



# Spin transport related to spin-orbit coupling

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# Next Generation Devices Desired

Non-volatile and higher performance devices

- Current stage of information storage

Volatile



Random Access Memory

Non-Volatile



Cassette Tape



Hard Disk Drive

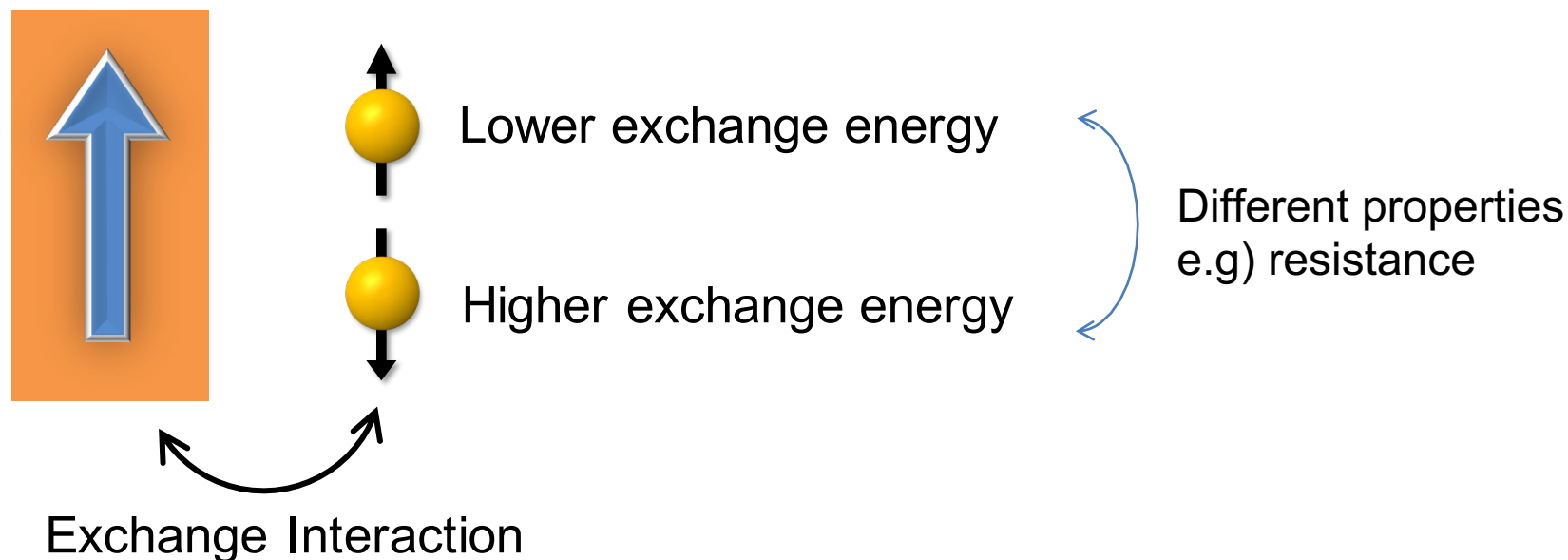
- (Pros) Higher Performance
- (Cons) Data lost when power is off
- (Pros) Data persist when power is off
- (Cons) Lower Performance

*Non-volatile with high performance? → Next generation devices*

# Why Magnetism?

Promising candidate for the next generation devices

- Magnetic state
  - Persists without power consumption (non-volatile)
  - Can be manipulated electrically (potentially fast)
- Coupling between electrons and magnetism



