Effects of curvature in low dimensional ferromagnetic nanosystems

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Outline

1. Introduction or is the game worth the candle? Possible research directions.

2. General approach for curvilinear wires.

3. Applications for 1D systems:

- Magnon spectrum in presence of curvature and torsion;
- Domain wall dynamics in curvilinear wire.

4. General approach for curvilinear shells.

5. Applications for 2D systems:

- Coupling of chiralities in Spin and Physical Spaces (Möbius Ring, twisted stripe);
- Curvature induced skyrmions. Interaction of skyrmion with curvilinear defects.

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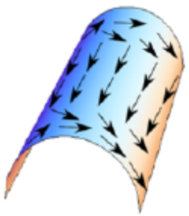
Can a curvature really affect the magnetic system?

Typical interactions in the system



$$E = L\int_{\mathcal{S}}\left[A\sum_{i=x,y,z}(\nabla m_i)^2 + K(\boldsymbol{m}\cdot\boldsymbol{n})^2\right]\mathrm{d}\mathcal{S} - \frac{M_S}{2}\int_V(\boldsymbol{m}\cdot\boldsymbol{H}_d)\mathrm{d}V$$

Is the game worth candles?

Expecting magnitude of the curvature effects $\sim \ell/\mathcal{R} \sim 10^{-3}$

$\ell = \sqrt{\dfrac{A}{|K|}} \lesssim 10\,\mathrm{nm}$ (characteristic length in magnetic system)

$\mathcal{R} \gtrsim 10^3\,\mathrm{nm}$ (curvature radius)

Possible directions in the curvilinear magnetism

Topologically induced patterns

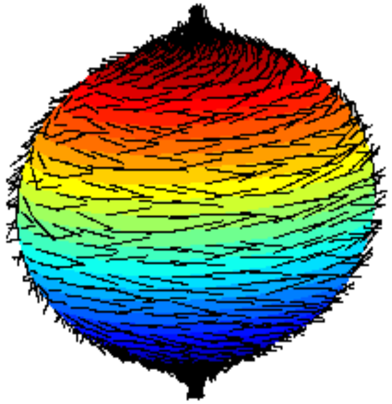
Chirality symmetry breaking

Nonlocal effects

Topologically induced patterns

Equilibrium magnetization states of films with strong anisotropy $|K| \gg A/\mathcal{R}^2$ are determined by the geometry

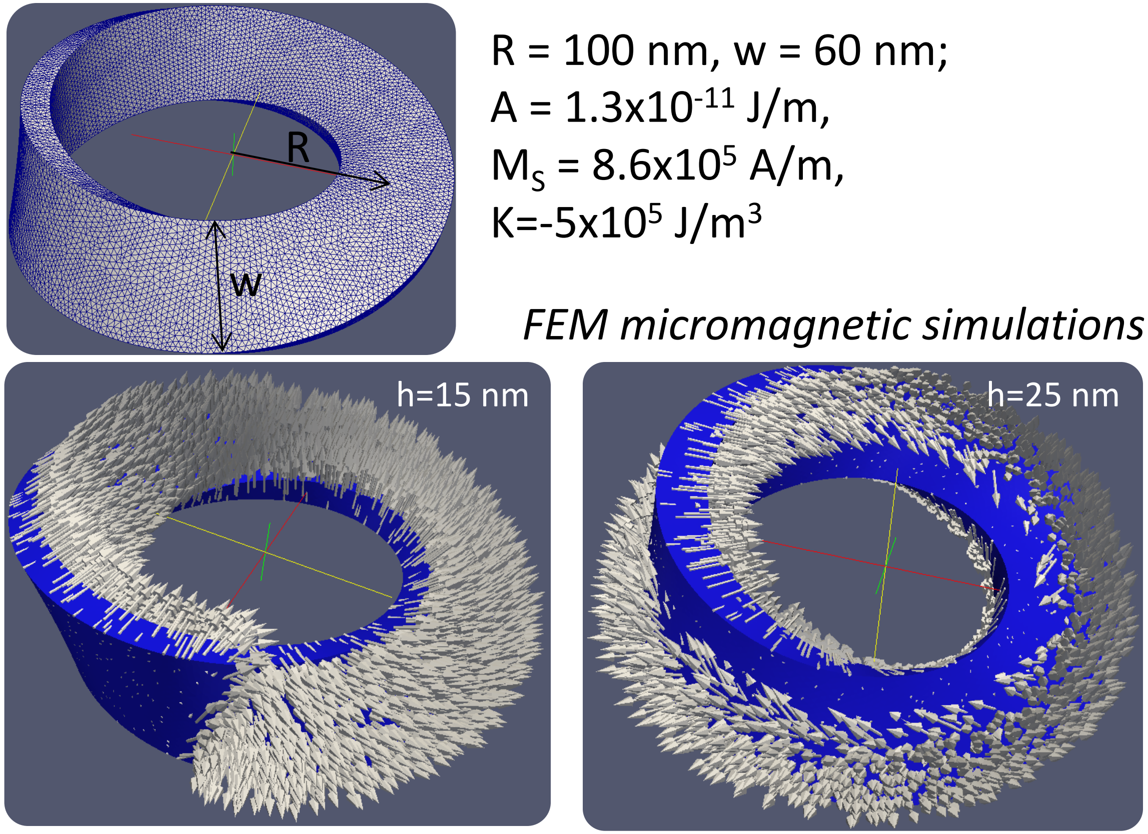
Easy surface anisotropy (K>0)



Vortices on a sphere are induced due to the **Hairy ball theorem**

Easy normal anisotropy (K<0)

Domain wall is induced due to the Möbius topology

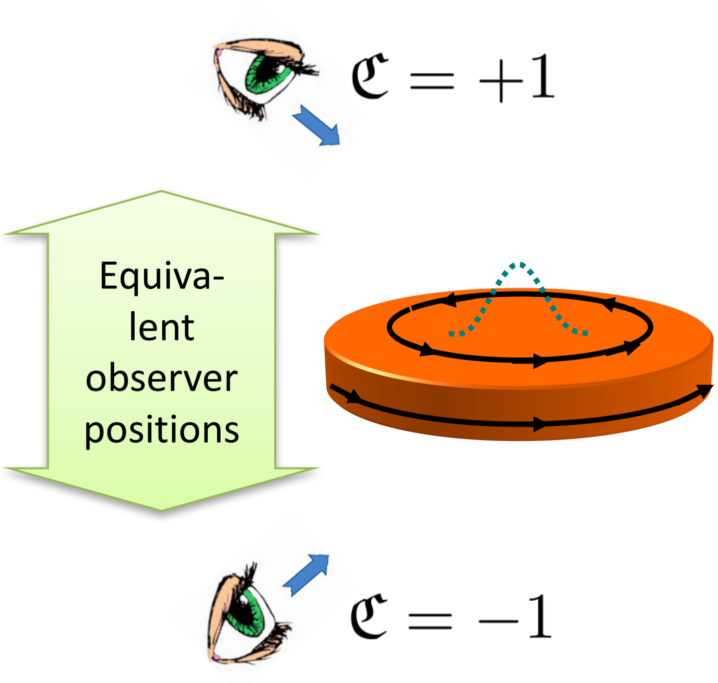


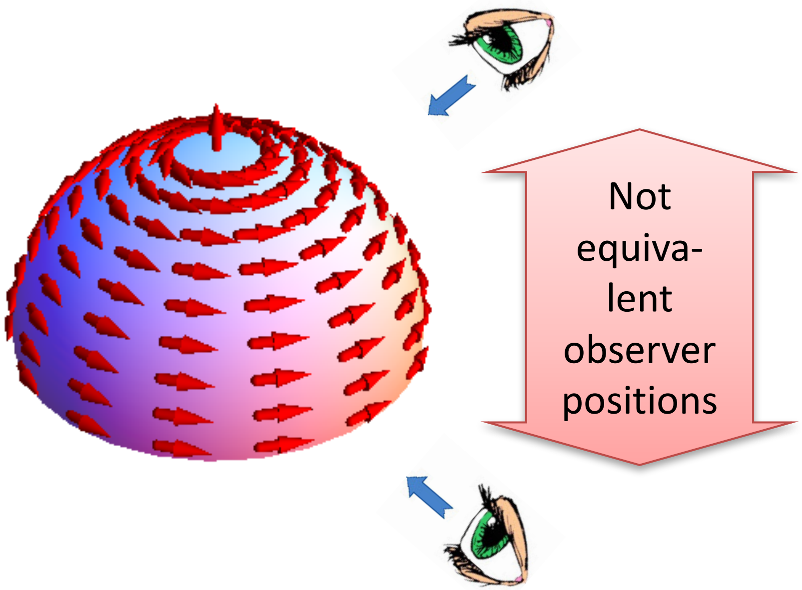
R = 100 nm, w = 60 nm;
$A = 1.3\times10^{-11}$ J/m,
$M_S = 8.6\times10^{5}$ A/m,
$K=-5\times10^{5}$ J/m^3

FEM micromagnetic simulations

Chirality symmetry breaking

The curvature brakes the chirality symmetry



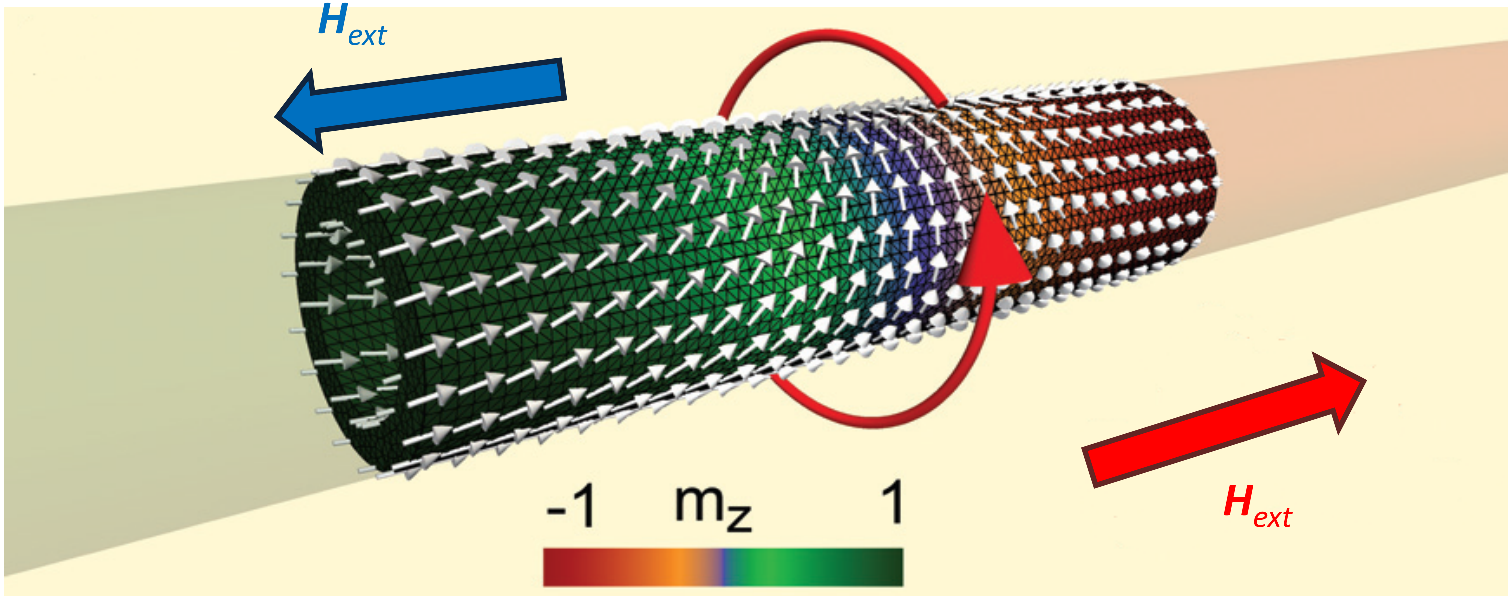


(Selected chirality on a sphere?)

In planar films the chirality *does not figure* in effects.

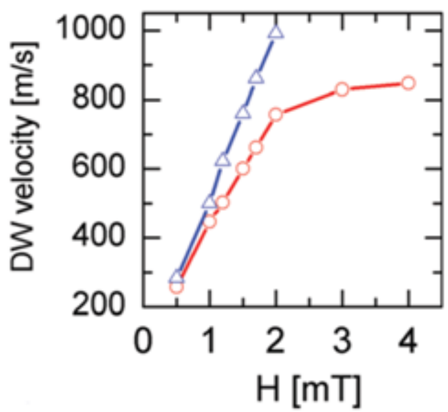
In curvilinear films chirality *does affect* the magnetization behavior.

