Radboud Universiteit Nijmegen

Spin-orbit and spin-transfer torques in two dimensions

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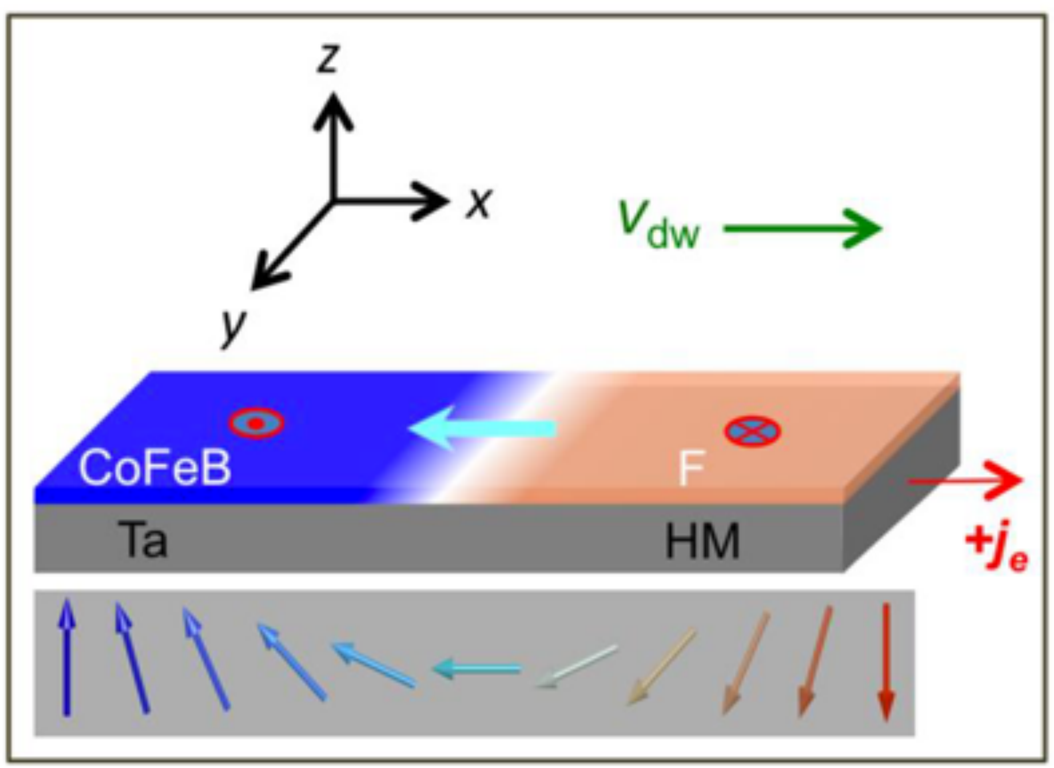
Microscopic theory of spin-orbit torque
I. A. Ado, O. A. Tretiakov, and M. Titov, arXiv:1511.07413 (2016)

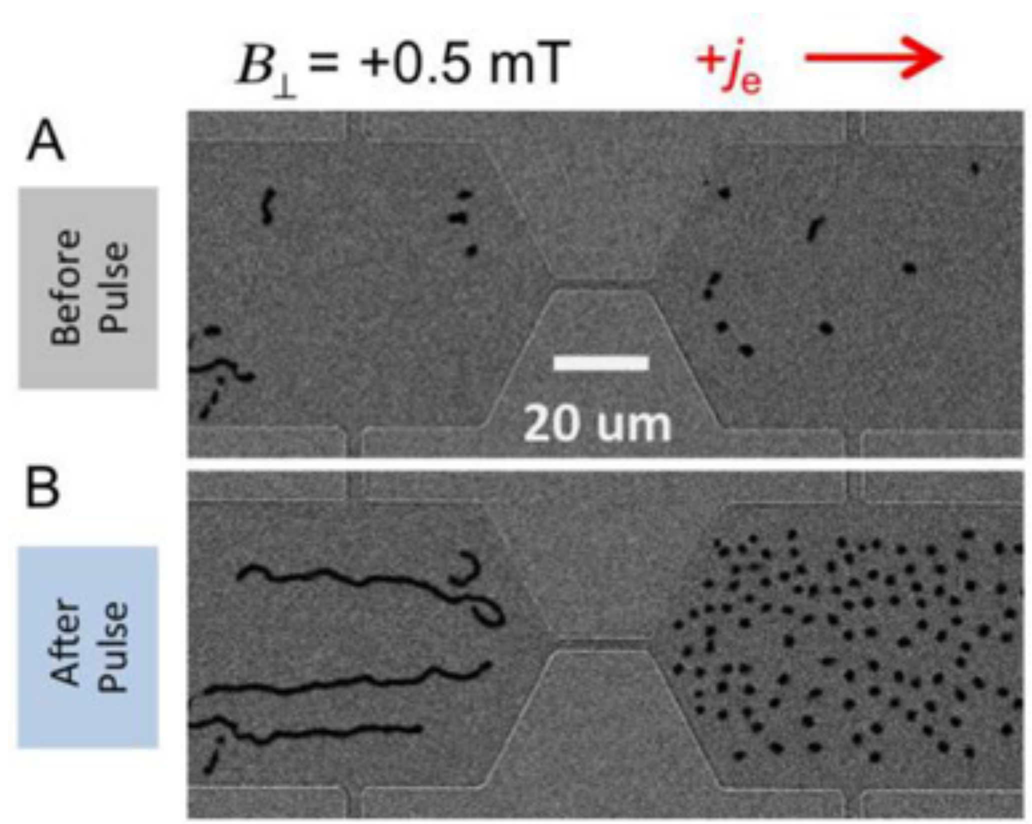
Experimental Motivation

creating and driving magnetic bubbles in FM/HM bilayer by electric current

FM = ferromagnet that supports magnetic bubbles or Skyrmions

HM = heavy metal with strong spin-orbit interaction (SOI)





Axel Hoffmann *et.al.*, *Blowing magnetic skyrmion bubbles*, Science (2015)

Outline

- Experimental motivation
- General microscopic theory of spin torques
- Spin-orbit torques in 2D Bychkov-Rashba model
- Applications to Skyrmion dynamics
- Open questions

Towards the microscopic theory

$\boldsymbol{m}(\boldsymbol{r},t)$ - magnetic texture with a constraint $|\boldsymbol{m}| = 1$

$$\boxed{\frac{\partial \boldsymbol{m}}{\partial t} = \boldsymbol{f} \times \boldsymbol{m}}$$ - magnetization dynamics

external mag. field

electric field or current

out of plane vector: SOI

gradients in space and time

$$\boldsymbol{f} = \boldsymbol{f}[\boldsymbol{H}_{\rm ext},\ \boldsymbol{m},\ \{\boldsymbol{E},\boldsymbol{J}\},\ \hat{\boldsymbol{z}},\ \nabla_\alpha m_\beta,\ \partial_t \boldsymbol{m}]$$

precession

SOT

STT

Gilbert damping

e.g. $\boldsymbol{f} = \gamma \boldsymbol{H}_{\rm ext} - \alpha_G\, \partial_t \boldsymbol{m} + A_1 \hat{\boldsymbol{z}} \times \boldsymbol{J} + A_2 \boldsymbol{m} \times (\hat{\boldsymbol{z}} \times \boldsymbol{J}) + A_3 (\boldsymbol{J} \cdot \boldsymbol{\nabla}) \boldsymbol{m} + \dots$

Our task is to compute *f* microscopically!