# Example: Electrical Detection of Magnetization

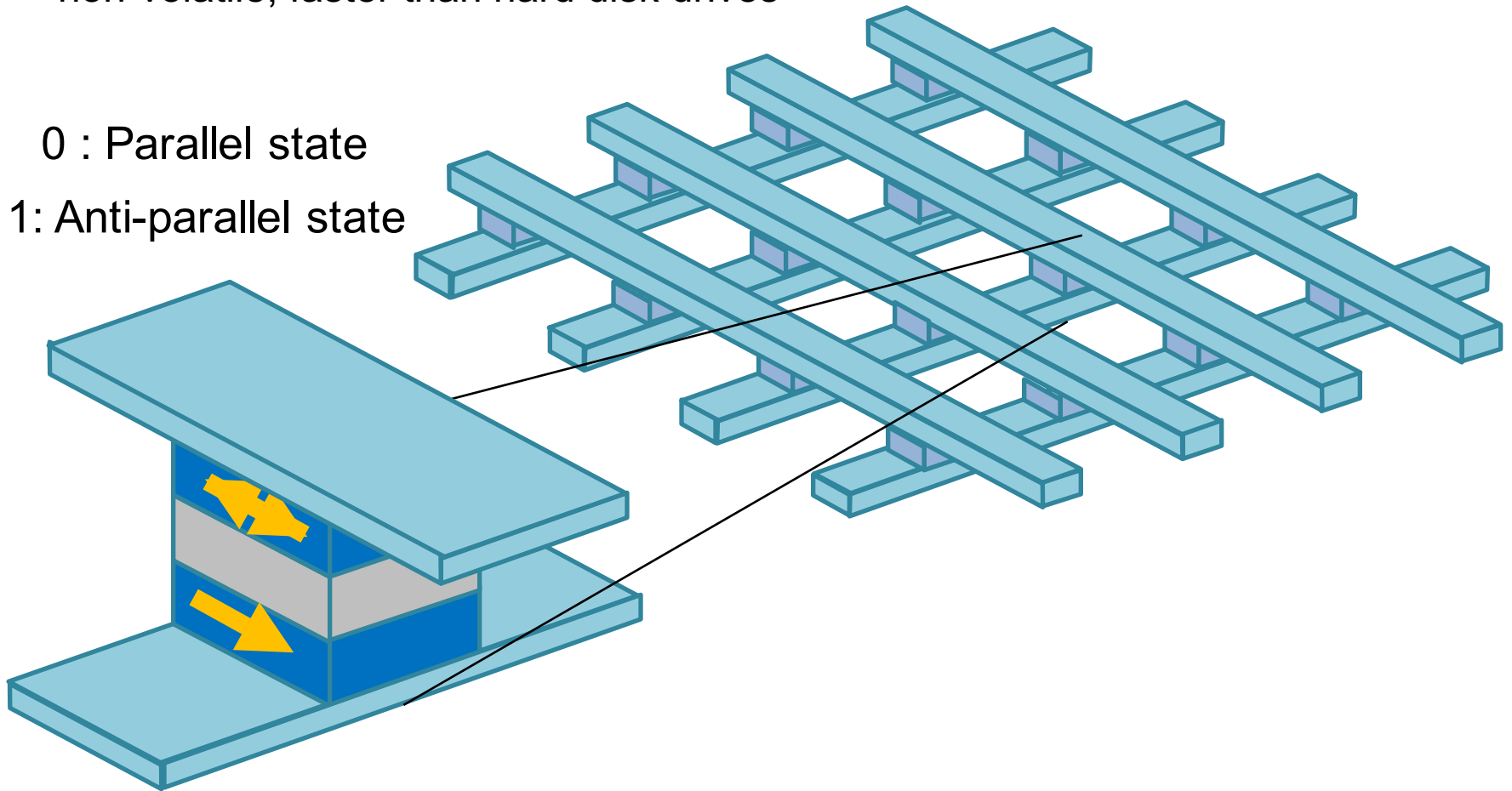
Exchange coupling allows to detect magnetization electrically