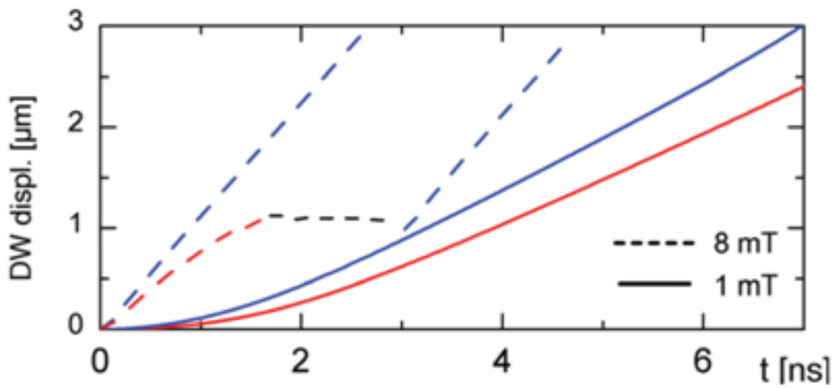
Chirality symmetry breaking in moving vortex DW



Permalloy,
R=60 nm,
h=10 nm

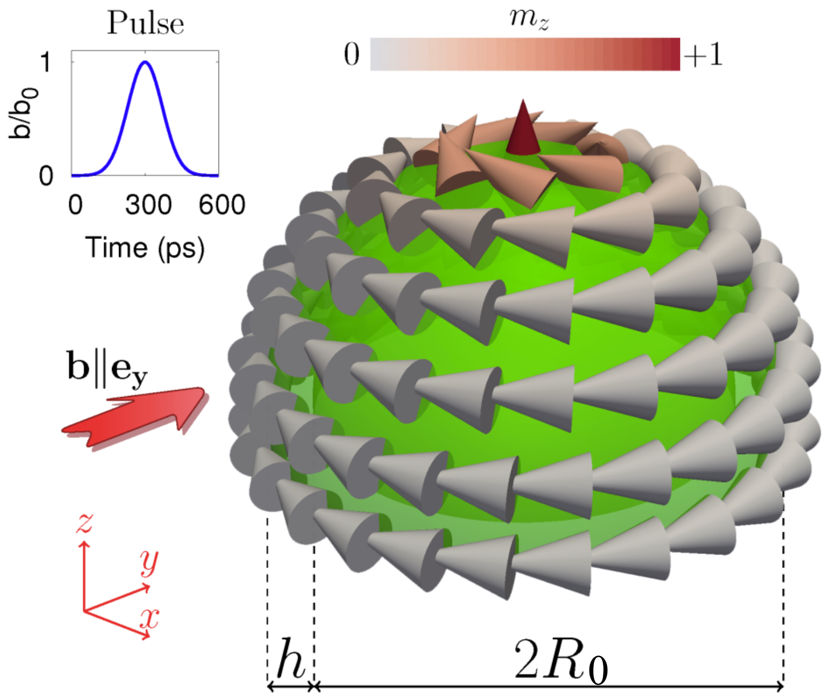
[Appl. Phys. Lett., **100**, 252401, (2012)]
R. Hertel group





[Appl. Phys. Lett., **100**, 072407, (2012)]
P. Landeros group

Chirality symmetry breaking in vortex polarity switching



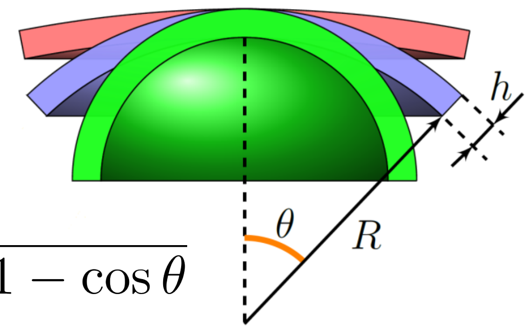
$$\boldsymbol{b} = \boldsymbol{e}_y b_0 \exp\left[-(t-3\tau)^2/\tau^2\right]$$

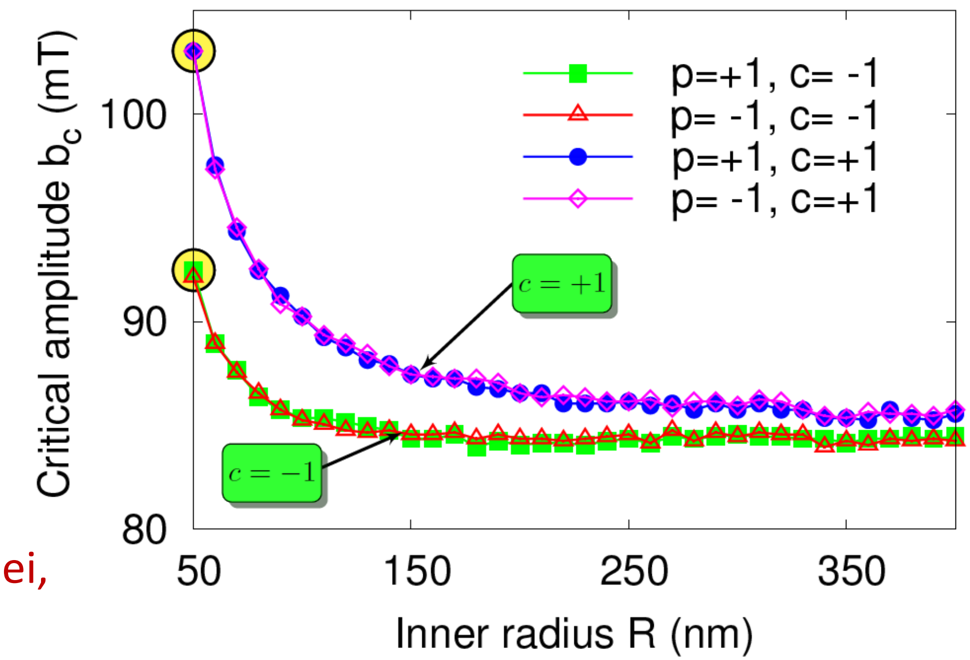
$$\tau = 100 \text{ ps}$$

Volume and thickness are constant

$h = 10$ nm

$$R = R_0/\sqrt{1-\cos\theta}$$



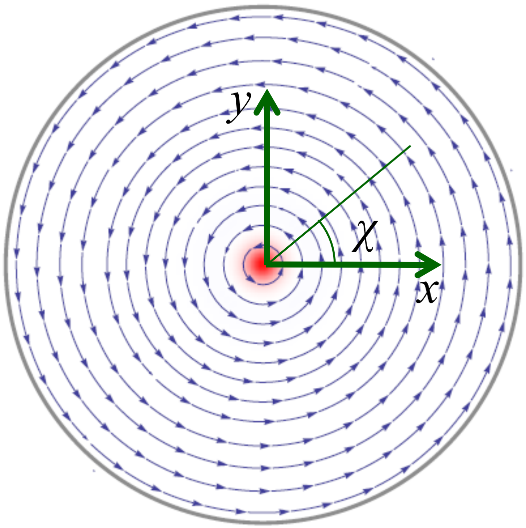


[M. Sloika, V. Kravchuk, D. Sheka, Yu. Gaididei, Appl. Phys. Lett., **104**, 252403 (2014)]

Nonlocal effects

Being of small amplitude some effects can be spatially nonlocal.

Vortex on a planar disk

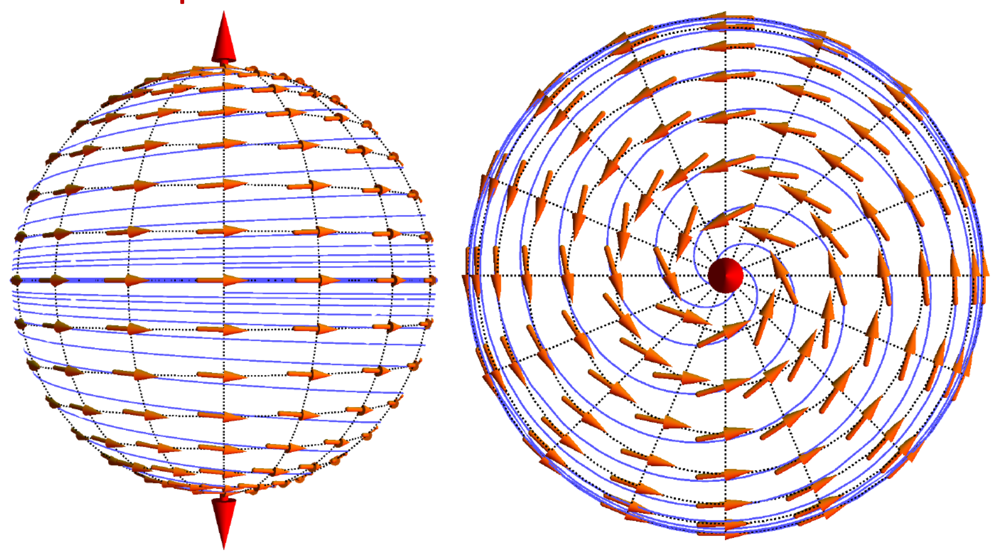


$$\varphi = \chi + \mathfrak{C}\frac{\pi}{2}$$

$\mathfrak{C} = \pm 1$ - chirality,

$$\tan\varphi = m_y/m_x$$

Vortex on a sphere



$$\varphi = \chi + \mathfrak{C}\frac{\pi}{2}\left(1 - p\frac{\ell}{\mathcal{R}}\xi \ln\tan\frac{\vartheta}{2}\right)$$

p - polarity,

$\mathcal{R}$ - sphere radius,

ϑ - polar angle

[V.P. Kravchuk, et al.,
Phys. Rev. B, **85**, 144433 (2012)]

Can the nonlocal magnetization deformation modify the vortex-vortex coupling on a sphere?

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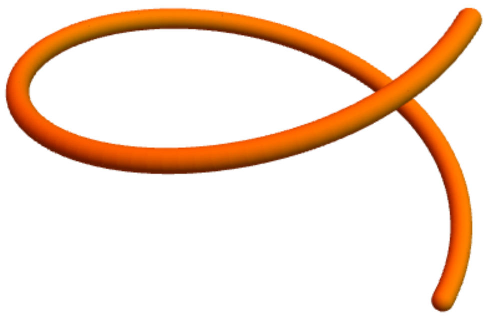
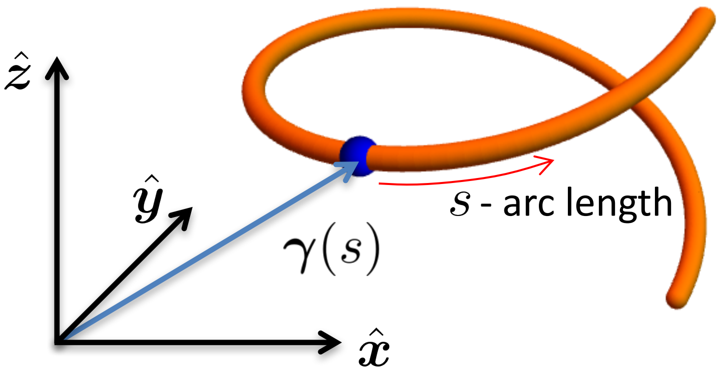
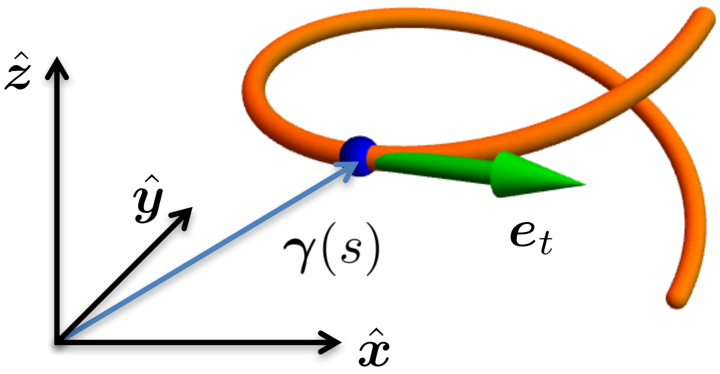
Frenet–Serret basis