Non-equilibrium quantum theory

$$\boldsymbol{f} = \gamma(\boldsymbol{H}_{\mathrm{ext}} + 2\mu_{\mathrm{B}}\boldsymbol{s})$$

See also: Tartara, Kohno, Shibata (2008)

$\boldsymbol{s}$ - local polarisation of "itinerant electrons"

$$\boldsymbol{s}(\boldsymbol{r},t) = \frac{1}{2}\langle\Psi^\dagger(\boldsymbol{r},t)\boldsymbol{\sigma}\Psi(\boldsymbol{r},t)\rangle = -i\,\mathrm{Tr}_\sigma\left[\frac{1}{2}\boldsymbol{\sigma}\,G^<(\boldsymbol{r},t;\boldsymbol{r},t)\right]$$

$G^<$ - non-equlibrium Green's function

$$G^<(\boldsymbol{r}_1,t_1;\boldsymbol{r}_2,t_2) = \frac{i}{2}(G^K - G^R + G^A)$$

Dyson Eq. on $G^<$ is reduced to the Boltzmann kinetic equation under the approximation called "*gradient expansion*"

Wigner Transform

$$G(\boldsymbol{r}_1, t_1; \boldsymbol{r}_2, t_2) = \int \frac{d\varepsilon}{2\pi} \int \frac{d^2\boldsymbol{p}}{(2\pi)^2} e^{-i\varepsilon(t_1-t_2)+i\boldsymbol{p}(\boldsymbol{r}_1-\boldsymbol{r}_2)} G(\underbrace{\varepsilon, t; \boldsymbol{p}, \boldsymbol{r}})$$

smooth function of all 4 variables

$$\boldsymbol{r} = \frac{\boldsymbol{r}_1 + \boldsymbol{r}_2}{2}$$ ***- physical coordinate***

$$\boldsymbol{t} = \frac{\boldsymbol{t}_1 + \boldsymbol{t}_2}{2}$$ ***- physical time***

Equlibrium in time:
$$\begin{cases} G(t_1, t_2) = G(t_1 - t_2) \;\to\; G(\varepsilon) \\ G^<(\varepsilon) = i f(\varepsilon)\big(G^R(\varepsilon) - G^A(\varepsilon)\big) \end{cases}$$

no *t* - dependence

$f(\varepsilon)$ ***- Fermi distribution function***

Linear response

Boltzmann kinetics is difficult for spin systems!

For linear response we take on an alternative direct route through generalised "Kubo" formulas

Consider response to electric field: $\boldsymbol{E} = -\dfrac{\partial \boldsymbol{A}}{\partial t}$

$H = H_0 - \hat{\boldsymbol{j}} \cdot \boldsymbol{A}$ $\quad$ $\boldsymbol{A}(t) = \dfrac{\boldsymbol{E} e^{-i\Omega t}}{i\Omega}$ $\quad$ ***dc limit:*** $\Omega \to 0$

Perturbation theory:

$$G = G_0 - G_0 \underbrace{(\hat{\boldsymbol{j}} \cdot \boldsymbol{A})}_{\textit{linear response:}} G_0 + \ldots$$

$$G(\boldsymbol{r}_1, t_1; \boldsymbol{r}_2, t_2) = G_0(\boldsymbol{r}_1, t_1; \boldsymbol{r}_2, t_2) - \int dt' \int d^2\boldsymbol{r}' G_0(\boldsymbol{r}_1, t_1; \boldsymbol{r}', t') \hat{\boldsymbol{j}}(\boldsymbol{r}') \cdot \boldsymbol{A}(t') G_0(\boldsymbol{r}', t'; \boldsymbol{r}_2, t_2) + \ldots$$

G_0 **- corresponds to an equilibrium in time**

Most general result for linear response

Contribution from the Fermi surface:
responsible for classical (Drude) results, weak localisation corrections, etc

$$\delta G^{<}(\varepsilon,t;\boldsymbol{r},\boldsymbol{r}) = \boxed{-i\int d^2\boldsymbol{r}'\left(G^R_{\boldsymbol{r},\boldsymbol{r}'}\,\hat{\boldsymbol{j}}\boldsymbol{E}\,G^A_{\boldsymbol{r}',\boldsymbol{r}} - \frac{1}{2}G^R_{\boldsymbol{r},\boldsymbol{r}'}\,\hat{\boldsymbol{j}}\boldsymbol{E}\,G^R_{\boldsymbol{r}',\boldsymbol{r}} - \frac{1}{2}G^A_{\boldsymbol{r},\boldsymbol{r}'}\,\hat{\boldsymbol{j}}\boldsymbol{E}\,G^A_{\boldsymbol{r}',\boldsymbol{r}}\right)\left(-\frac{\partial f}{\partial\varepsilon}\right)}$$

$$\boxed{+\frac{1}{2i}\int d^2\boldsymbol{r}'\left(G^R_{\boldsymbol{r},\boldsymbol{r}'}\,\hat{\boldsymbol{j}}\boldsymbol{E}\,\frac{\partial G^R_{\boldsymbol{r}',\boldsymbol{r}}}{\partial\varepsilon} - \frac{\partial G^R_{\boldsymbol{r},\boldsymbol{r}'}}{\partial\varepsilon}\,\hat{\boldsymbol{j}}\boldsymbol{E}\,G^R_{\boldsymbol{r}',\boldsymbol{r}} - G^A_{\boldsymbol{r},\boldsymbol{r}'}\,\hat{\boldsymbol{j}}\boldsymbol{E}\,\frac{\partial G^A_{\boldsymbol{r}',\boldsymbol{r}}}{\partial\varepsilon} + \frac{\partial G^A_{\boldsymbol{r},\boldsymbol{r}'}}{\partial\varepsilon}\,\hat{\boldsymbol{j}}\boldsymbol{E}\,G^A_{\boldsymbol{r}',\boldsymbol{r}}\right)f(\varepsilon)}$$

Topological contribution from the entire Fermi sea:
responsible for QHE & AQHE, related to Chern number and Berry curvature

Non-locality is fully taken into account!

Conductivity

Consider electric current: $\boldsymbol{J} = \langle \hat{j} \rangle = -i\,\mathrm{Tr}\,\hat{j}G^{<} = \hat{\sigma}\boldsymbol{E}$

$$\hat{\sigma} = \hat{\sigma}^{I} + \hat{\sigma}^{II}$$

Kubo & Streda

$\sigma^{II}_{xx} = 0$

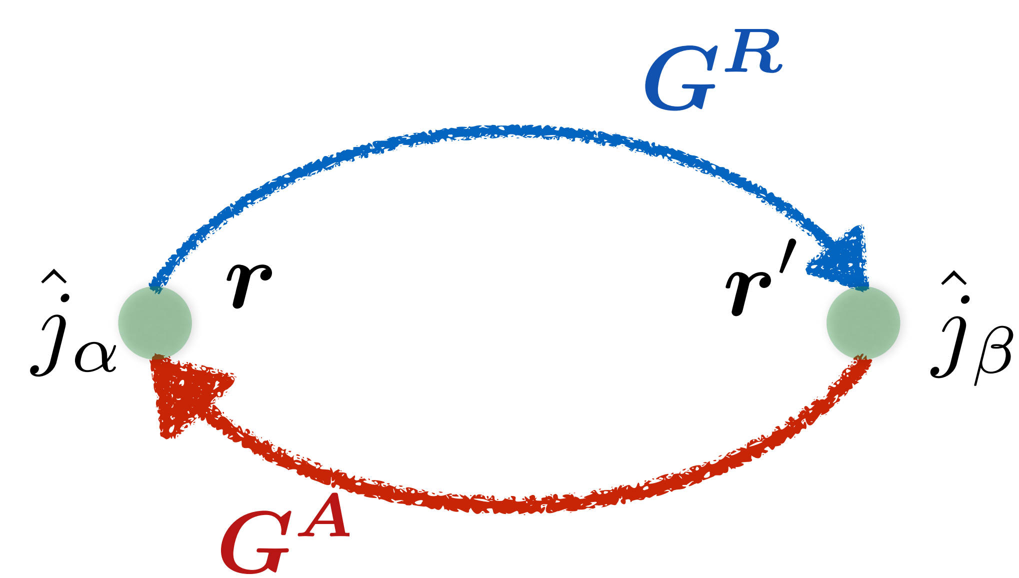
σ^{II}_{xy} **- non-zero for topologically non-trivial systems (QHE etc.)**

$$\sigma^{I}_{\alpha\beta} = \frac{i}{2}\int\frac{d\varepsilon}{2\pi}\left(-\frac{\partial f}{\partial\varepsilon}\right)\mathrm{Tr}\left[\hat{j}_\alpha G^R \hat{j}_\beta \mathcal{A} - \hat{j}_\alpha \mathcal{A} \hat{j}_\beta G^A\right]$$

averaging over space & disorder assumed

$$\mathcal{A} = \frac{i}{2\pi}(G^R - G^A)$$

Main contribution: $\sigma^{I}_{\alpha\beta} =$



“Kubo-Streda” for local spin polarization

$$\boldsymbol{s}(\boldsymbol{r},t) = \boldsymbol{s}^I(\boldsymbol{r},t) + \boldsymbol{s}^{II}(\boldsymbol{r},t)$$