- Magnetic Random Access Memory

- non-volatile, faster than hard disk drives

0 : Parallel state

1 : Anti-parallel state

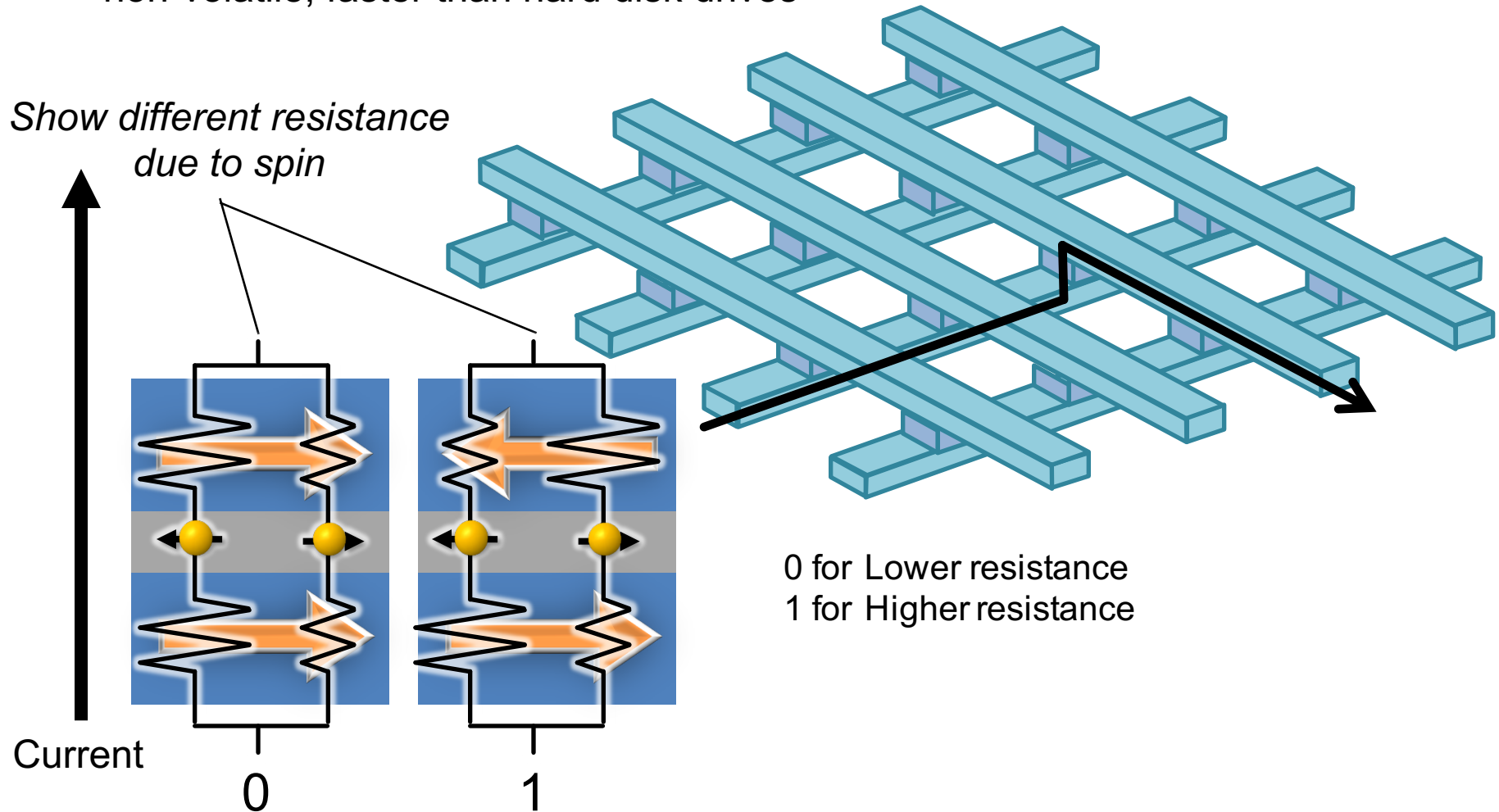




# Example: Electrical Detection of Magnetization

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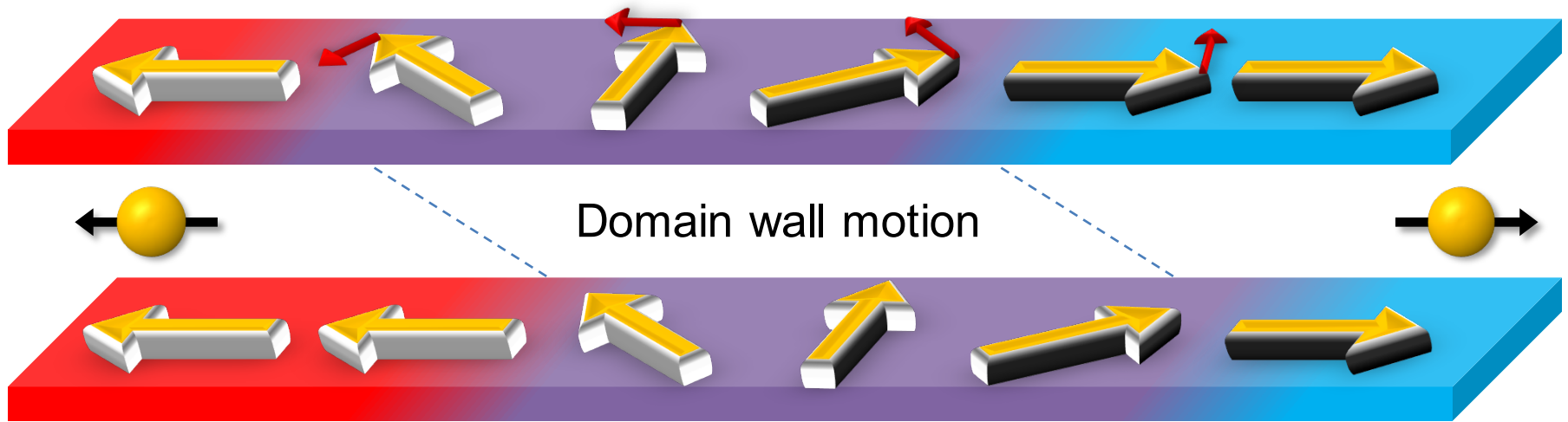
- Magnetic Random Access Memory
  - non-volatile, faster than hard disk drives