Frenet–Serret basis

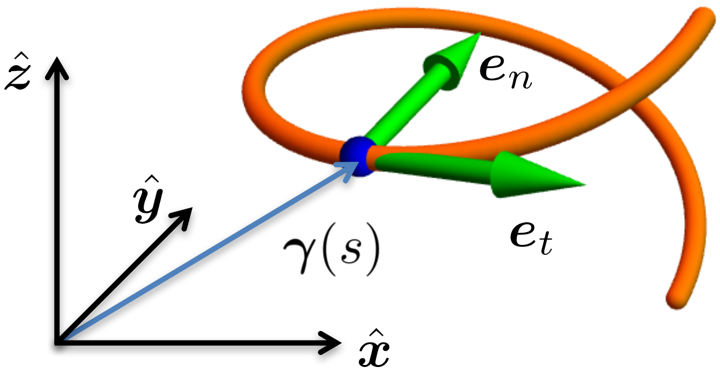


Frenet–Serret basis



$$e_t = \gamma'(s)$$

Frenet–Serret basis

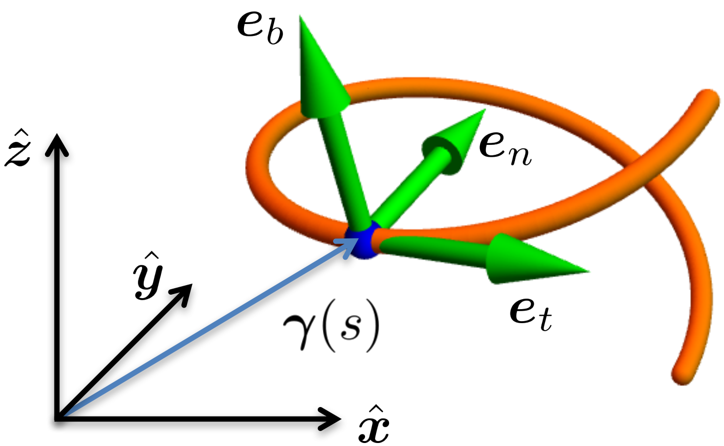


$$\boldsymbol{e}_t = \boldsymbol{\gamma}'(s)$$

$$\boldsymbol{e}_n = \boldsymbol{\gamma}''(s)/|\boldsymbol{\gamma}''(s)|$$

Frenet–Serret basis

$$\boldsymbol{r}(s,\rho,\chi) = \boldsymbol{\gamma}(s) + \rho\cos\chi\,\boldsymbol{e}_n(s) + \rho\sin\chi\,\boldsymbol{e}_b(s)$$



$$\left.\begin{aligned} \boldsymbol{e}_t &= \boldsymbol{\gamma}'(s) \\ \boldsymbol{e}_n &= \boldsymbol{\gamma}''(s)/|\boldsymbol{\gamma}''(s)| \\ \boldsymbol{e}_b &= \boldsymbol{e}_t \times \boldsymbol{e}_n \end{aligned}\right\} \text{Frenet-Serret basis}$$

$$\hat{\boldsymbol{x}}_i \in \{\hat{\boldsymbol{x}}_1, \hat{\boldsymbol{x}}_2, \hat{\boldsymbol{x}}_3\} = \{\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}, \hat{\boldsymbol{z}}\}$$

$$\boldsymbol{e}_\alpha \in \{\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3\} = \{\boldsymbol{e}_t, \boldsymbol{e}_n, \boldsymbol{e}_b\}$$

$$\boldsymbol{e}'_\alpha = F_{\alpha\beta}\boldsymbol{e}_\beta, \qquad ||F_{\alpha\beta}|| = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix}$$

κ - curvature

τ - torsion

Exchange energy of a curvilinear wire

Isotropic exchange

$$\mathscr{E}_{\text{ex}} = (\boldsymbol{\nabla} m_i)(\boldsymbol{\nabla} m_i)$$

$$m_i = m_\alpha \left(\boldsymbol{e}_\alpha \cdot \hat{\boldsymbol{x}}_i\right)$$

$$\boldsymbol{\nabla} \equiv \boldsymbol{e}_1 \partial_s$$

$$\mathscr{E}_{\text{ex}} = (m_\alpha \boldsymbol{e}_\alpha)' (m_\beta \boldsymbol{e}_\beta)' = \mathscr{E}_{\text{ex}}^0 + \mathscr{E}_{\text{ex}}^D + \mathscr{E}_{\text{ex}}^A$$

$$\mathscr{E}_{\text{ex}}^0 = m_\alpha' m_\alpha' = |\boldsymbol{m}'|^2$$

Isotropic part (rectilinear case)

$$\mathscr{E}_{\text{ex}}^D = F_{\alpha\beta}\left(m_\alpha m_\beta' - m_\alpha' m_\beta\right)$$

Effective Dzyaloshinskii interaction

$$\mathscr{E}_{\text{ex}}^A = K_{\alpha\beta} m_\alpha m_\beta, \quad K_{\alpha\beta} = F_{\alpha\gamma} F_{\beta\gamma}, \quad \|K_{\alpha\beta}\| = \begin{pmatrix} \kappa^2 & 0 & -\kappa\tau \\ 0 & \kappa^2 + \tau^2 & 0 \\ -\kappa\tau & 0 & \tau^2 \end{pmatrix}$$

Effective anisotropy

General solution for a strong easy-tangential anisotropy

Angular parameterization

$$|\boldsymbol{m}| = 1 \quad \Longrightarrow \quad \boldsymbol{m} = \sin\theta\cos\phi\,\boldsymbol{e}_t + \sin\theta\sin\phi\,\boldsymbol{e}_n + \cos\theta\,\boldsymbol{e}_b$$

$$\mathscr{E}_{\text{ex}} = \left[\theta' - \tau\sin\phi\right]^2 + \left[\sin\theta(\phi' + \kappa) - \tau\cos\theta\cos\phi\right]^2$$

$$\mathscr{E} = \ell^2\mathscr{E}_{\text{ex}} + \lambda\sin^2\theta\cos^2\phi$$

high anisotropy case

$$\lambda < 0, \quad |\lambda| \gg 1 \quad \Longrightarrow \quad \theta = \pi/2 + \vartheta, \quad |\vartheta|, |\phi| \ll 1$$

$$\mathscr{E} \approx \mathscr{E}_{\text{ex}}^0 + \underbrace{2\ell^2(\tau\kappa\vartheta - \kappa'\phi)}_{\mathscr{E}^F = -(\boldsymbol{F}\cdot\boldsymbol{m})} + |\lambda|(\vartheta^2 + \phi^2) + \text{const}$$

$$\boldsymbol{F} = 2\ell^2\left(\kappa'\boldsymbol{e}_2 + \tau\kappa\boldsymbol{e}_3\right)$$

Effective magnetic field

$$\theta = \frac{\pi}{2} - \frac{\ell^2}{|\lambda|}\kappa\tau + \mathcal{O}\left(\frac{1}{|\lambda|^2}\right), \qquad \phi = \frac{\ell^2}{|\lambda|}\kappa' + \mathcal{O}\left(\frac{1}{|\lambda|^2}\right)$$

$\kappa' = 0, \quad \tau = 0$
(circle or rectilinear wire) $\Longrightarrow$ Strictly tangential solution

[D. Sheka, V. Kravchuk, Yu. Gaididei, J. Phys. A, **48**, 125202 (2015)]

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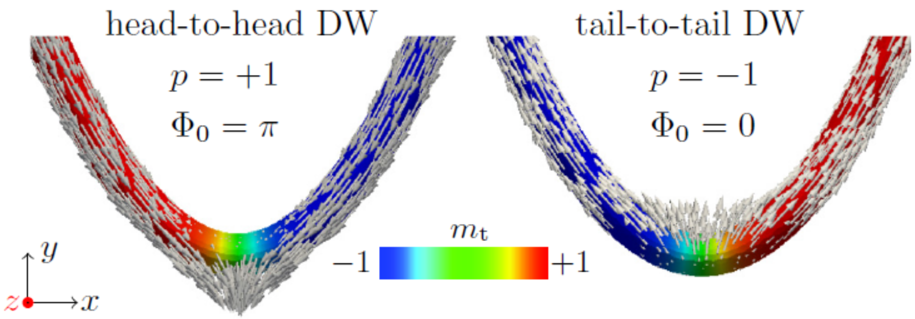
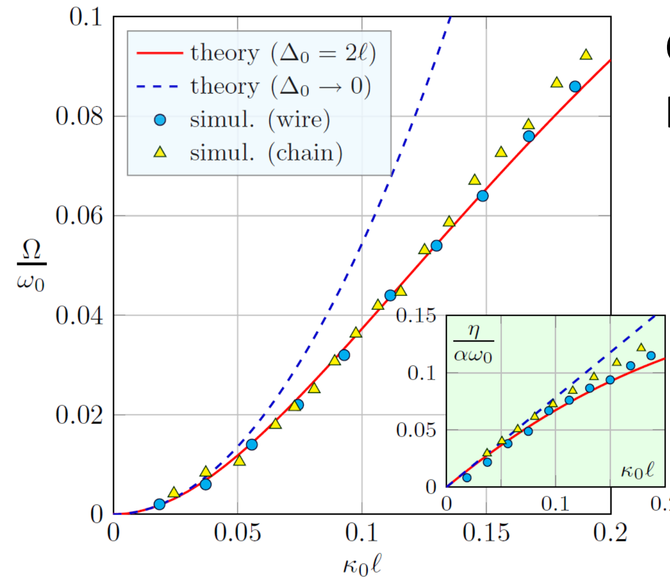
Domain wall pinning at a local wire bend



q-Φ model ➡ harmonic oscillations

$$\Omega \approx \omega_0 \ell^2 \pi \sqrt{|\kappa(q_0)\kappa''(q_0)|}$$ DW frequency

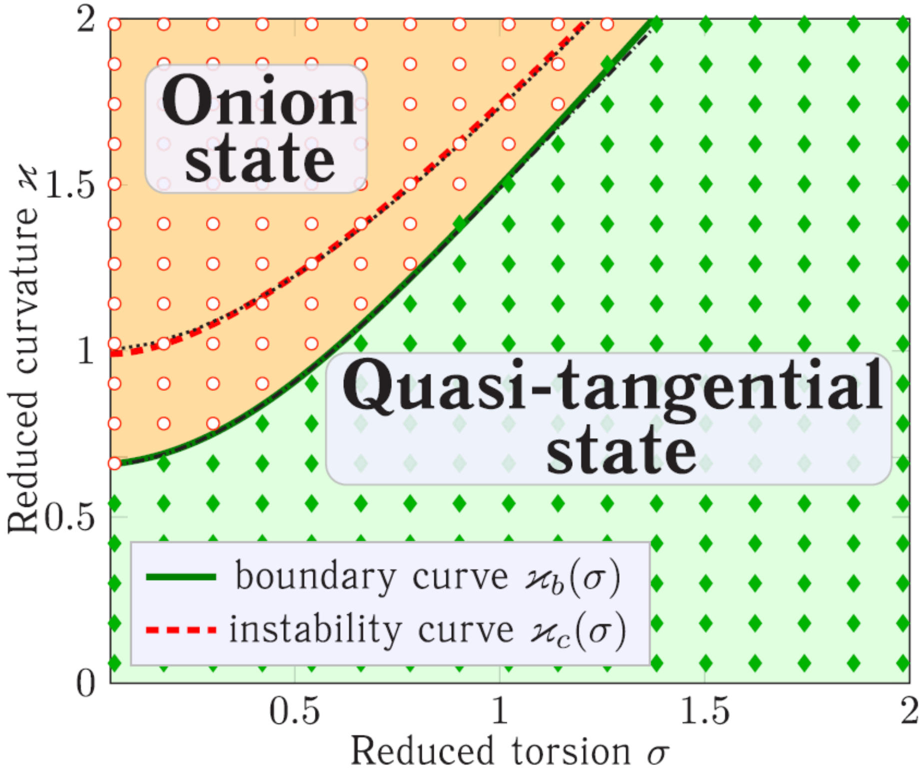
$$\eta \approx \alpha\omega_0 \frac{\pi}{2} \frac{\ell^2 \kappa(q_0)}{\Delta_0}$$ Effective damping

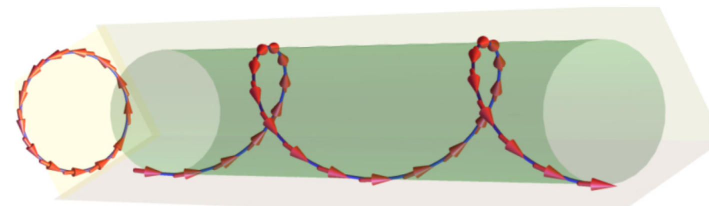


Case of parabola

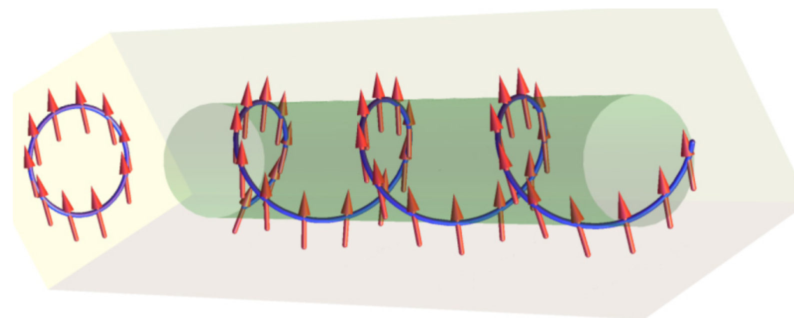
[K. Yershov, V. Kravchuk, D. Sheka, Yu. Gaididei, Phys. Rev. B, **92**, 104412 (2015)]