$\boldsymbol{s}^I$ **- responsible for normal torques (spin-orbit, spin transfer etc.)**

$\boldsymbol{s}^{II}$ **- responsible for topological (quantised) torques**

$$\boldsymbol{s}^I(\boldsymbol{r}) = \frac{1}{2}\int d^2\boldsymbol{r}'\, \mathrm{Tr}_\sigma \left[\boldsymbol{\sigma}\left(G^R_{\boldsymbol{r},\boldsymbol{r}'}\, \boldsymbol{j}\cdot\boldsymbol{E}\, G^A_{\boldsymbol{r}',\boldsymbol{r}} - \tfrac{1}{2}G^R_{\boldsymbol{r},\boldsymbol{r}'}\, \boldsymbol{j}\cdot\boldsymbol{E}\, G^R_{\boldsymbol{r}',\boldsymbol{r}} - \tfrac{1}{2}G^A_{\boldsymbol{r},\boldsymbol{r}'}\, \boldsymbol{j}\cdot\boldsymbol{E}\, G^A_{\boldsymbol{r}',\boldsymbol{r}}\right)\right]$$

where $G^R_{\boldsymbol{r},\boldsymbol{r}'} = G^R(\varepsilon, t, \boldsymbol{r}, \boldsymbol{r}')$ **is Wigner transform in time**

The formula is still fully non-local!

$$\left(i\frac{\partial}{\partial t_1} - H_1 + i0\right) G^R(t_1, \boldsymbol{r}_1; t_2, \boldsymbol{r}_2) = \delta(t_1 - t_2)\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

Spin texture: gradient expansion

$$H = h(\boldsymbol{p}) + \boldsymbol{\sigma}\boldsymbol{M}(\boldsymbol{r}) \qquad\qquad \boldsymbol{M} = M\boldsymbol{m}$$

*exchange coupling between texture **M(r,t)** and the spin of itinerant electrons*

Perform Wigner transform in space assuming that *M* is a smooth function of the coordinate: gradient expansion!

If $H = h(\boldsymbol{p}) + V(\boldsymbol{r})$ **then**

$$G(\boldsymbol{r},\boldsymbol{p}) = G_0 + \frac{i}{2}\left(G_0\,\boldsymbol{\nabla}V\,G_0\,\boldsymbol{v}\,G_0 - G_0\,\boldsymbol{v}\,G_0\,\boldsymbol{\nabla}V\,G_0\right) + \ldots$$

up to the first order in the spacial gradient

where

$$G_0^R(\boldsymbol{r},\boldsymbol{p}) = \frac{1}{\varepsilon - h(\boldsymbol{p}) - V(\boldsymbol{r}) + i0} \qquad\qquad \boldsymbol{v} = \nabla_{\boldsymbol{p}} h(\boldsymbol{p})$$

Also use

$$(A\circ B)(\boldsymbol{r},\boldsymbol{p}) = A(\boldsymbol{r},\boldsymbol{p})B(\boldsymbol{r},\boldsymbol{p}) + \frac{i}{2}\left\{(\nabla_{\boldsymbol{r}}A)(\nabla_{\boldsymbol{p}}B) - (\nabla_{\boldsymbol{p}}A)(\nabla_{\boldsymbol{r}}B)\right\} + \ldots$$

Poisson bracket

K & R tensors

Up to the first-order gradients in space:

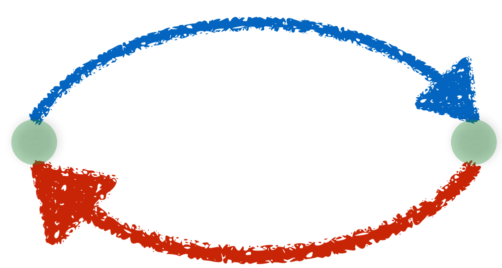
$$s_\alpha = \sum_\beta K_{\alpha\beta} E_\beta + \sum_{\beta,\gamma,\delta} R^{\gamma\delta}_{\alpha\beta} (\nabla_\delta M_\gamma) E_\beta$$

tensors responsible for

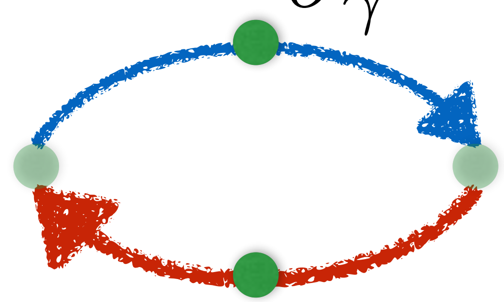
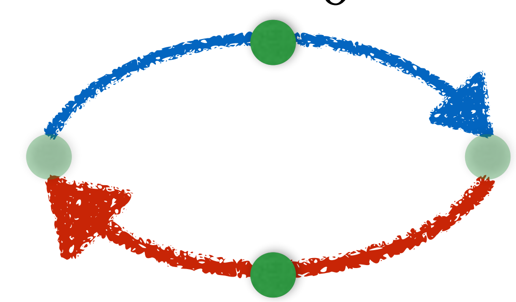
SOT

STT

diagrammatic representation of tensors:

$K^I_{\alpha\beta} = \sigma_\alpha$  $\hat{j}_\beta$

notion of spin-current is not needed!

$(R^I)^{\gamma\delta}_{\alpha\beta} = \frac{i}{2}\Big[\sigma_\alpha$  $\hat{j}_\beta - \sigma_\alpha$  $\hat{j}_\beta\Big] + \ldots$

σ & P tensors

One applies electric current!

Electric field must be excluded from the relations:

$$s_\alpha = \sum_\beta K_{\alpha\beta} E_\beta + \sum_{\beta,\gamma,\delta} R_{\alpha\beta}^{\gamma\delta} (\nabla_\delta M_\gamma) E_\beta$$

$$J_\alpha = \sum_\beta \sigma_{\alpha\beta} E_\beta + \sum_{\beta,\gamma,\delta} P_{\alpha\beta}^{\gamma\delta} (\nabla_\delta M_\gamma) E_\beta$$

To compute SOT & STT torques one needs to compute, in addition to the conductivity tensor, also the tensors *K*, *R*, and *P* !!!

