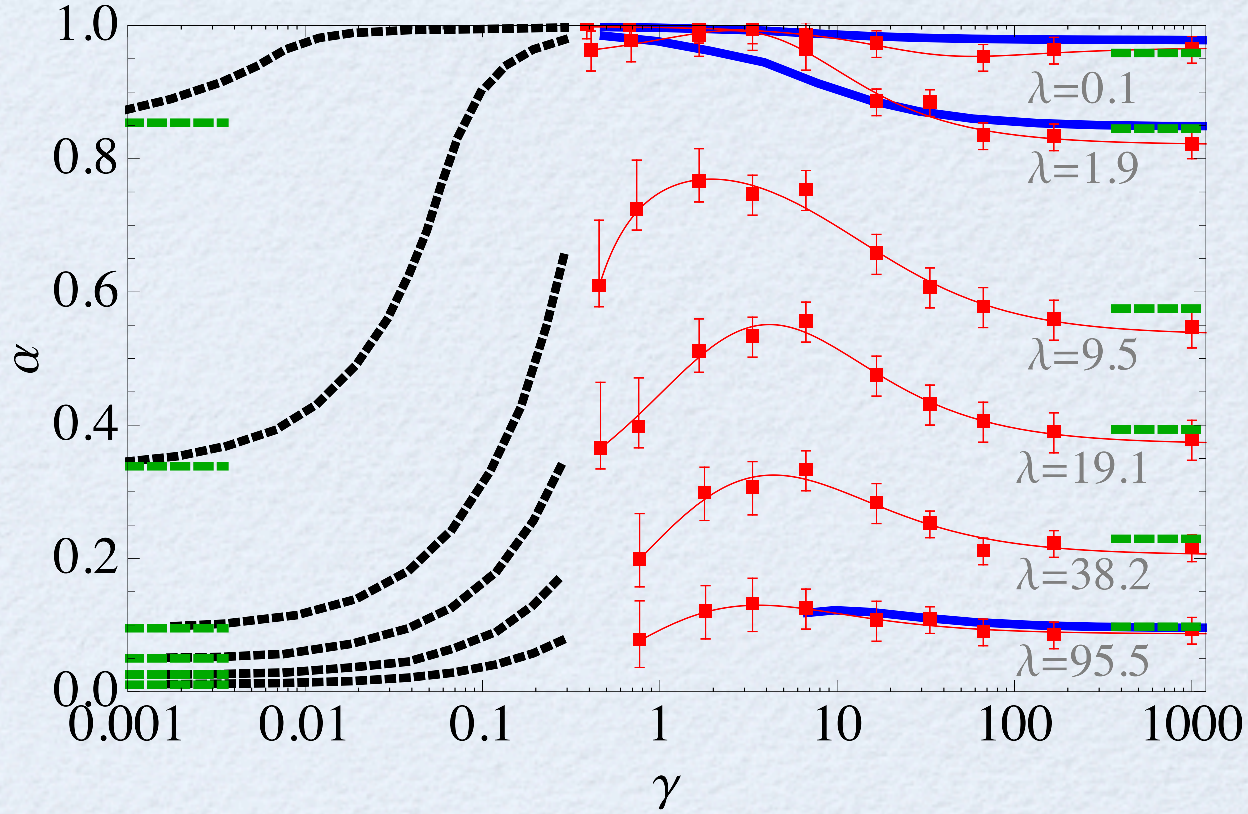

❖existence of an optimal regime where the defects are less influential !

❖ need to choose extremals for an effective quantum state manipulation !