Ground states of a helix with easy tangential anisotropy





Quasi-tangential state: $\varkappa = \sigma = 0.1$



Onion state: $\varkappa = 1.5,\ \sigma = 1$

$$\kappa = \frac{\mathcal{R}}{\mathcal{R}^2 + \mathfrak{p}^2} \qquad \varkappa = \kappa\ell$$

$$\tau = \frac{\mathcal{C}\mathfrak{p}}{\mathcal{R}^2 + \mathfrak{p}^2} \qquad \sigma = \tau\ell$$

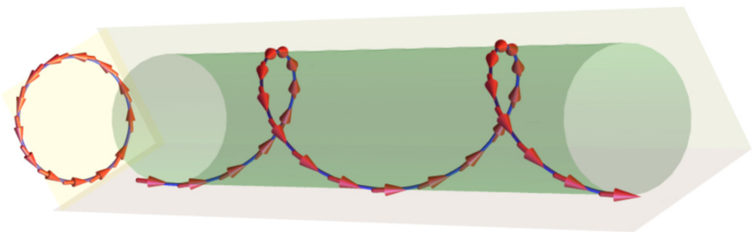
$\mathcal{R}$ - helix radius

$\mathcal{P}$ - helix pitch

$\mathfrak{p} = \mathcal{P}/(2\pi)$

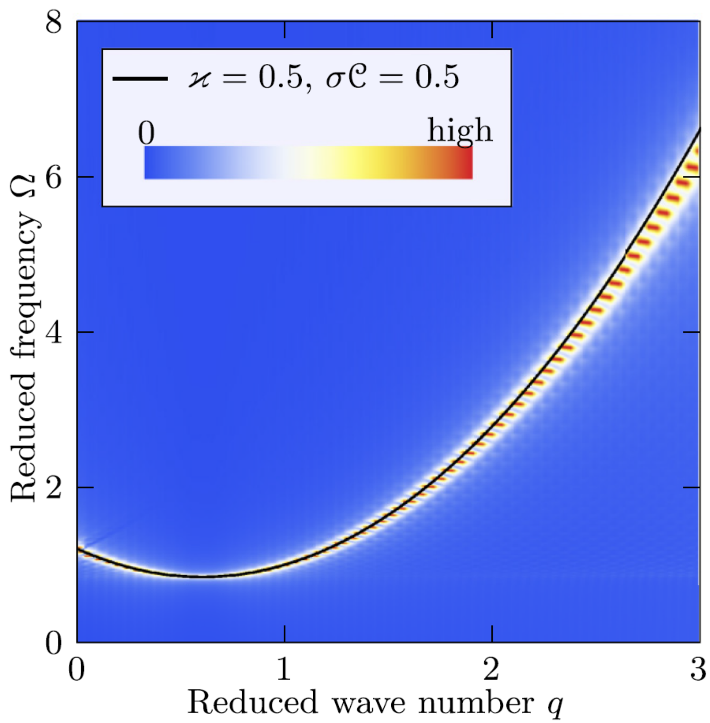
[D. Sheka, V. Kravchuk, K. Yershov, Yu. Gaididei, Phys. Rev. B, **92**, 054417 (2015)]

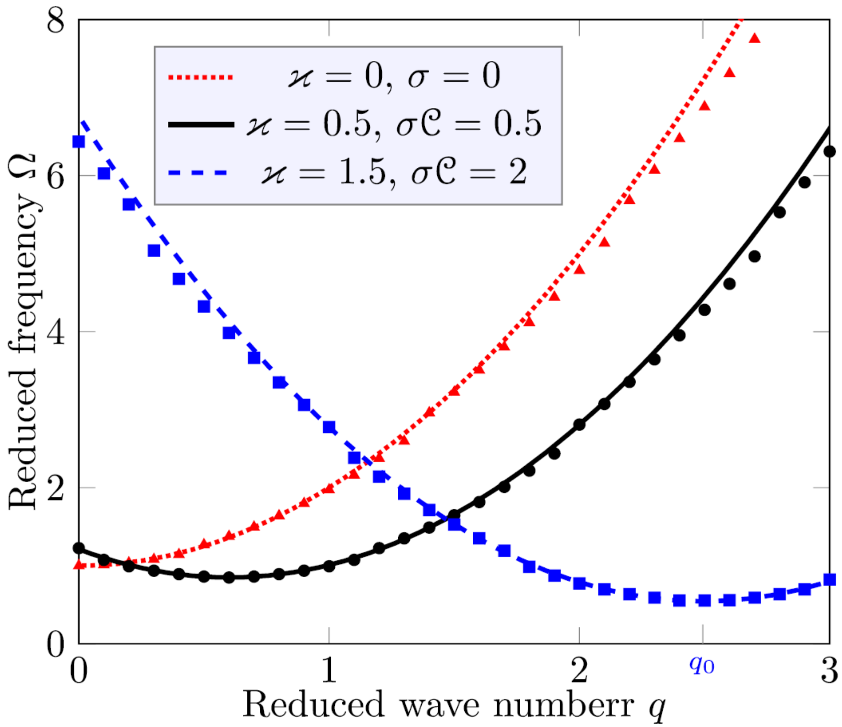
Magnon spectrum on the ground of easy-tangential state



$$\Omega(q) = \Omega_{\text{gap}} + (q - \mathcal{C}\sigma)^2 + \mathcal{O}(\varkappa^2, \sigma^2, \varkappa\sigma),$$

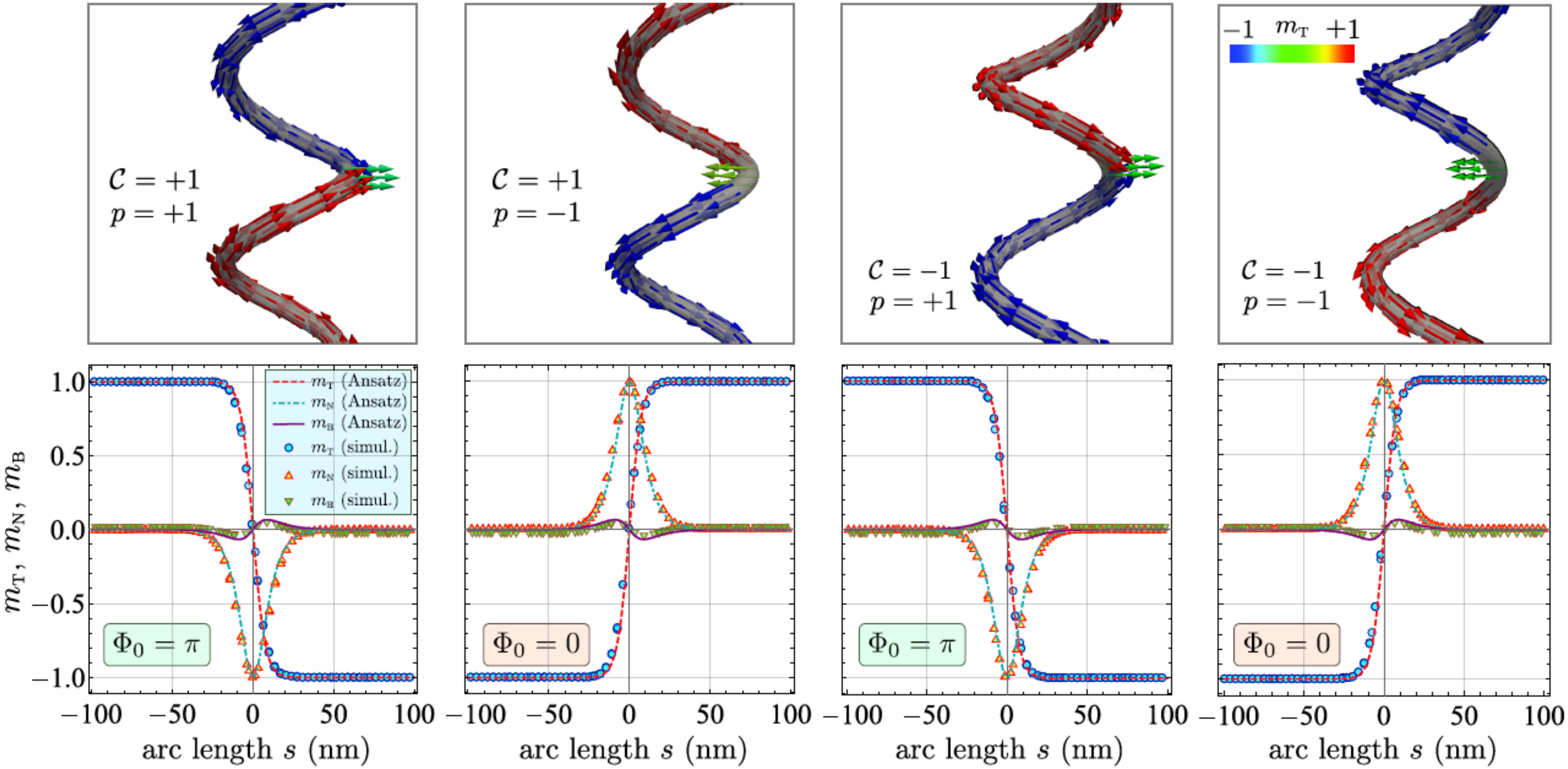
$$\Omega_{\text{gap}} = 1 - \frac{\varkappa^2}{2} + \mathcal{O}(\varkappa^2, \sigma^2, \varkappa\sigma).$$





[D. Sheka, V. Kravchuk, K. Yershov, Yu. Gaididei, Phys. Rev. B, **92**, 054417 (2015)]

Static domain wall on an easy-tangential helix



$$\theta = 2\arctan e^{p(s-q)/\Delta}, \qquad \phi = \Phi + a\frac{s-q}{\Delta}$$

$$\boldsymbol{m} = \cos\theta \boldsymbol{e}_t + \sin\theta\cos\phi \boldsymbol{e}_n + \sin\theta\sin\phi \boldsymbol{e}_b$$

Generalized q-Φ model

[K. Yershov, V. Kravchuk, D. Sheka, Yu. Gaididei, PRB, **93,** 094418, (2016)]

Domain wall dynamics on an easy-tangential helix

Equation of motion

Bazaliy-Zhang-Li mechanism

$$\boxed{\dot{\boldsymbol{m}} = \omega_0 \boldsymbol{m} \times \frac{\delta \mathcal{E}}{\delta \boldsymbol{m}} + \alpha \boldsymbol{m} \times \dot{\boldsymbol{m}} + \boldsymbol{m} \times \left[\boldsymbol{m} \times (\boldsymbol{u} \cdot \nabla) \boldsymbol{m}\right] + \beta \boldsymbol{m} \times (\boldsymbol{u} \cdot \nabla) \boldsymbol{m}}$$

$$\omega_0 = 4\pi\gamma_0 M_s, \qquad \mathcal{E} = \mathcal{S} \int_{-\infty}^{+\infty} \left[\ell^2 \mathscr{E}_{\mathrm{ex}} - k_{\mathrm{t}} (\boldsymbol{m} \cdot \boldsymbol{e}_t)^2\right] \mathrm{d}s, \qquad \boldsymbol{u} = \boldsymbol{j} P \mu_B / (|e| M_s), \quad \boldsymbol{j} || \boldsymbol{e}_t$$

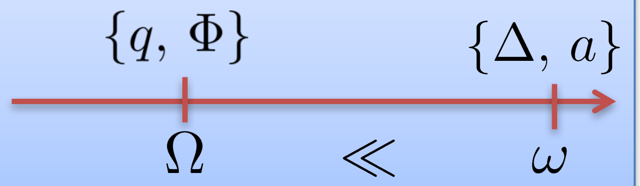
Generalized q-Φ model

$$\theta = 2 \arctan e^{p(s-q)/\Delta}, \qquad \phi = \Phi + a \frac{s-q}{\Delta}$$

with time scales separation condition

$$\frac{u}{v_0} |\beta - \beta^\star - \alpha| \ll \frac{2}{c} \sqrt{k_t}$$

$$v_0 = \ell \omega_0 \qquad c = \pi^2/12$$



$$\alpha^2 \ll 1, \qquad \alpha|\beta^\star| \ll 1,$$
$$\beta^2 \ll 1, \qquad \kappa\ell \ll 1,$$

$$\boxed{\begin{aligned} \dot{q} - \alpha p \Delta_0 \dot{\Phi} &= u - \pi v_0 \ell \kappa \sin \Phi, \\ \alpha p \dot{q} + \Delta_0 \dot{\Phi} &= u p (\beta - \beta^\star). \end{aligned}}$$

$$\beta^\star = p \tau \Delta_0$$

[K. Yershov, V. Kravchuk, D. Sheka, Yu. Gaididei, PRB, **93,** 094418, (2016)]