Models of disorder

Any computation of torques induced by *dc*-currents must take into account electron scattering on impurities!

$$V(\boldsymbol{r}) = \sum_i u(\boldsymbol{r} - \boldsymbol{R}_i)$$

$\boldsymbol{R}_i$ **- random impurity coordinates**

$u(\boldsymbol{r})$ **- impurity potential**

$$u(\boldsymbol{r}) = u_0 \delta(\boldsymbol{r})$$ ***- scalar spin-independent disorder***

$$u(\boldsymbol{r}) = \frac{1 + \boldsymbol{\sigma m}}{2} u_\uparrow(\boldsymbol{r}) + \frac{1 - \boldsymbol{\sigma m}}{2} u_\downarrow(\boldsymbol{r})$$ ***- paramagnetic disorder***

$$u(\boldsymbol{r}) = u_0(\boldsymbol{r}) + u_1(\boldsymbol{r})\, \boldsymbol{S} \cdot \boldsymbol{\sigma}$$ ***- random magnetic disorder***

Gaussian limit (for spin-independent disorder): $n_{\mathrm{imp}} \to \infty \quad u_0 \to 0$

$$\langle V(\boldsymbol{r}) V(\boldsymbol{r}') \rangle_{\mathrm{dis}} = \frac{1}{m\tau} \delta(\boldsymbol{r} - \boldsymbol{r}')$$

τ **- scattering time**

m **- effective electron mass**

Metal parameter

for FM/good heavy-metal bilayer

$$\frac{\sigma_{xx}}{\sigma_{xy}} \propto \varepsilon_F \tau \gg 1$$

That is why the effect of AHE is often disregarded in the system

What are the implications of this condition for spin torques?

$\sigma_{xx} \propto \varepsilon_F \tau$ **- dissipative and even in magnetisation**

$\sigma_{xy} \propto 1$ **- non-dissipative and odd in magnetisation**

Can we make similar statements for the components of all other tensors: *K*, *R*, and *P* in a given model?

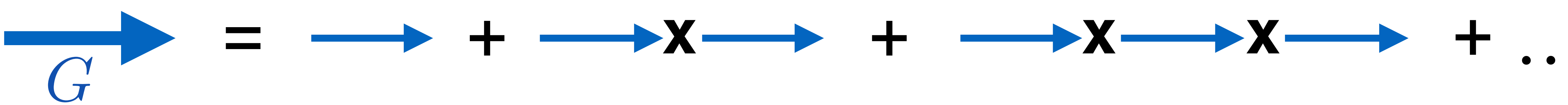
Self-consistent Born approximation (SCBA)

is justified by the condition $\varepsilon_F \tau \gg 1$

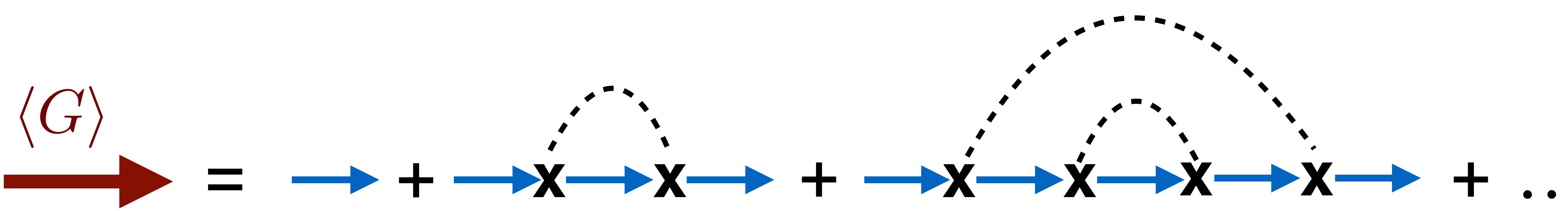
(but only for those quantities which are proportional to $\varepsilon_F \tau$ **)**

$$H = H_0 + V$$

Dyson series: $$G = G_0 + G_0 V G_0 + G_0 V G_0 V G_0 + \ldots$$

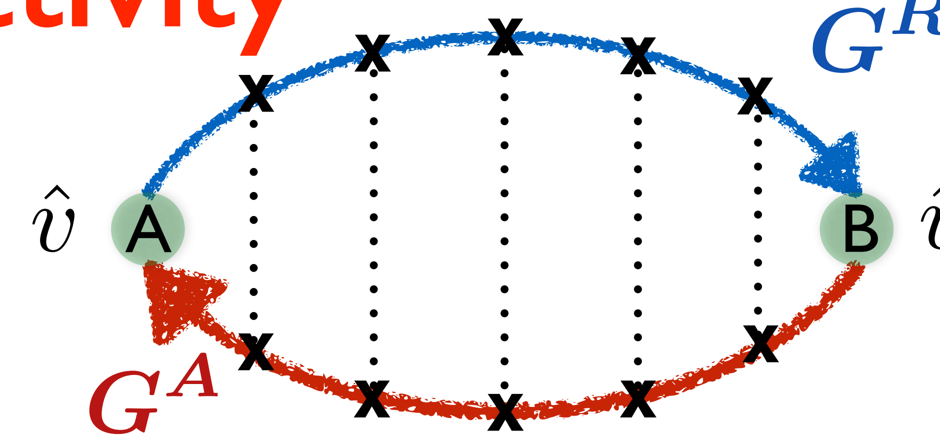


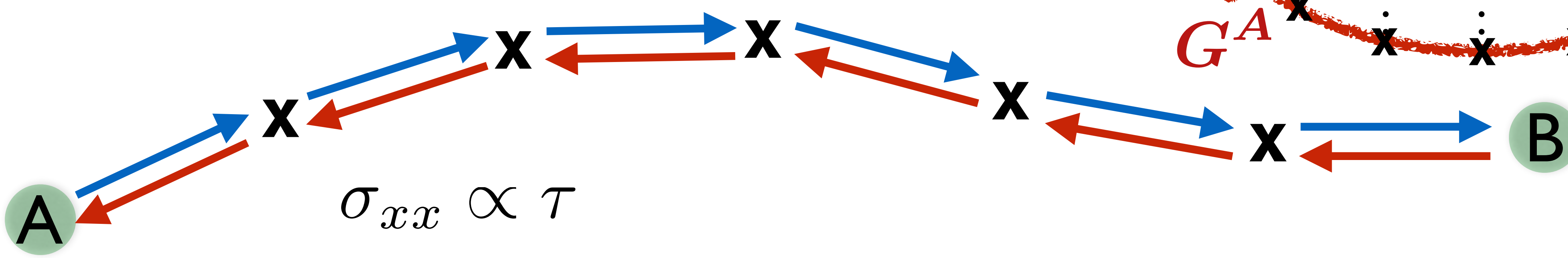
Averaging (gaussian disorder): $$\langle V \rangle = 0, \quad \langle VV \rangle \propto \tau^{-1} \delta(\boldsymbol{r} - \boldsymbol{r}')$$



Standard theory of conductivity

classical diffusion:
non-crossing (ladder) approx.