# Example: Electrical Manipulation of Magnetization

Exchange coupling allows to manipulate magnetization electrically

- Current-induced magnetic domain wall motion

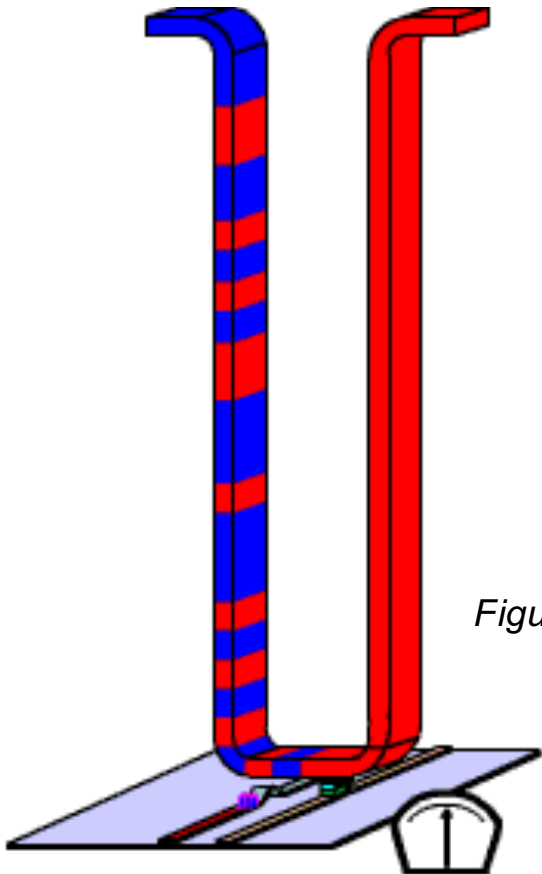


- Angular momentum conservation
  - **Spin-transfer torque**

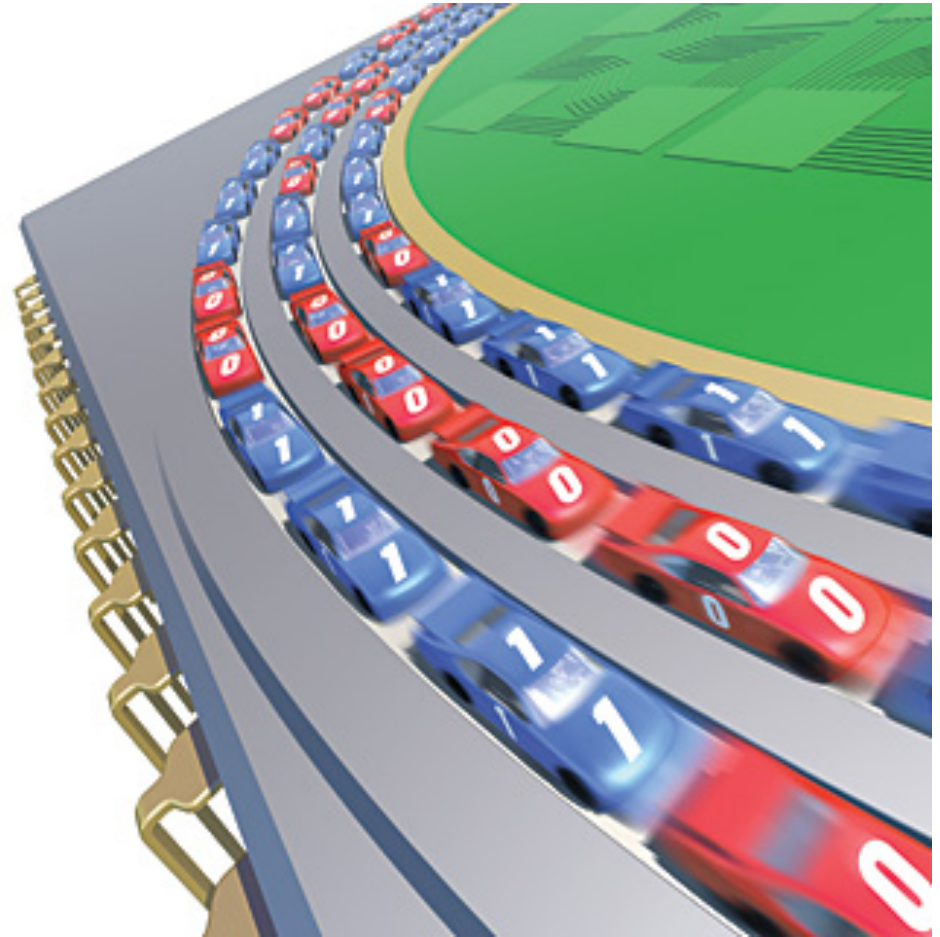
# Example: Electrical Manipulation of Magnetization

Exchange coupling allows to manipulate magnetization electrically

- Magnetic information transferred by electric fields
- Velocity  $\approx 100$  m/s



*Figure from IBM*



*Figure from ScientificAmerican*

# Why Spin-Orbit Coupling?

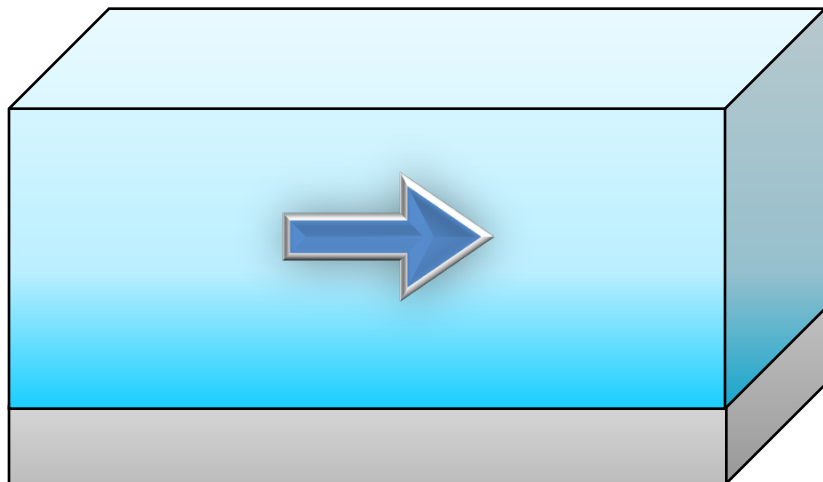
Spin-orbit coupling becomes important in low dimensional systems, enriches physics, and advances device application