A couple of new effects

$$\dot{q} - \alpha p \Delta_0 \dot{\Phi} = u - \pi v_0 \ell \kappa \sin\Phi,$$
$$\alpha p \dot{q} + \Delta_0 \dot{\Phi} = up(\beta - \beta^\star).$$

Torsion induced shift of nonadiabatic parameter

$$\beta \to \beta - \beta^\star$$

$$\beta^\star = p\tau\Delta_0$$

Curvature induced Walker limit

$$u_c \approx v_0 \frac{\pi\alpha\kappa\ell}{|\alpha - \beta + \beta^\star|}$$

$u < u_c$ **traveling wave motion**

$$q = Vt, \quad \Phi = \text{const}$$

$$V \approx u\frac{\beta - \beta^\star}{\alpha}$$

$u > u_c$ **precession motion**

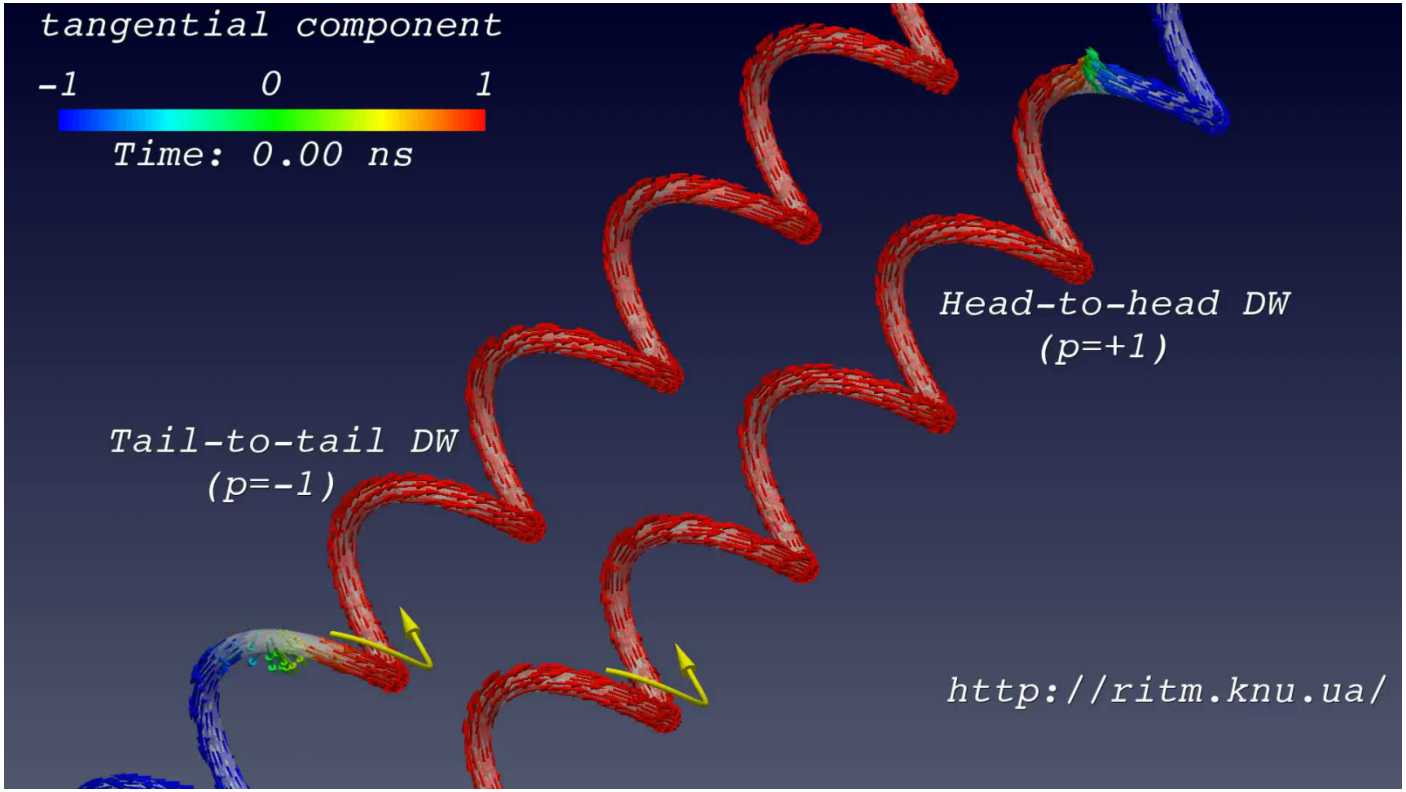
$$\Omega \approx p(\beta - \beta^\star - \alpha)\frac{\sqrt{u^2 - u_c^2}}{\Delta_0}$$

[K. Yershov, V. Kravchuk, D. Sheka, Yu. Gaididei, PRB, **93,** 094418, (2016)]

Traveling wave domain wall motion

Permalloy wire, length $L = 4\,\mu\mathrm{m}$, radius of round cross-section $R = 5\,\mathrm{nm}$,

helix radius $\mathcal{R} = 30\,\mathrm{nm}$, helix pitch $\mathcal{P} = 93\,\mathrm{nm}$.



$A = 13\,\mathrm{pJ/m}$

$M_s = 860\,\mathrm{kA/m}$

$v_0 \approx 704\,\mathrm{m/s}$

$\alpha = 0.01$

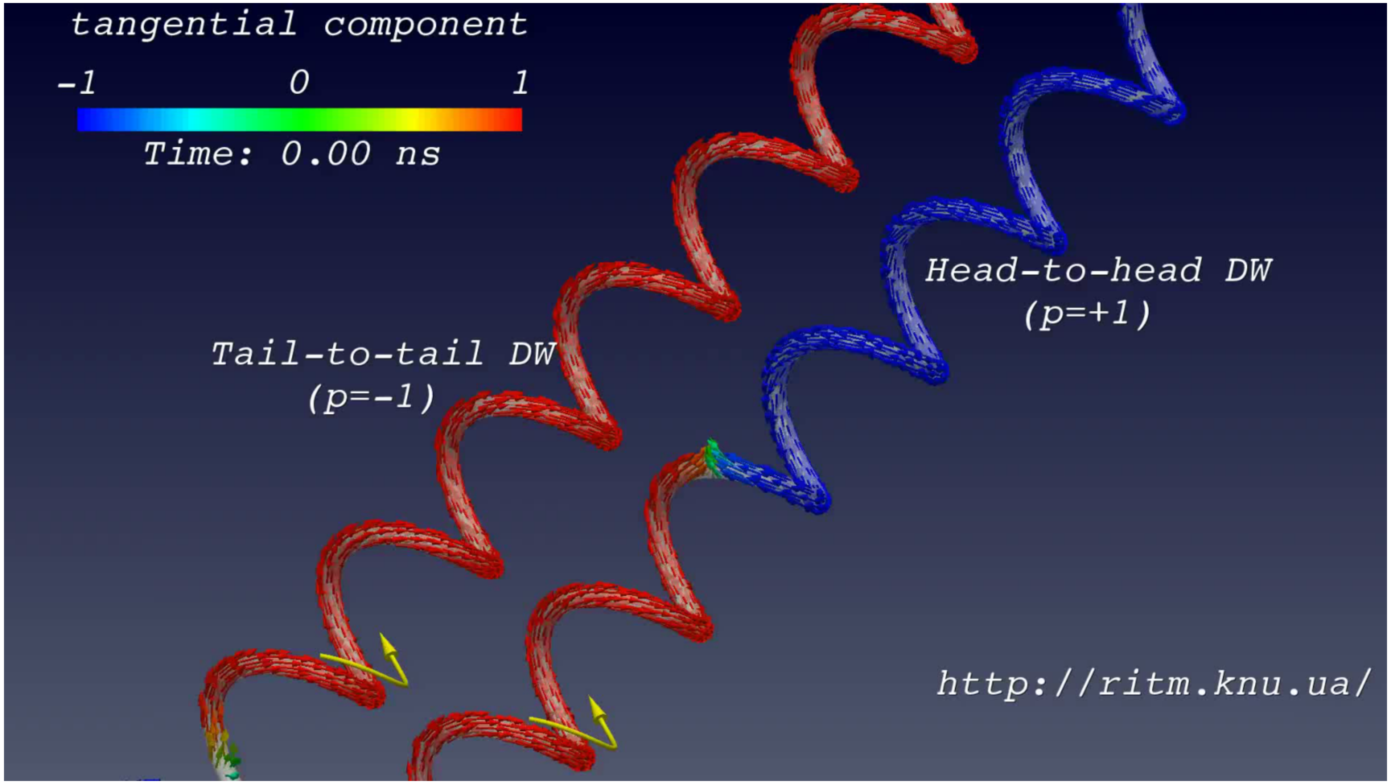
$\beta = 0$

$u = 0.02 v_0$

$j = 2 \times 10^{11}\,\mathrm{A/m^2}$

Precessoin domain wall motion

Permalloy wire, length $L = 4\,\mu\mathrm{m}$, radius of round cross-section $R = 5\,\mathrm{nm}$, helix radius $\mathcal{R} = 30\,\mathrm{nm}$, helix pitch $\mathcal{P} = 93\,\mathrm{nm}$.