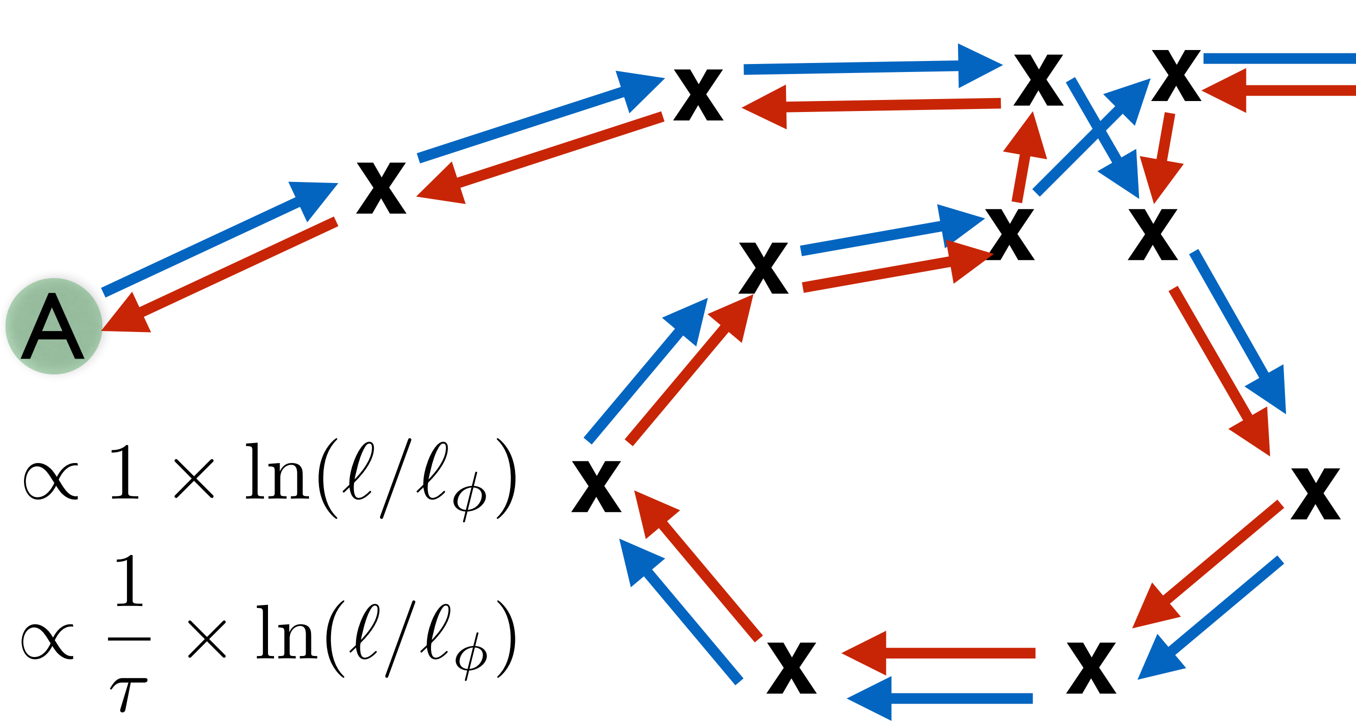


$\sigma_{xx} \propto \tau$

describes classical diffusion propagation from A to B

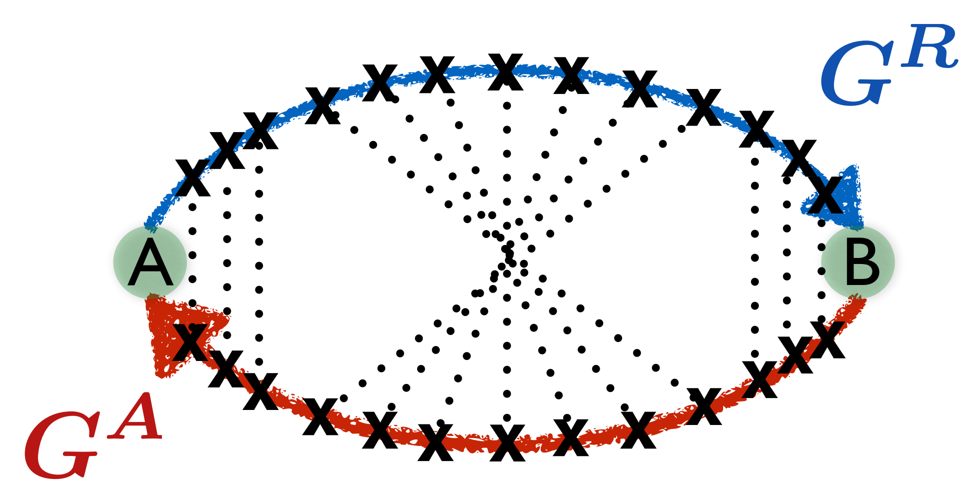
weak localisation correction to classical diffusion

leading interference correction to classical diffusion



$$\sigma_{xx} \propto 1 \times \ln(\ell/\ell_\phi)$$

$$\sigma_{xy} \propto \frac{1}{\tau} \times \ln(\ell/\ell_\phi)$$



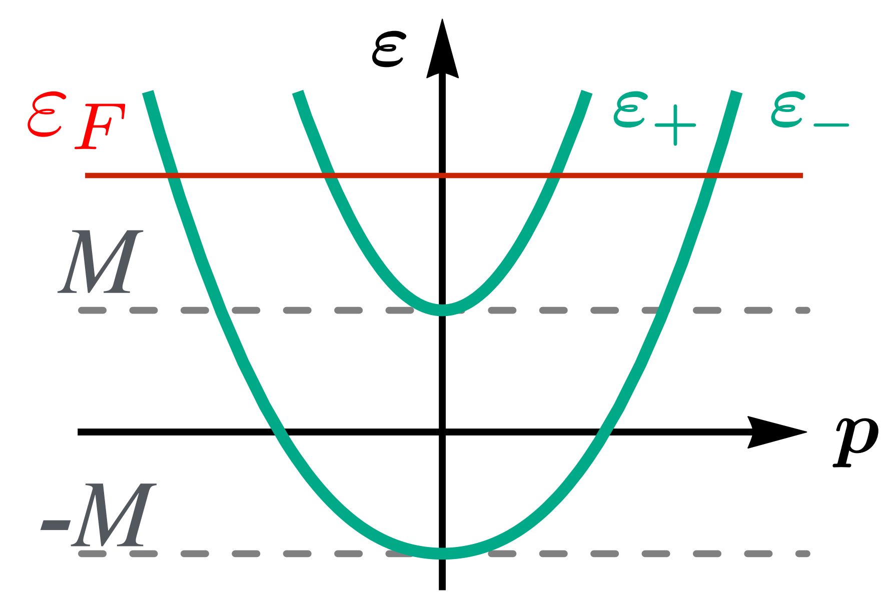
Model Hamiltonian

We want to apply this powerful machinery to the analysis of spin susceptibility tensors that define spin-torques!

Consider FM/heavy-metal bilayer

Integrate over z to get an effective 2D model for conduction electrons

$$H = \xi_{\boldsymbol{p}} + \alpha_R \underbrace{(\boldsymbol{\sigma} \times \boldsymbol{p})_z}_{\textit{Rashba SOI}} + \underbrace{\boldsymbol{M} \cdot \boldsymbol{\sigma}}_{\textit{exchange}}$$



Assume $\varepsilon_F > M$ (two Fermi surfaces)

We start with the symmetry analysis for K tensor

Symmetry of K

For the symmetry analysis we choose normal coordinates

$$H = \xi_p + \alpha_R(\boldsymbol{\sigma} \times \boldsymbol{p})_z + M_z\sigma_z + \underline{M_x\sigma_x} \qquad \boldsymbol{M} = M\boldsymbol{m}$$

anisotropy in the plane

$$\hat{\boldsymbol{j}} = e\hat{\boldsymbol{v}} \qquad \hat{\boldsymbol{v}} = \nabla_{\boldsymbol{p}}\xi_p + \alpha_R\hat{\boldsymbol{z}} \times \boldsymbol{\sigma}$$

symmetry I:

$$\sigma_x H[-p_x]\sigma_x = H[-M_z]$$
$$\sigma_x v_x[-p_x]\sigma_x = -v_x$$
$$\sigma_x v_y[-p_x]\sigma_x = v_y$$

symmetry II:

$$\sigma_z H[-\boldsymbol{p}]\sigma_z = H[-M_x]$$
$$\sigma_z \boldsymbol{v}[-\boldsymbol{p}]\sigma_z = -\boldsymbol{v}$$

Use the symmetries in the formula for *K*

$$K_{\alpha\beta} = \frac{e}{2}\int \frac{d^2\boldsymbol{p}}{(2\pi)^2}\,\mathrm{Tr}\left[\sigma_\alpha G^R_{\boldsymbol{p}}\hat{v}_\beta G^A_{\boldsymbol{p}}\right]$$

$$\hat{K} = \begin{pmatrix} m_z\kappa_{xx} & \kappa_{xy} \\ \kappa_{yx} & m_z\kappa_{yy} \\ m_x\kappa_{zx} & m_x m_z\kappa_{zy} \end{pmatrix}$$