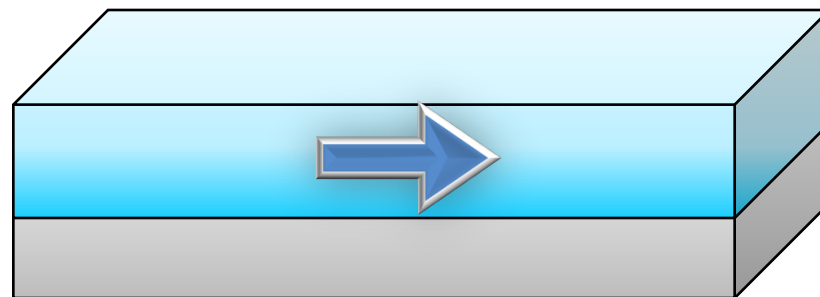
# Why spin-orbit coupling?

Nanostructures naturally accompany 'broken symmetry'

- Application: towards small size



Large size

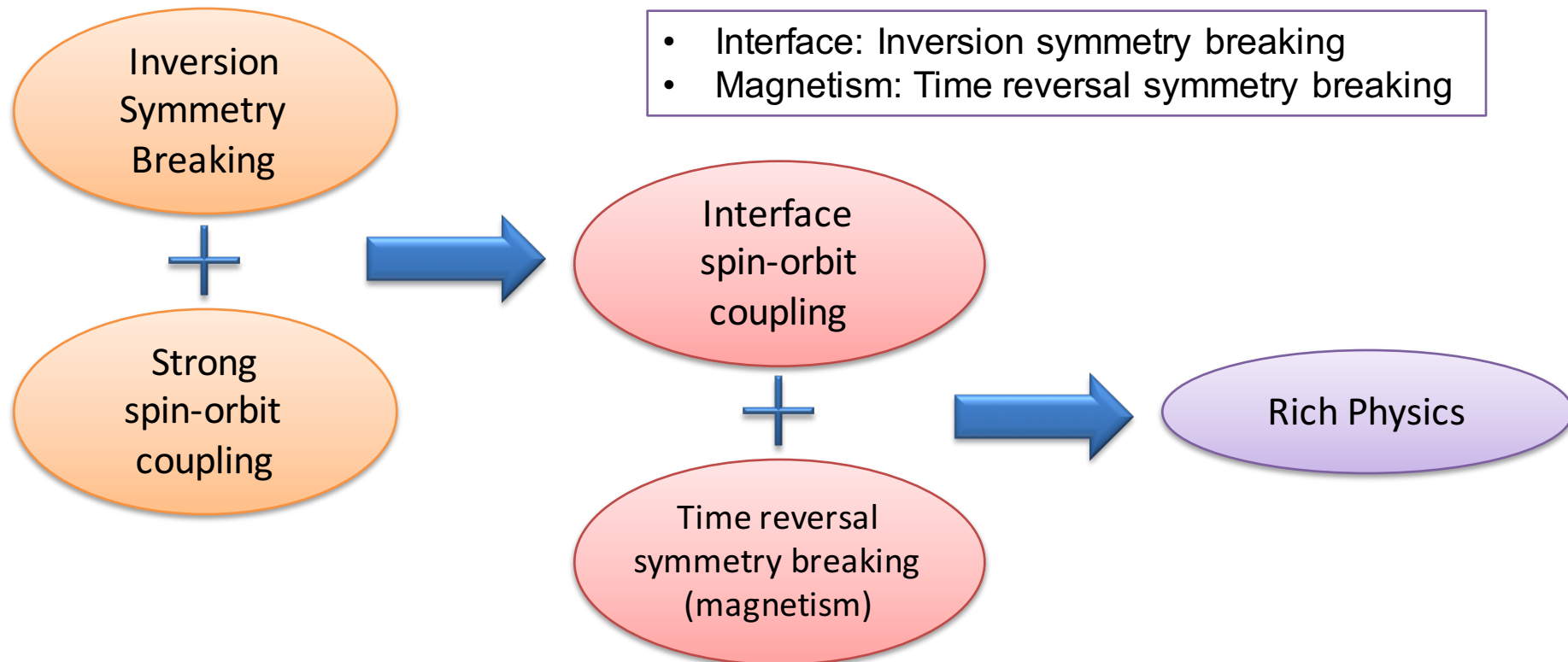
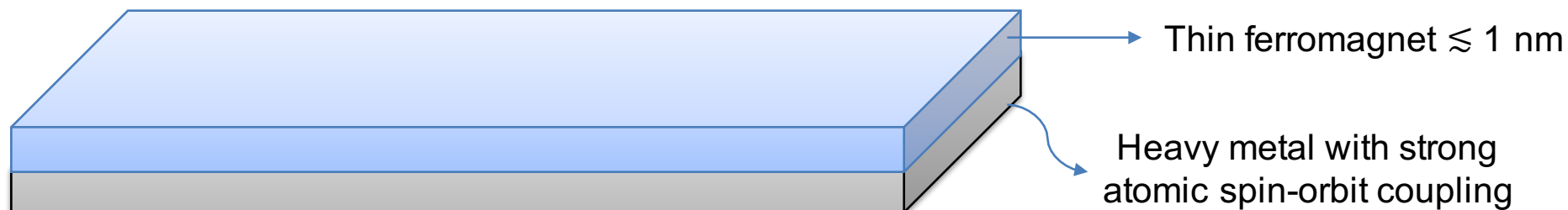