$A = 13\,\mathrm{pJ/m}$

$M_s = 860\,\mathrm{kA/m}$

$v_0 \approx 704\,\mathrm{m/s}$

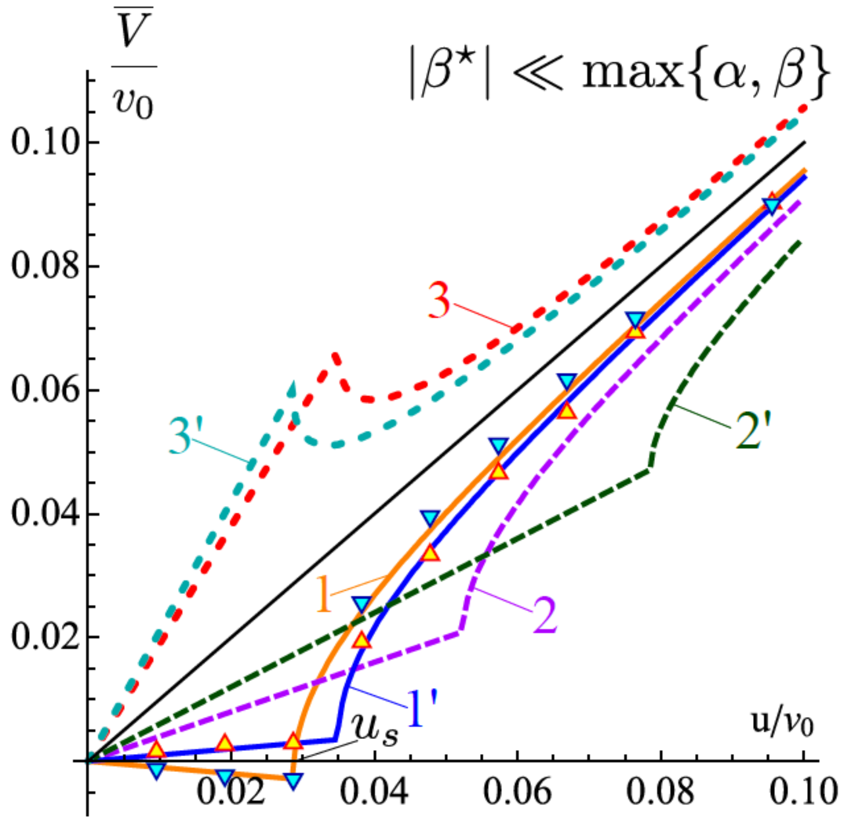
$\alpha = 0.01$

$\beta = 0$

$u = 0.115 v_0$

$j = 1.2 \times 10^{12}\,\mathrm{A/m^2}$

Averaged domain wall velocity via applied current

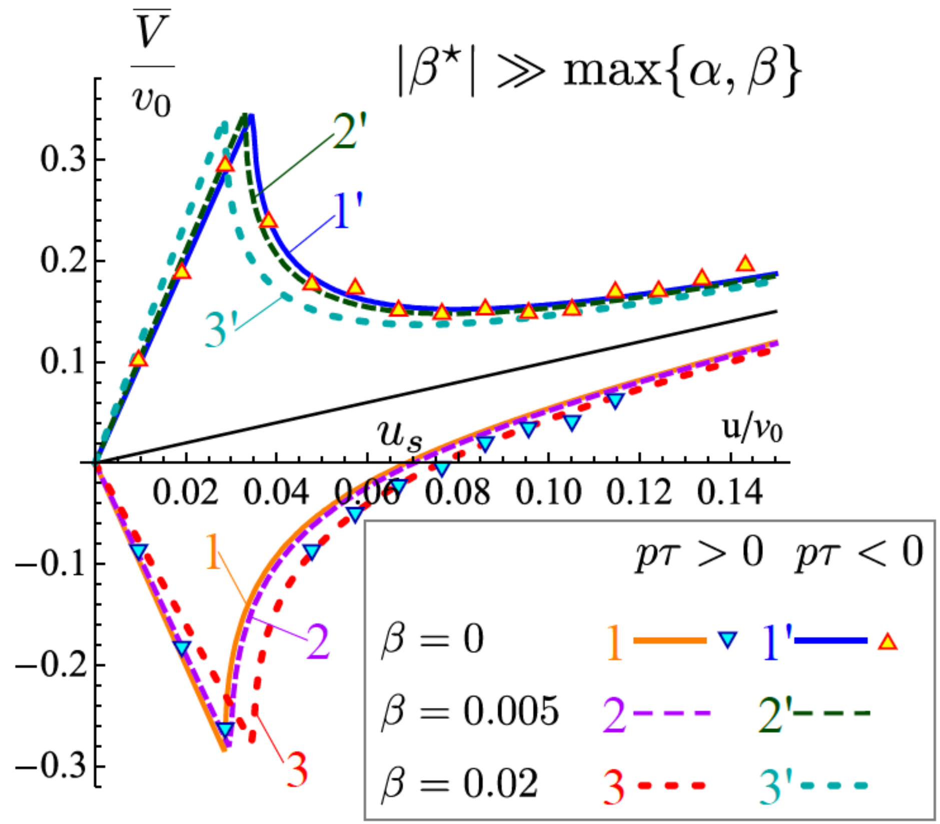


$\kappa\ell = 0.01$

$\tau\ell = 0.0005$

$|\beta^\star| = 0.001$

$\alpha = 0.01$



$\kappa\ell = 0.1$

$\tau\ell = 0.05$

$|\beta^\star| = 0.1$

$\alpha = 0.01$

Outline

1. Introduction or is the game worth the candle? Possible research directions.

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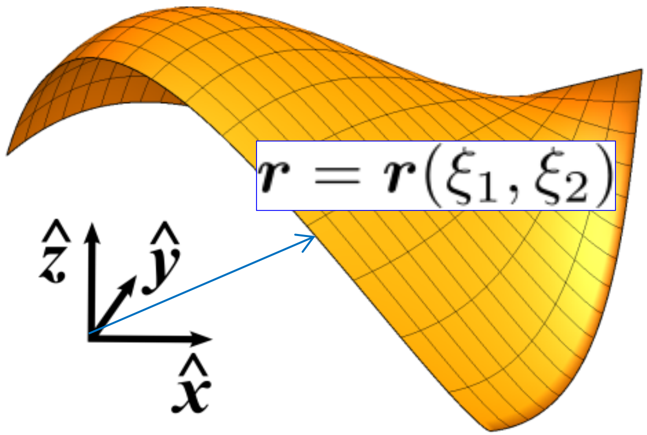
Short thesaurus for curvilinear surfaces



$$\boldsymbol{g}_\alpha = \partial_\alpha \boldsymbol{r}$$
$$\partial_\alpha \equiv \partial / \partial \xi_\alpha$$
$$\alpha,\ \beta,\ \gamma = 1, 2$$

$$g_{\alpha\beta} = \boldsymbol{g}_\alpha \cdot \boldsymbol{g}_\beta$$

metric tensor (**diagonal!**)

Orthonormal basis $\boldsymbol{e}_\alpha = \boldsymbol{g}_\alpha / |\boldsymbol{g}_\alpha|$, $\boldsymbol{n} = [\boldsymbol{e}_1 \times \boldsymbol{e}_2]$

Gradient on the surface $\boldsymbol{\nabla} \equiv (g_{\alpha\alpha})^{-1/2} \boldsymbol{e}_\alpha \partial_\alpha$

Weingarten map $h_{\alpha\beta} = \boldsymbol{e}_\beta \cdot (\boldsymbol{e}_\alpha \cdot \boldsymbol{\nabla}) \boldsymbol{n}$

Spin connection $\Omega_\gamma = \dfrac{1}{2} \epsilon_{\alpha\beta} \boldsymbol{e}_\alpha \cdot \nabla_\gamma \boldsymbol{e}_\beta$

Mean curvature $\mathcal{H} = \mathrm{tr} || h_{\alpha\beta} ||$

Gauß curvature $\mathcal{K} = \det || h_{\alpha\beta} ||$

Basis differentiation

Gauß-Codazzi formula

$$\nabla_\alpha \boldsymbol{e}_\beta = h_{\alpha\beta} \boldsymbol{n} - \Omega_\alpha \epsilon_{\beta\gamma} \boldsymbol{e}_\gamma$$

Weingarten formula

$$\nabla_\alpha \boldsymbol{n} = -h_{\alpha\beta} \boldsymbol{e}_\beta$$

Energy representation in curvilinear basis

Energy functional

$$\mathcal{E} = AL \int\limits_{\mathcal{S}} \left[\mathscr{E}_{ex} + \frac{1}{\ell^2}(\boldsymbol{m}\cdot\boldsymbol{n})^2 \right] \mathrm{d}\mathcal{S}$$

$$\mathscr{E}_{ex} = (\boldsymbol{\nabla} m_i)(\boldsymbol{\nabla} m_i)$$

$$m_i = m_\alpha(\boldsymbol{e}_\alpha \cdot \hat{\boldsymbol{x}}_i) + m_n(\boldsymbol{n}\cdot\hat{\boldsymbol{x}}_i)$$

Exchange energy density

$$\mathscr{E}_{ex} = \mathscr{E}_{ex}^{0} + \mathscr{E}_{ex}^{D} + \mathscr{E}_{ex}^{A}$$

$$\mathscr{E}_{ex}^{0} = \boldsymbol{\nabla} m_\alpha \cdot \boldsymbol{\nabla} m_\alpha$$

$$\mathscr{E}_{ex}^{D} = D_{\alpha\beta\gamma} m_\beta \nabla_\gamma m_\alpha, \qquad D_{\alpha\beta\gamma} = -D_{\beta\alpha\gamma}$$

$$\mathscr{E}_{\mathrm{ex}}^{A} = K_{\alpha\beta} m_\alpha m_\beta, \qquad K_{\alpha\beta} = K_{\beta\alpha}$$

Angular parameterization

$$\boldsymbol{m} = \sin\theta\cos\phi\,\boldsymbol{e}_1 + \sin\theta\sin\phi\,\boldsymbol{e}_2 + \cos\theta\,\boldsymbol{n}$$

$$\mathscr{E}_{ex} = \left[\boldsymbol{\nabla}\theta - \boldsymbol{\Gamma}\right]^2 + \left[\sin\theta(\boldsymbol{\nabla}\phi - \boldsymbol{\Omega}) - \cos\theta\,\partial_\phi \boldsymbol{\Gamma}\right]^2$$

$$\boldsymbol{\Gamma} = ||h_{\alpha\beta}|| \cdot \boldsymbol{\varepsilon}, \qquad \boldsymbol{\varepsilon} = \cos\phi\,\boldsymbol{e}_1 + \sin\phi\,\boldsymbol{e}_2$$

[Yu. Gaididei, V. Kravchuk, D.Sheka, *Phys. Rev. Lett*. **112**, 257203 (2014)]

[D. Sheka, V. Kravchuk, Yu. Gaididei, *J. Phys. A: Math. Theor.*, **48**, 125202 (2015)]

Story of the question

Incomplete energy expression

$$\mathscr{E}_{ex} = (\boldsymbol{\nabla}\varphi - \boldsymbol{\Omega})^2$$

- [A.M. Turner, et al., Rev. Mod. Phys., **82**, 1301, (2010)]
- [M.J. Bowik, et al., Advances in Physics, **58**, 449, (2009)]
- [V. Vitelli, et al., Phys. Rev. E, **74,** 021711, (2006)]
- [V. Vitelli, et al., Phys. Rev. Lett., **93,** 215301, (2004)]

$$\mathscr{E}_{ex} = \boldsymbol{\Gamma}^2 + (\boldsymbol{\nabla}\varphi - \boldsymbol{\Omega})^2$$

Strictly tangential distribution

$$(\theta = \pi/2)$$

- [G. Napoli, et al., Int. J. Non-Linear Mechanics **49**, 66, (2013)]
- [G. Napoli, et al., Phys. Rev. Lett. **108**, 207803, (2012)]
- [G. Napoli, et al., Phys. Rev. E. **85**, 061701, (2012)]

Certain case of a <u>cylindrical</u> shell
[P. Landeros et al.,
J. Appl. Phys., **108**, 033917 (2010)]

Certain case of a <u>spherical</u> shell
[V. Kravchuk et al.,
Phys. Rev. B ., **85,** 144433 (2012)]

General case for an arbitrary curvilinear shell
[Yu. Gaididei, V. Kravchuk, D.Sheka,
Phys. Rev. Lett. **112**, 257203 (2014)]

General approach for an arbitrary 1D and 2D curvilinear magnets
[D. Sheka, V. Kravchuk, Yu. Gaididei,
J. Phys. A: Math. Theor., **48**, 125202 (2015)]

Curvature induced effective fields

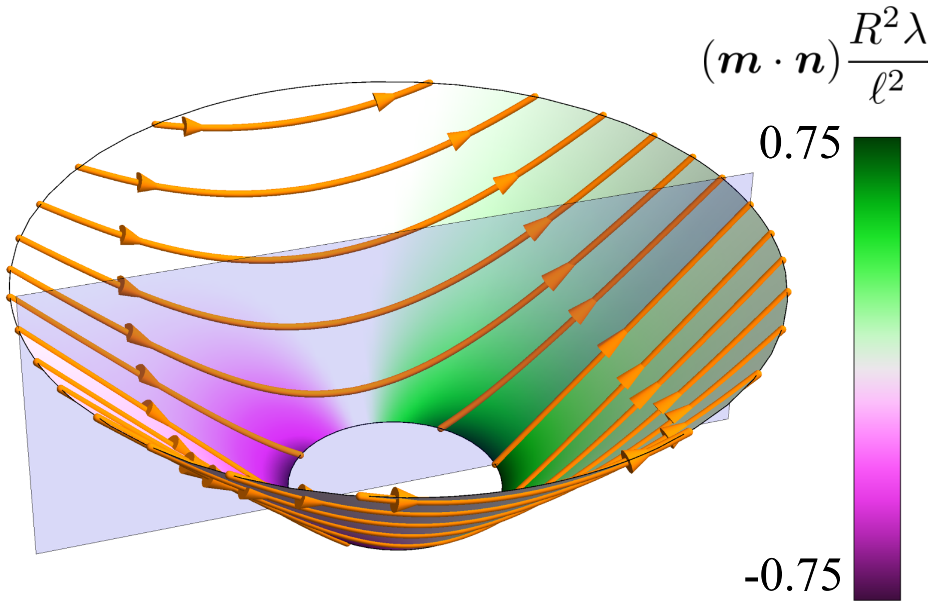
case of strong easy-surface anisotropy $\lambda \gg \ell/\mathcal{R}$

$$\theta = \frac{\pi}{2} + \vartheta, \qquad \vartheta \ll 1$$

$$\mathscr{E}_{ex} \approx \mathscr{E}^{\mathrm{t}} + \mathrm{F}^{\mathrm{t}}\vartheta + \lambda\vartheta^2$$

$$\mathscr{E}^{\mathrm{t}} = \ell^2 \left[\boldsymbol{\Gamma}^2 + (\boldsymbol{\nabla}\varphi - \boldsymbol{\Omega})^2\right]$$

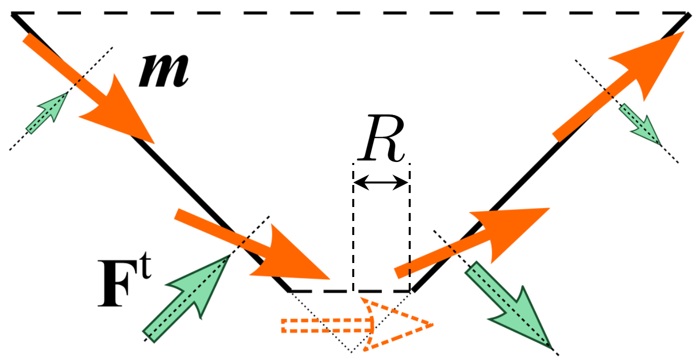
Energy of the strictly tangential distribution $E^{\mathrm{t}} = \int_{\mathcal{S}} \mathscr{E}^{\mathrm{t}} \mathrm{d}\mathcal{S}$