Take-Home message

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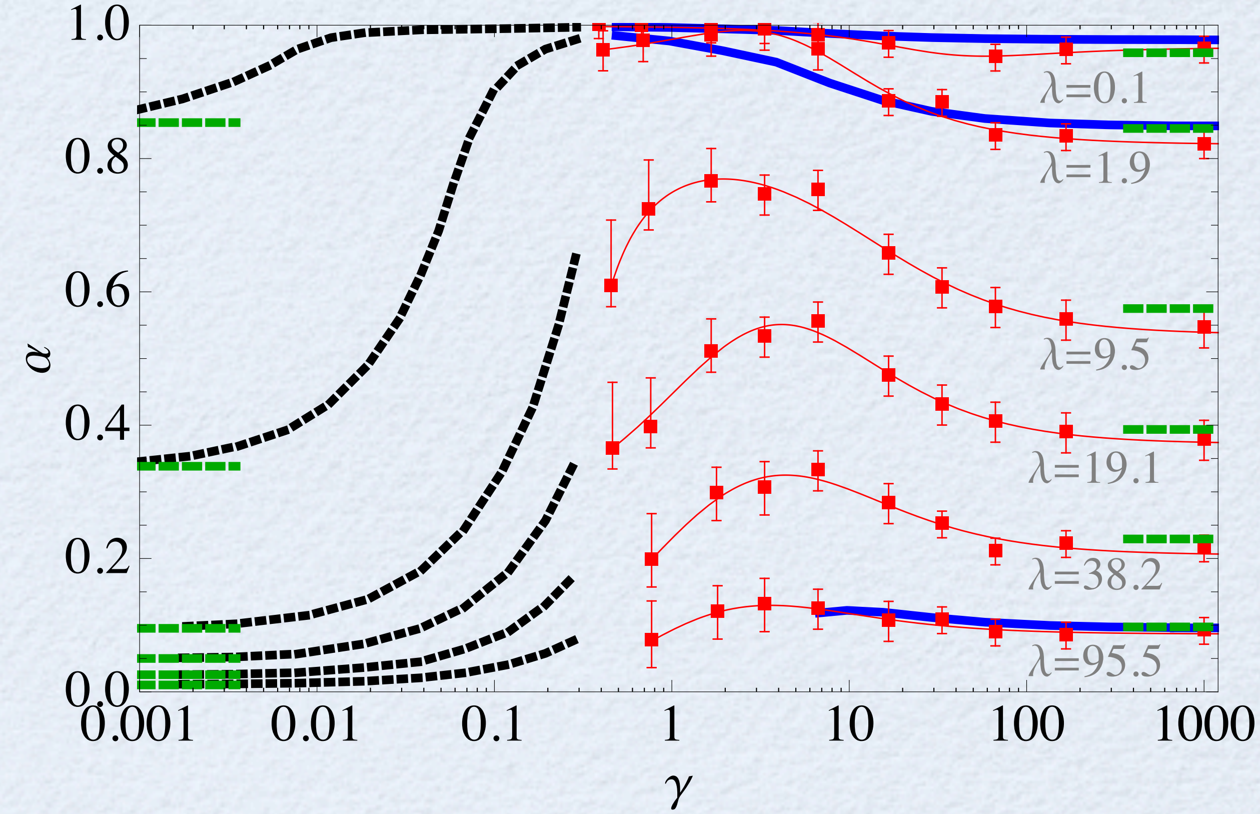
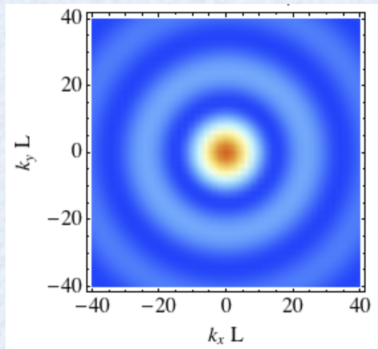


❖existence of an optimal regime where the defects are less influential !

❖ need to choose extremals for an effective quantum state manipulation !

- effects visible in time-of-flight momentum distributions (in progress)

$$n(\mathbf{k}) = \int \mathrm{d}\mathbf{x} \int \mathrm{d}\mathbf{x}' \; e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \rho_1(\mathbf{x}, \mathbf{x}')$$



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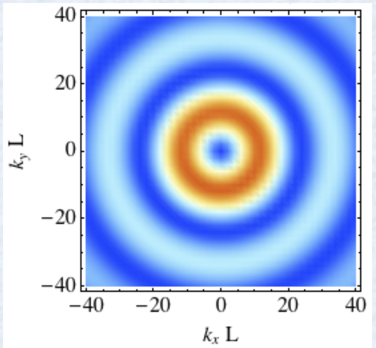

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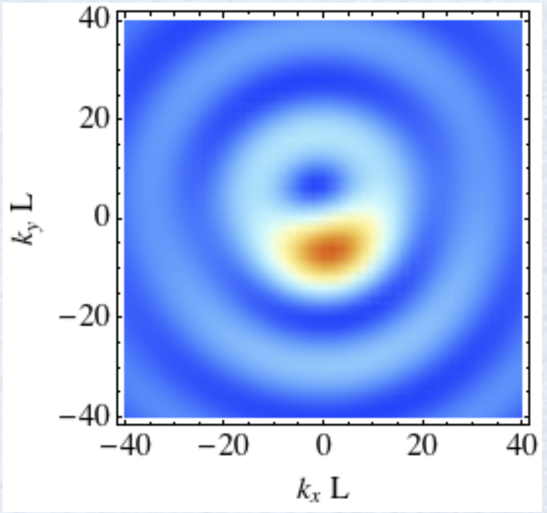

Thanks to ...

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

MATERIALS IN MAINZ SCIENCE

Contact me !!
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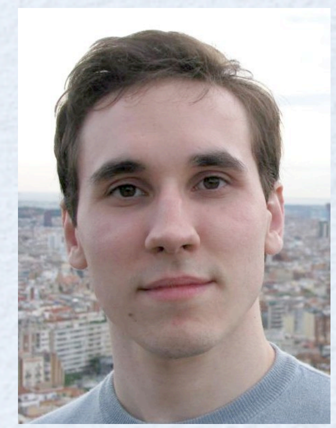
LPMMC, Grenoble, FR



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Davide Rossini
SNS, Pisa, IT

... all of you for your attention !

M.Cominotti, D. Rossini, M. Rizzi, F. Hekking, A. Minguzzi, arXiv:1310.0382