$\kappa_{\alpha\beta}$ **are the functions of**

$$m_x^2 = 1 - m_z^2$$

SOT: general results

$$\frac{\partial \boldsymbol{m}}{\partial t} = \boldsymbol{f} \times \boldsymbol{m}$$ - magnetization dynamics

$$\boldsymbol{f}_{\mathrm{SOT}} = \hat{K}\boldsymbol{E}$$ - non-equlibrium polarization due to SOI

$$\boldsymbol{T}_{\mathrm{SOT}} = \boldsymbol{f}_{\mathrm{SOT}} \times \boldsymbol{m}$$ - spin-orbit torque

Symmetries of the tensor in the normal coordinates:

$$\hat{K} = \begin{pmatrix} m_z\kappa_{xx} & \kappa_{xy} \\ \kappa_{yx} & m_z\kappa_{yy} \\ m_x\kappa_{zx} & m_x m_z\kappa_{zy} \end{pmatrix}$$

$$\boxed{\begin{aligned} &a, \bar{a} \propto \tau \\ &b, \bar{b} \propto 1 \\ &a/b \propto \tau\Delta_s \end{aligned}}$$

Alternatively one can write: $\boldsymbol{T}_{\mathrm{SOT}} = \boldsymbol{T}^{\parallel} + \boldsymbol{T}^{\perp}$

$$\boldsymbol{T}^{\parallel} = a\,\boldsymbol{m} \times (\hat{\boldsymbol{z}} \times \boldsymbol{E}) - \bar{a}\,\boldsymbol{m} \times (\boldsymbol{m} \times \hat{\boldsymbol{z}})\,(\boldsymbol{m} \cdot \boldsymbol{E})$$

$$\boldsymbol{T}^{\perp} = b\,\boldsymbol{m} \times (\boldsymbol{m} \times (\hat{\boldsymbol{z}} \times \boldsymbol{E})) - \bar{b}\,\boldsymbol{m} \times \hat{\boldsymbol{z}}\,(\boldsymbol{m} \cdot \boldsymbol{E})$$

$$\Delta_s = \sqrt{M_z^2 + 2\varepsilon_F m_{\mathrm{e}} \alpha_{\mathrm{R}}^2}$$

A one-to-one correspondence exists between $a, b, \bar{a}, \bar{b}$ and $\kappa_{\alpha\beta}$

$\boldsymbol{T}^{\parallel}$ - changes sign under TR, hence it is anti-damping torque

$\boldsymbol{T}^{\perp}$ - invariant under TR, hence it is field-like torque

Response to electric current

same symmetries in normal coordinates:

$$\hat{K}_{\mathrm{J}} = \begin{pmatrix} m_z\tilde{\kappa}_{xx} & \tilde{\kappa}_{xy} \\ \tilde{\kappa}_{yx} & m_z\tilde{\kappa}_{yy} \\ m_x\tilde{\kappa}_{zx} & m_x m_z\tilde{\kappa}_{zy} \end{pmatrix}$$

introduce the tensor: $\hat{K}_J = \hat{K}\hat{\sigma}^{-1}$

$$\boldsymbol{f}_{\mathrm{SOT}} = \hat{K}_J \boldsymbol{J}$$

spin-orbit torque $\boldsymbol{T}_{\mathrm{SOT}} = \boldsymbol{f}_{\mathrm{SOT}} \times \boldsymbol{m} = \boldsymbol{T}_{\mathrm{J}}^{||} + \boldsymbol{T}_{\mathrm{J}}^{\perp}$

$\boldsymbol{T}_{\mathrm{J}}^{||} = a_{\mathrm{J}}\,\boldsymbol{m} \times (\hat{\boldsymbol{z}} \times \boldsymbol{J}) - \bar{a}_{\mathrm{J}}\,\boldsymbol{m} \times (\boldsymbol{m} \times \hat{\boldsymbol{z}})\,(\boldsymbol{m} \cdot \boldsymbol{J}),$ ***- field-like !***

$\boldsymbol{T}_{\mathrm{J}}^{\perp} = b_{\mathrm{J}}\,\boldsymbol{m} \times (\boldsymbol{m} \times (\hat{\boldsymbol{z}} \times \boldsymbol{J})) - \bar{b}_{\mathrm{J}}\,\boldsymbol{m} \times \hat{\boldsymbol{z}}\,(\boldsymbol{m} \cdot \boldsymbol{J}),$ ***- anti-damping !***

Note that classification to ADL and FL is now reversed!

(notion of spin current is not needed for the theory)

Now: $a_{\mathrm{J}}, \bar{a}_{\mathrm{J}} \propto 1$ $\qquad b_{\mathrm{J}}, \bar{b}_{\mathrm{J}} \propto 1/\tau$

$$a_{\mathrm{J}} = \tilde{\kappa}_{xy} - m_x^2\tilde{\kappa}_{zy}, \qquad \bar{a}_{\mathrm{J}} = \tilde{\kappa}_{zy} - (\tilde{\kappa}_{xy} + \tilde{\kappa}_{yx})/m_x^2,$$

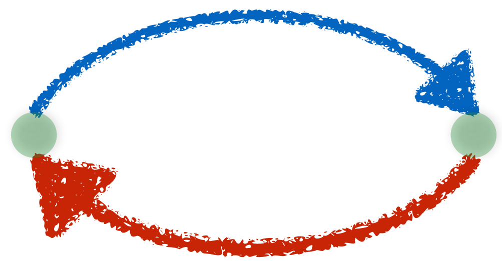
$$b_{\mathrm{J}} = \tilde{\kappa}_{yy}, \qquad \bar{b}_{\mathrm{J}} = \tilde{\kappa}_{xx} + \tilde{\kappa}_{zx} + (\tilde{\kappa}_{yy} - \tilde{\kappa}_{xx})/m_x^2,$$

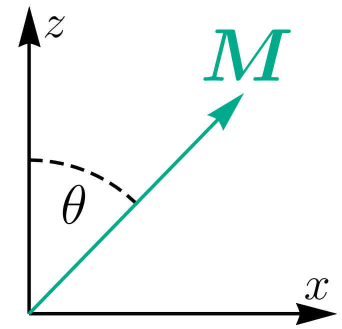
General:

$$\frac{a_J}{b_J} \propto \tau\Delta_s$$

Bare results

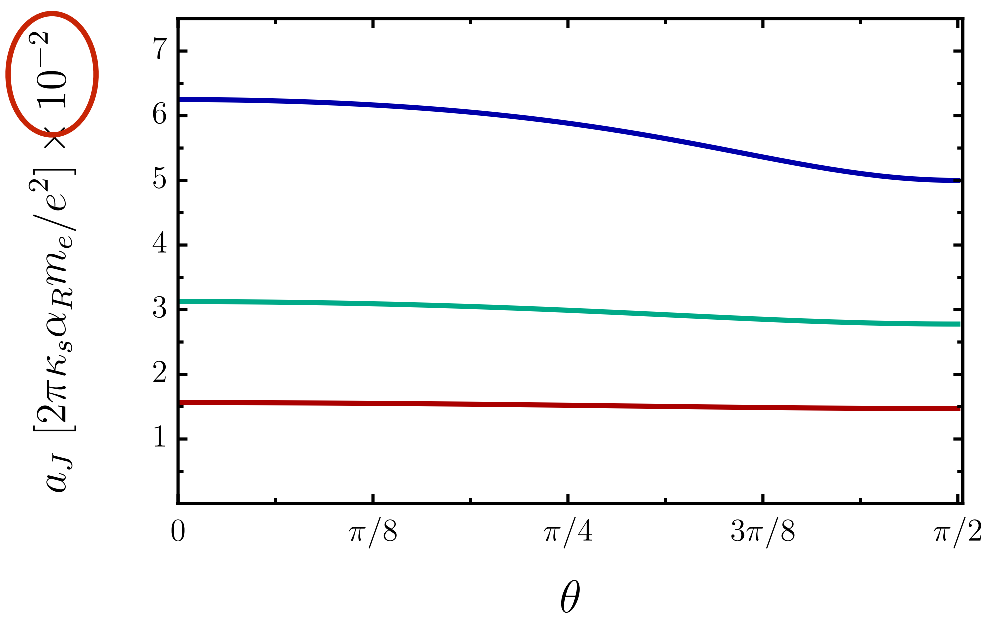
Use $\xi_p = \dfrac{p^2}{2m_\mathrm{e}}$