small size

**Interface** becomes dominant  
**Symmetry breaking** at the interface

# Why spin-orbit coupling?

Broken symmetry gives rise to interface spin-orbit coupling, enriching physics

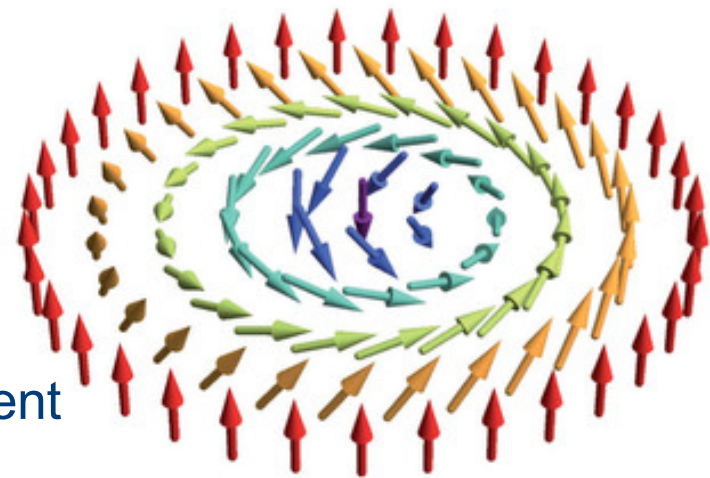


[Park, PRB (2013)]

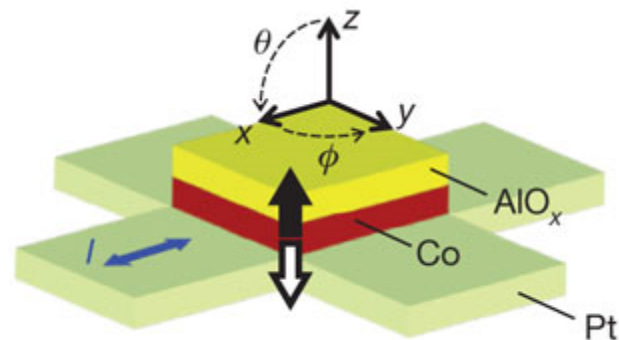
# Why spin-orbit coupling?

Examples of the 'rich physics'

- The Dzyaloshinskii-Moriya interaction
  - Skyrmions and Néel domain walls
- Spin-orbit torque
  - Magnetization reversal by an in-plane current
    - [Miron, Nat. (2011)], [Liu, Science (2012)]
  - Reversed domain wall motion direction
    - [Emori, Nat. Mater. (2013)], [Ryu, Nat. Nano. (2013)]



[Duine, Nat. Nano. (2013)]



- Perpendicular magnetic anisotropy
  - [Barnes, Sci. Rep. (2014)]
- Rashba spin-motive force
  - [Kim, PRL (2012)]

# Contents

- Quantum transport at spin-orbit coupled interfaces
  - Two-dimensional Rashba Model
  - Beyond the two-dimensional model
  - Results and implications
  
- *Intrinsic* non-adiabatic spin-transfer torque
  - Current-induced domain wall motion
  - *Intrinsic* spin-orbit torque
  - Chiral derivatives
  - *Intrinsic* non-adiabatic spin-transfer torque