Curvature induced effective field

$$\mathbf{F}^{\mathrm{t}} = 2\boldsymbol{n}\ell^2 \left[\nabla\cdot\boldsymbol{\Gamma} + (\nabla\varphi - \boldsymbol{\Omega})\frac{\partial\boldsymbol{\Gamma}}{\partial\varphi}\right]$$

$$\delta E^{\mathrm{t}}/\delta\varphi = 0$$



Curvature induced effective fields

case of strong easy-normal anisotropy $\lambda \ll -\ell/\mathcal{R}$

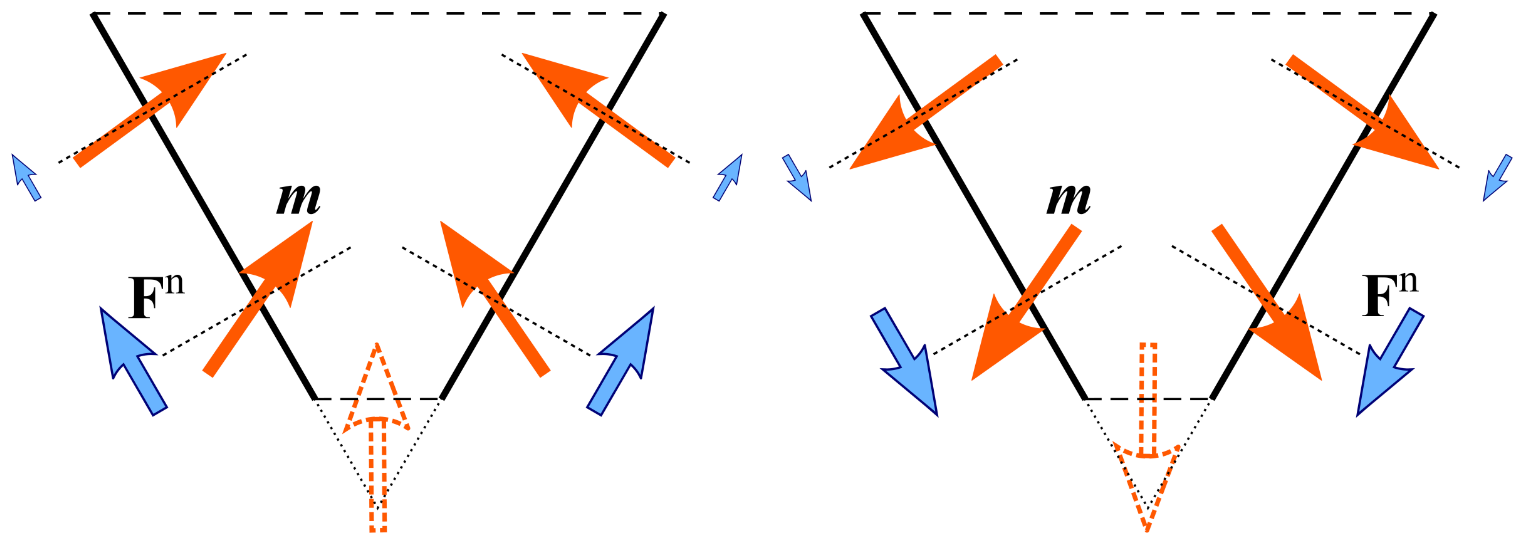
$$\theta = \vartheta, \qquad \vartheta \ll 1 \quad \Longrightarrow \quad \mathscr{E}_{ex} \approx \mathrm{F}^{\mathrm{n}}\vartheta + |\lambda|\vartheta^2 + \text{const}$$

Curvature induced effective field

$$\mathbf{F}^{\mathrm{n}} = 2\varepsilon\ell^2\left[(\nabla\cdot h)\cdot\varepsilon + \mathbf{\Omega}\cdot\left(h\,\frac{\partial\varepsilon}{\partial\varphi}\right)\right]$$

$$\tan\varphi = \frac{(\nabla\cdot h)_2 - (h\mathbf{\Omega})_1}{(\nabla\cdot h)_1 + (h\mathbf{\Omega})_2}$$

$$(\nabla\cdot h)_\alpha = \frac{1}{\sqrt{g}}\partial_\beta\left(h_{\beta\alpha}\sqrt{g/g_{\beta\beta}}\right)$$



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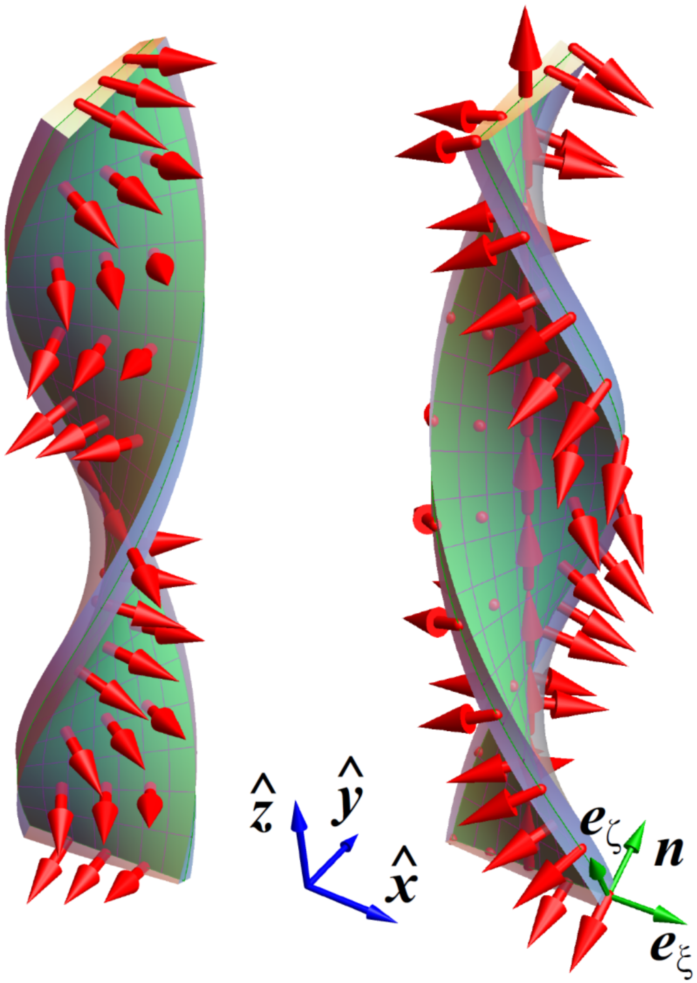
Torsion → Effective DMI → Chirality symmetry breaking



$$x + iy = \xi e^{i\mathcal{C}\pi\zeta/\Lambda}, \qquad -\frac{w}{2} \le \xi \le \frac{w}{2},$$

$$z = \zeta, \qquad -\infty < \zeta < +\infty$$

Transversal domain wall

$$\theta^t = 2\arctan\exp\frac{\zeta}{d^t}, \qquad \varphi = \mathfrak{c}\frac{\pi}{2}$$

$$\frac{E^t}{2Ahw} \approx \frac{1}{d^t} - \frac{\pi^2}{\Lambda}\mathfrak{c}\mathcal{C} + \frac{d^t}{\ell^2} + \text{const}$$

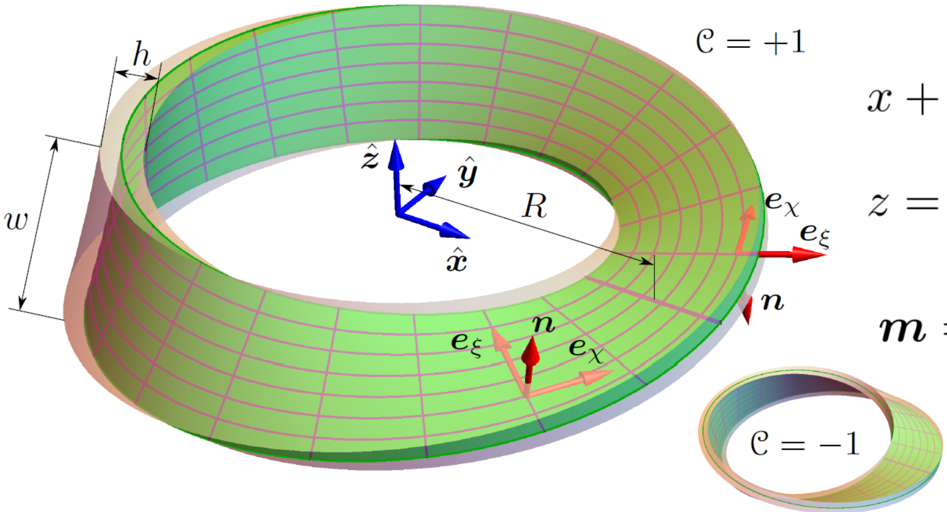
Longitudinal domain wall

$$\theta^l = 2\arctan\exp\frac{\xi}{d^l}, \qquad \varphi = \pi\frac{\mathfrak{c}+1}{2}$$

$$\frac{E^l}{2AhL} \approx \frac{1}{d^l} + \frac{\pi^2}{\Lambda}\mathfrak{c}\mathcal{C} + \frac{d^l}{\ell^2} + \text{const}$$

$$\boldsymbol{m} = \sin\theta\cos\phi\,\boldsymbol{e}_\zeta + \sin\theta\sin\phi\,\boldsymbol{e}_\xi + \cos\theta\,\boldsymbol{n}$$

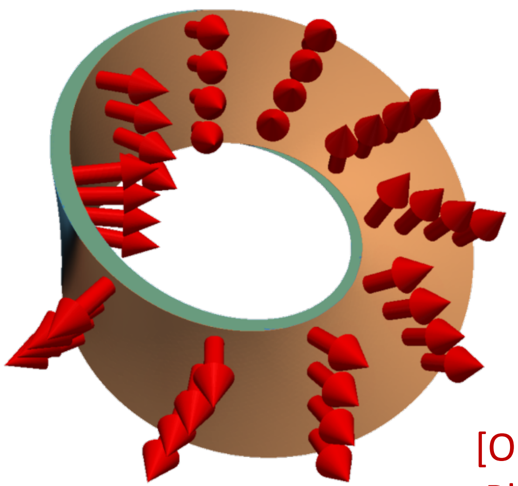
Transverse domain wall on a Möbius strip



$$x+iy=\left(R+\xi\cos\frac{\chi}{2}\right)e^{i\chi},\qquad 0\le\chi<2\pi,$$

$$z=\mathcal{C}\,\xi\sin\frac{\chi}{2},\qquad -\frac{w}{2}\le\xi\le\frac{w}{2}$$

$$\boldsymbol{m}=\sin\theta\cos\phi\,\boldsymbol{e}_\chi+\sin\theta\sin\phi\,\boldsymbol{e}_\xi+\cos\theta\,\boldsymbol{n}$$

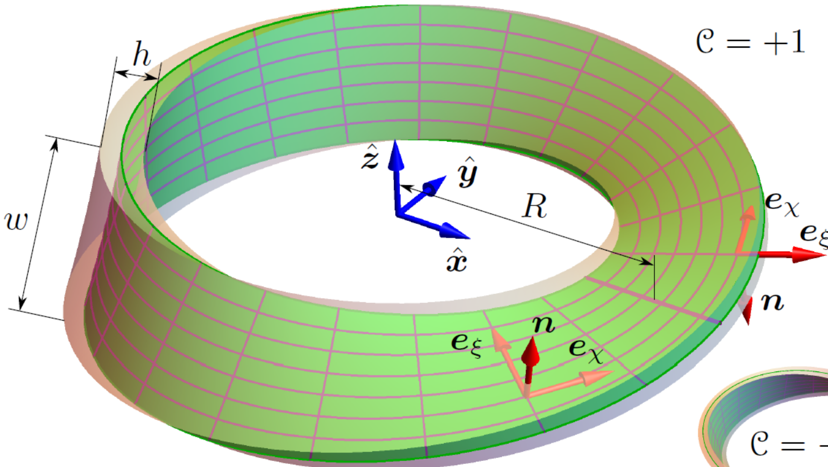


$$\theta^t=2\arctan e^{p\frac{\chi-X}{\sigma}},\qquad \varphi^t=\mathfrak{c}p\frac{\pi}{2}$$

$$\frac{E^t}{2Ah}\approx\frac{w}{R}\left[\frac{1}{\sigma}+\mathfrak{c}\mathcal{C}\frac{\pi}{2}+\sigma\left(\cos X-\frac{1}{4}\right)\right]+\sigma\frac{Rw}{\delta^2}$$
$$+\,\text{const},$$