$K^I_{\alpha\beta} = \sigma_\alpha$  $\hat{j}_\beta$

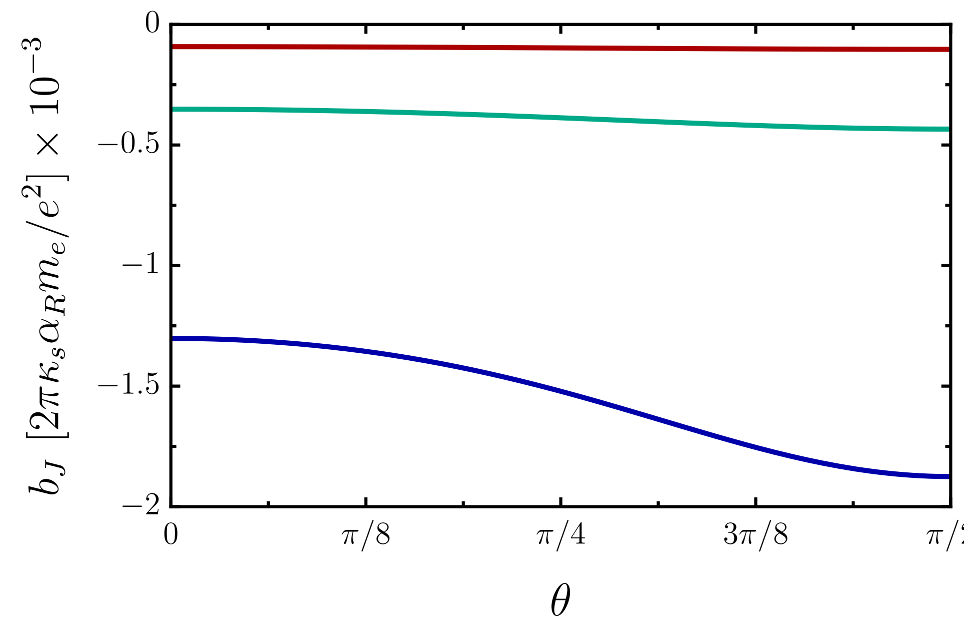


blue	$M = m\alpha_R^2 = 10/\tau = \varepsilon/2$
green	$M = m\alpha_R^2 = 10/\tau = \varepsilon/4$
red	$M = m\alpha_R^2 = 10/\tau = \varepsilon/8$