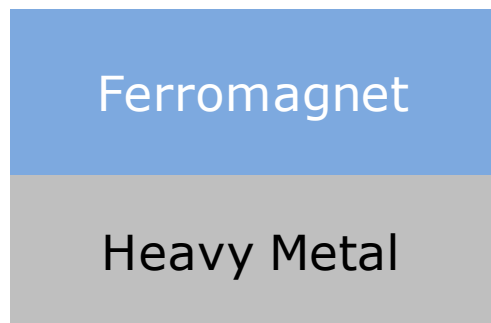


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# Motivation

A careful treatment is required for interpretation of experiments



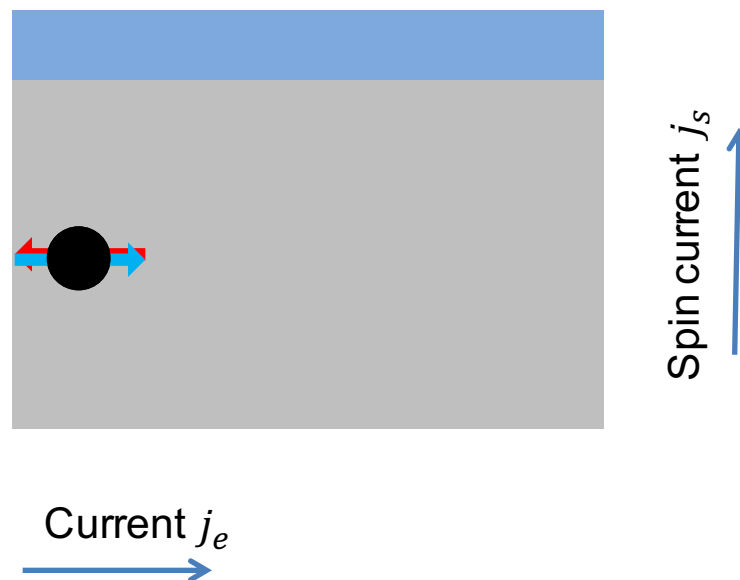
## Spin-orbit coupling at the interface

- Interface Rashba effect
- Two-dimensional Rashba model

## Spin-orbit coupling in the bulk

- Spin Hall effect
- Drift-diffusion equation in three-dimension

- Models completely different
  - A way to reconcile these two?
  - A new model required for the interface spin-orbit coupling
- Experimental situation
  - Spin Hall angle  $\theta = j_s/j_e$  overestimated
  - Reports on **roles of the interface**  
[Allen, PRB (2015)], [Zhang, Nat. Phys. (2015)], [Wang arXiv (2015)]

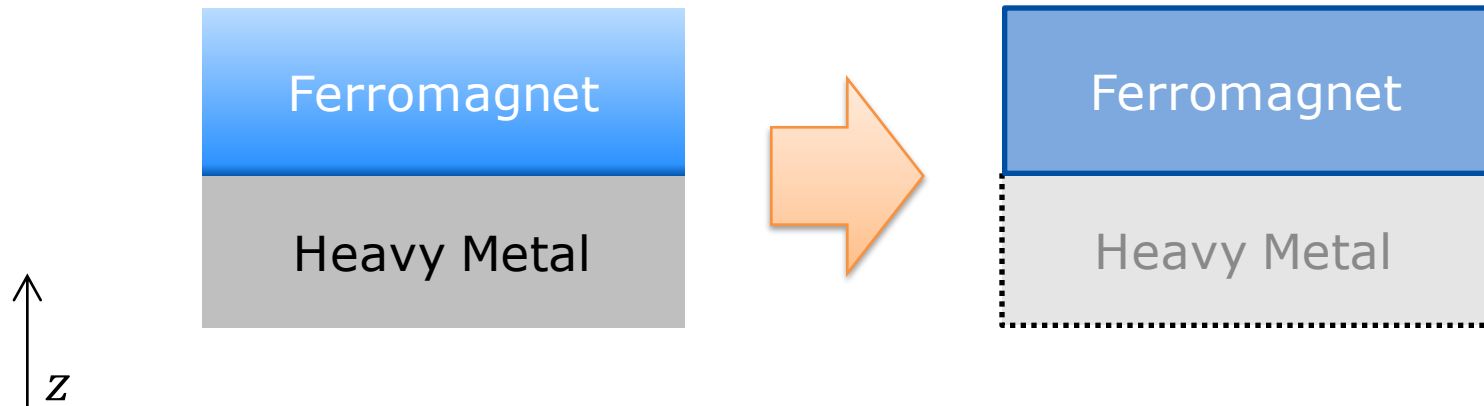


## Two-Dimensional Rashba Model

A simple model for magnetic bilayers

# Model for Spin-Orbit Coupled Interface

Two-dimensional (2D) Rashba model gives a simple description



$$H = \frac{\hbar^2 \mathbf{k}^2}{2m_e} + J \boldsymbol{\sigma} \cdot \mathbf{m} + \alpha_R \boldsymbol{\sigma} \cdot (\mathbf{k} \times \hat{\mathbf{z}})$$

- Treating electrons in the ferromagnet as 2D electron gas
  - Works well for thin films
  - Simplifies the situation a lot, but some information lost



# Results from 2D Rashba model

- Equilibrium features

- The Dzyaloshinskii-Moriya interaction [Kim PRL (2013)]
- Perpendicular magnetic anisotropy [Barnes Sci. Rep. (2014)]