[O. Pylypovskyi, V. Kravchuk, D. Sheka, D. Makarov, O. Schmidt, Yu.Gaididei, Phys. Rev. Lett. **114**, 197204 (2015)]

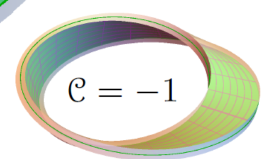
Longitudinal domain wall on a Möbius strip

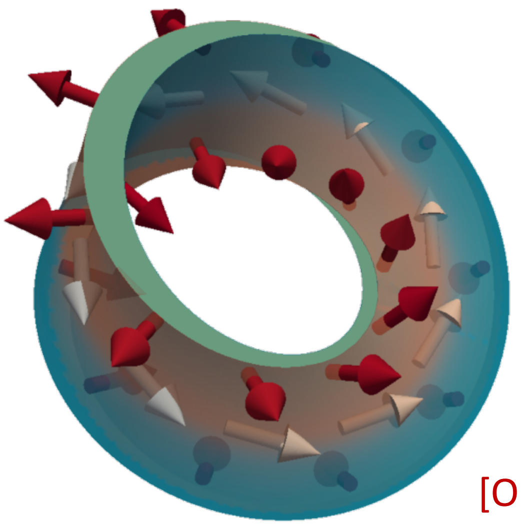


$$x + iy = \left(R + \xi \cos\frac{\chi}{2}\right) e^{i\chi}, \qquad 0 \le \chi < 2\pi,$$

$$z = \mathcal{C}\, \xi \sin\frac{\chi}{2}, \qquad -\frac{w}{2} \le \xi \le \frac{w}{2}$$

$$\boldsymbol{m} = \sin\theta\cos\phi\, \boldsymbol{e}_\chi + \sin\theta\sin\phi\, \boldsymbol{e}_\xi + \cos\theta\, \boldsymbol{n}$$





$$\theta^l = 2\arctan e^{p\frac{\xi}{d}}, \qquad \phi^l = \pi\frac{\mathfrak{c} + p}{2}$$

$$\frac{E^l}{4\pi A h} \approx \frac{R}{d} - \frac{\pi}{2}\mathcal{C}\mathfrak{c} + \varepsilon_0\frac{d}{R} + \frac{dR}{\ell^2} + \text{const}$$

$$\varepsilon_0 = 1/4 + \pi^2/96$$

[O. Pylypovskyi, V. Kravchuk, D. Sheka, D. Makarov, O. Schmidt, Yu.Gaididei, Phys. Rev. Lett. **114**, 197204 (2015)]

Skyrmion solution on a planar film

$$E = L\int\left[A\,\mathscr{E}_{\text{ex}} - K(\boldsymbol{m}\cdot\boldsymbol{n})^2 + D\,\mathscr{E}_{\text{D}}\right]\mathrm{d}\mathcal{S}$$ - uniaxial magnet with DMI

$$\mathscr{E}_{\text{ex}} = \partial_i\boldsymbol{m}\cdot\partial_i\boldsymbol{m} \qquad \mathscr{E}_{\text{D}} = m_n\boldsymbol{\nabla}\cdot\boldsymbol{m} - (\boldsymbol{m}\cdot\boldsymbol{\nabla})m_n$$

$\mathcal{C} = +1$

Is it possible to stabilize magnetic skyrmion with effective curvature induced DMI only?

Small radius skyrmion: $E_{\text{ex}} \approx 8\pi A h$, $\underbrace{E_{\text{a}} \propto K R_s^2, \quad E_{\text{D}} \propto \mathcal{C} D R_s,}_{R_s \propto |D|/K,}$

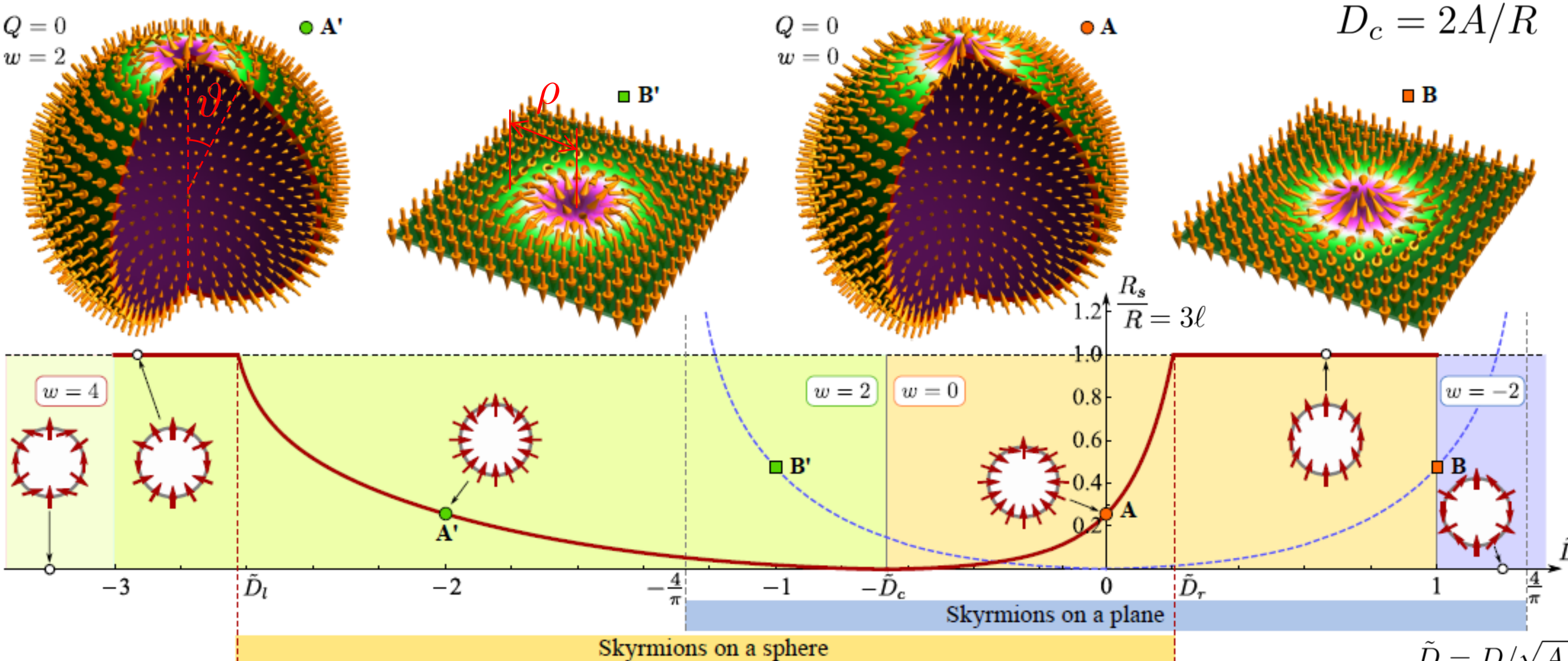
Skyrmion solution on a spherical shell

$$\cos\theta = \boldsymbol{m}\cdot\boldsymbol{n}$$

Plane: $$\theta_{\rho\rho} + \frac{1}{\rho}\theta_\rho - \sin\theta\cos\theta\left[\frac{1}{\rho^2}+\frac{1}{\ell^2}\right] + \frac{2}{\rho}\frac{D}{A}\sin^2\theta = 0$$

Sphere: $$\theta_{\vartheta\vartheta} + \cot\vartheta\,\theta_\vartheta - \sin\theta\cos\theta\left[\frac{\cos 2\vartheta}{\sin^2\vartheta}+\frac{R^2}{\ell^2}-\frac{4D}{D_c}\right] + 2\cot\vartheta\sin^2\theta\left(1+\frac{D}{D_c}\right) = 0$$

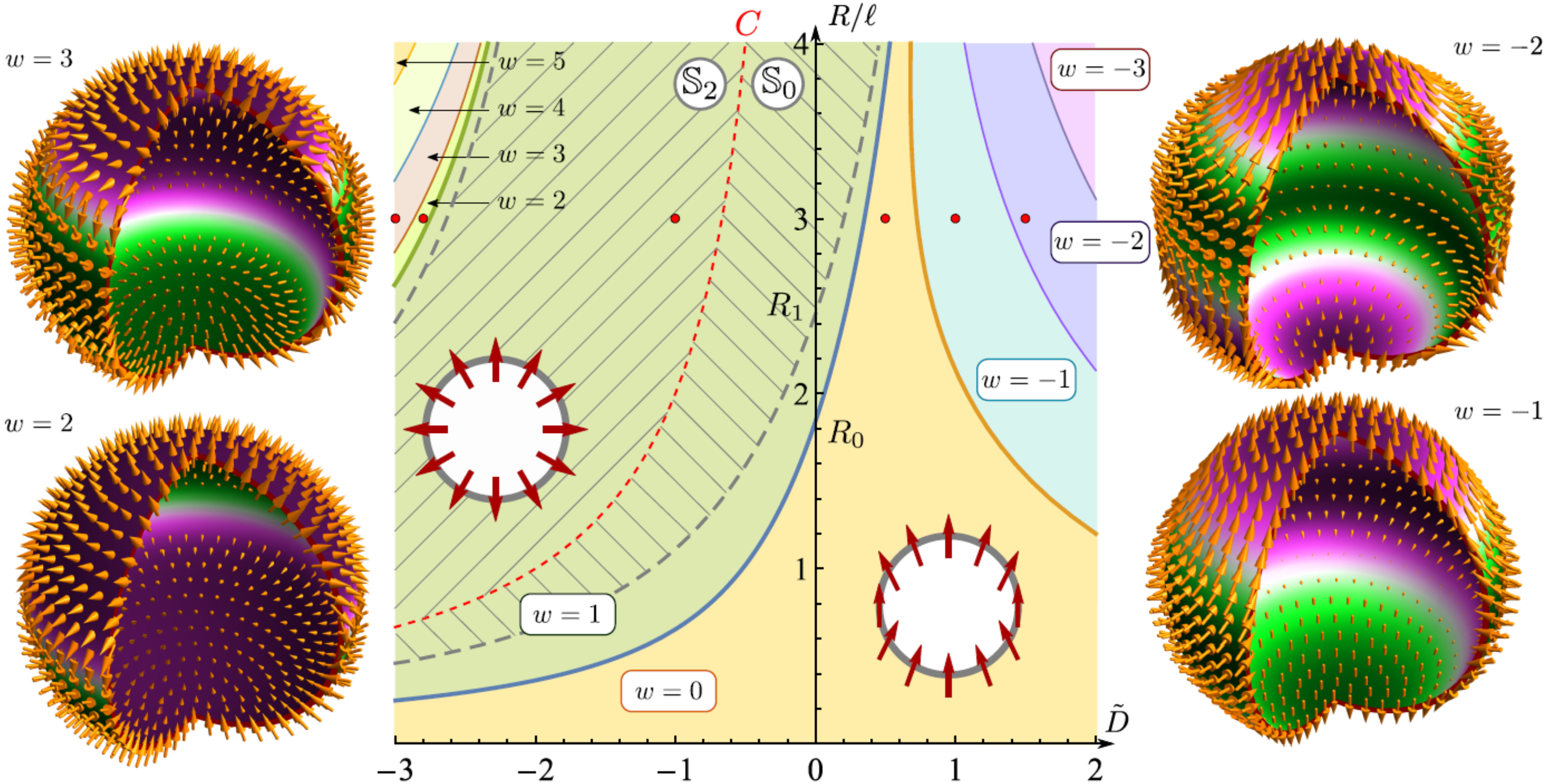
$$D_c = 2A/R$$



$$\tilde{D} = D/\sqrt{AK}$$

[V.P. Kravchuk et al., Phys. Rev. B **94**, 144402 (2016)]

Diagram of ground states



[V.P. Kravchuk et al., Phys. Rev. B **94**, 144402 (2016)]

Each state is doubly degenerate with respect to the transformation $\boldsymbol{m} \to -\boldsymbol{m}$

Conclusions

1. Domain wall is pinned at the local wire bend.

2. Torsion results in a shift of nonaidabatic spin torque parameter. This can lead to negative domain wall mobility.

3. Skyrmion can be stabilized by curvature effects only, without intrinsic DMI.

4. Curvature defect in form of surface of rotation can creates pinning as well as repulsing potential for the skyrmion.

5. Skyrmion on a curvilinear defect can be a ground state ?!

Thank you for your attention!