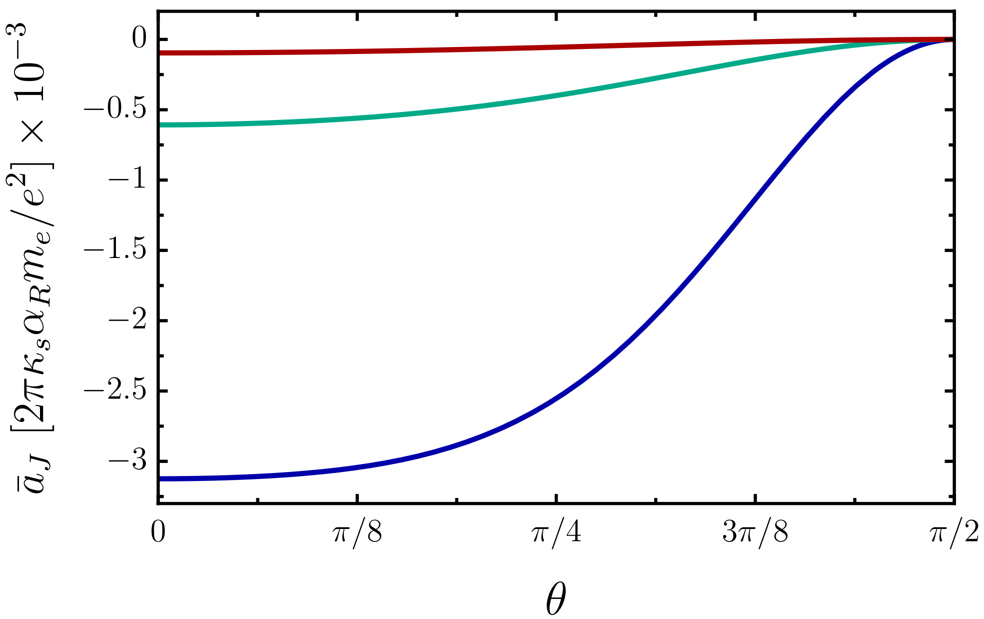
a_J



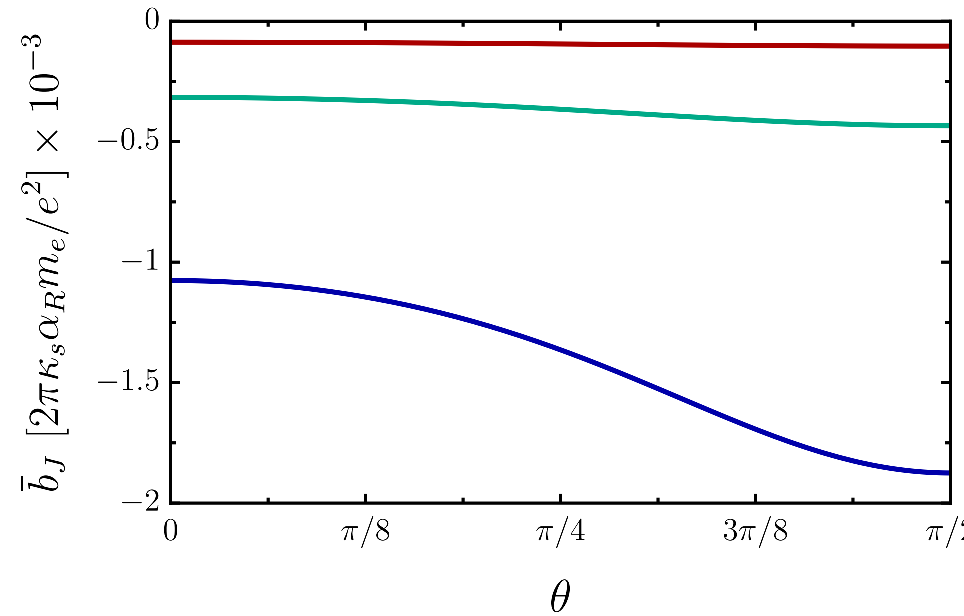
b_J



$\bar{a}_\mathrm{J}$

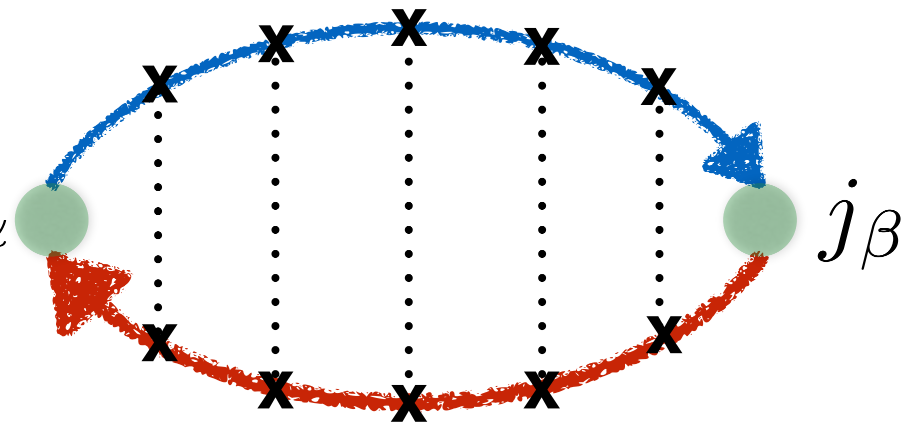


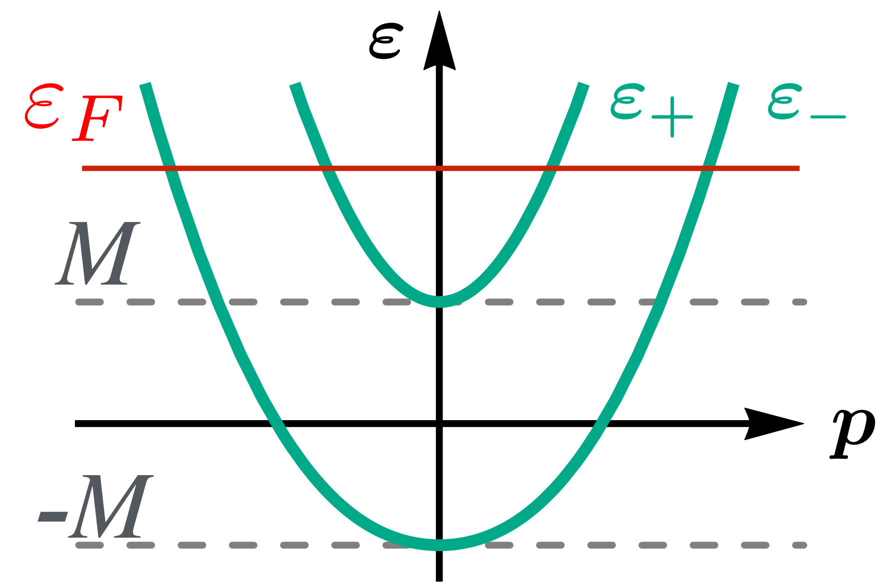
$\bar{b}_\mathrm{J}$



Final results for spin-independent disorder

$K^{I}_{\alpha\beta} =$





$$H = \frac{p^2}{2m_e} + \alpha_{\mathrm{R}}(\boldsymbol{\sigma} \times \boldsymbol{p})_z + \boldsymbol{M} \cdot \boldsymbol{\sigma} + V(\boldsymbol{r})$$

The "dressing" leads to immense cancelations!!!

$$\hat{K} = \kappa_s \alpha_{\mathrm{R}} m_e \begin{pmatrix} 0 & 2\tau \\ -2\tau & 0 \\ 0 & 0 \end{pmatrix}, \qquad \hat{\sigma} = \frac{e^2}{2\pi} 2\tau(\varepsilon + m_e \alpha_{\mathrm{R}}^2) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\hat{K}_{\mathrm{J}} = \hat{K}\hat{\sigma}^{-1} = a_{\mathrm{J}} \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad a_{\mathrm{J}} = \frac{2\pi \kappa_s \alpha_{\mathrm{R}} m_e}{e^2(\varepsilon + m_e \alpha_{\mathrm{R}}^2)} \qquad \kappa_s = \frac{\mu_B^2 \mu_0}{\pi d}$$

Only Edelstein effect survives!

$\boldsymbol{M} \cdot \boldsymbol{\sigma}$ **term is irrelevant!**

STT: results

$$\hat{R}^{\gamma\delta}_{\alpha\beta} = 0$$

for spin-independent disorder

SOI did not help STT torques

Thus, for the Bychkov-Rashba model with spin-independent disorder we are left with the only torque:

$$\boxed{\boldsymbol{T} = a_{\mathrm{J}}\, \boldsymbol{m} \times (\hat{\boldsymbol{z}} \times \boldsymbol{J})}$$

Spin-dependent disorder: stay tuned

Implications for domain motion

consider all 4 SOTs again:

$$\boldsymbol{T}_{\mathrm{J}}^{\|} = a_{\mathrm{J}}\, \boldsymbol{m} \times (\hat{\boldsymbol{z}} \times \boldsymbol{J}) - \bar{a}_{\mathrm{J}}\, \boldsymbol{m} \times (\boldsymbol{m} \times \hat{\boldsymbol{z}})\,(\boldsymbol{m} \cdot \boldsymbol{J}),$$

$$\boldsymbol{T}_{\mathrm{J}}^{\perp} = b_{\mathrm{J}}\, \boldsymbol{m} \times (\boldsymbol{m} \times (\hat{\boldsymbol{z}} \times \boldsymbol{J})) - \bar{b}_{\mathrm{J}}\, \boldsymbol{m} \times \hat{\boldsymbol{z}}\,(\boldsymbol{m} \cdot \boldsymbol{J}),$$

$$\frac{\partial \boldsymbol{m}}{\partial t} = -\gamma\, \boldsymbol{m} \times \boldsymbol{H}_{\mathrm{ext}} + \alpha_{\mathrm{G}}\, \boldsymbol{m} \times \frac{\partial \boldsymbol{m}}{\partial t} + \boldsymbol{T}$$

Solve Thiele equation for $\boldsymbol{m} = \boldsymbol{m}(\boldsymbol{r} - \boldsymbol{\nu} t)$

Result: $$\frac{\nu_y}{\nu_x} = \frac{DF_y - QF_x}{DF_x + QF_y}$$

$$Q = \frac{1}{4\pi} \int d^2\boldsymbol{r}\ \boldsymbol{m} \cdot [(\nabla_x \boldsymbol{m}) \times (\nabla_y \boldsymbol{m})]$$ **- topological charge**

$$D_{\alpha\beta} = \frac{\alpha_{\mathrm{G}}}{4\pi} \int d^2\boldsymbol{r}\ (\nabla_\alpha \boldsymbol{m}) \cdot (\nabla_\beta \boldsymbol{m}) = D\delta_{\alpha\beta}$$ **- damping**

$$F_\alpha = \frac{1}{4\pi} \int d^2\boldsymbol{r}\ (\nabla_\alpha \boldsymbol{m}) \cdot \boldsymbol{f}_{\mathrm{SOT}}$$ **- effective force**

Results for Skyrmion with Q=1

$$\boldsymbol{F} = 0, \text{ unless } Q = \pm 1$$

$$\frac{\nu_y}{\nu_x} = \frac{DF_y - QF_x}{DF_x + QF_y}$$

for azimutally-symmetric skyrmion: $m_z = \cos\theta(\rho)$

damping coefficient: $D = \alpha_G \int_0^\infty d\rho \, \frac{\sin^2\theta + (\rho\,\theta')^2}{4\rho}$

force:

$$\boldsymbol{F}^{\parallel} = \frac{\hat{\boldsymbol{z}} \times \boldsymbol{J}_\delta}{4} \int_0^\infty d\rho \left[\frac{\partial a_{\mathrm{J}}}{\partial \rho} - \bar{a}_{\mathrm{J}} \sin^2\theta\right] \sin\theta$$

$$\boldsymbol{F} = \boldsymbol{F}^{\perp} + \boldsymbol{F}^{\parallel}$$

$$\boldsymbol{F}^{\perp} = \frac{\boldsymbol{J}_\delta}{4} \int_0^\infty d\rho \left[\frac{b_{\mathrm{J}}}{2} \sin 2\theta + (b_{\mathrm{J}} - \bar{b}_{\mathrm{J}} \sin^2\theta)\rho\,\theta'\right]$$

δ **- skyrmion helicity** $\boldsymbol{J}_\delta = \boldsymbol{J}\cos\delta + (\boldsymbol{J} \times \hat{\boldsymbol{z}})\sin\delta$

Thus, we find no skyrmion drive when $a_{\mathrm{J}} = \mathrm{const}(\theta)$

and $\bar{a}_{\mathrm{J}} = b_{\mathrm{J}} = \bar{b}_{\mathrm{J}} = 0$

Open question

What is the role of thermal fluctuations of domain shapes!?

Things to do

Take a closer look to the spin-dependent disorder models!

Apply the theory to treat spin torques in anti-ferromagnets!

Thank you for your attention