- Nonequilibrium features

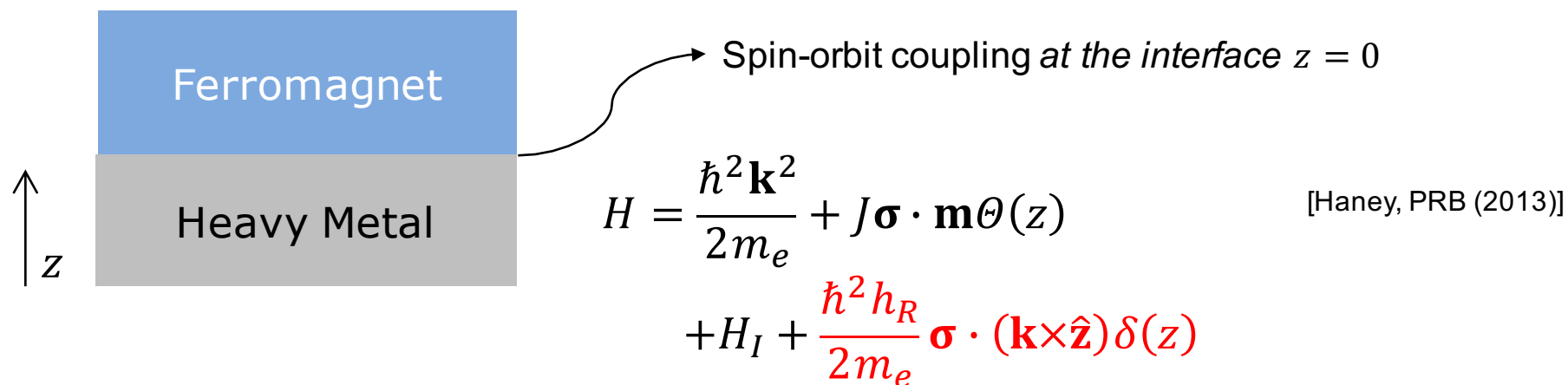
- Field-like spin-orbit torque [Manchon PRB (2008)]
- *Extrinsic* Damping-like spin-orbit torque  
[Wang, PRL (2012)], [Kim, PRB (2012)], [Pesin, PRB (2012)]
  - From spin relaxation
- *Intrinsic* Damping-like spin-orbit torque [Kurebayashi, Nat. Nano. (2014)]
- Correction to spin-motive force [Kim, PRL (2012)]

## Beyond the Two-Dimensional Model

Interpretations of experimental data require a three-dimensional model

# A Three-Dimensional (3D) Model

3D Rashba model for interface spin-orbit coupling is desirable



Spin-orbit coupling at the interface  $z = 0$

$$H = \frac{\hbar^2 \mathbf{k}^2}{2m_e} + J \boldsymbol{\sigma} \cdot \mathbf{m} \Theta(z)$$

[Haney, PRB (2013)]

$$+ H_I + \frac{\hbar^2 h_R}{2m_e} \boldsymbol{\sigma} \cdot (\mathbf{k} \times \hat{\mathbf{z}}) \delta(z)$$

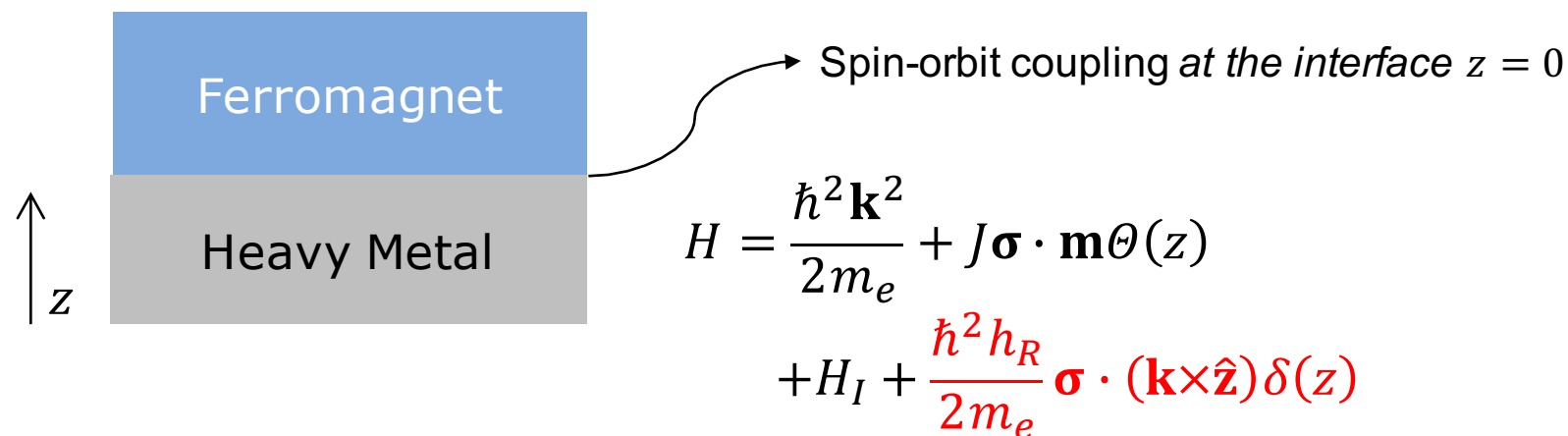
$H_I$ : interface potential  $z = 0 \propto \delta(z)$  other than spin-orbit coupling  
e.g.) interface magnetism, interface barrier

## ■ Previous attempts to this model

- Numerical works [Haney, PRB (2013)], [Amin, in preparation (2016)]
  - Restricted to the equal Fermi surface model
- Analytic works [Chen, PRL (2015)], [Zhang, PRB (2015)]
  - Focused on a few phenomena with different formalisms

# A Three-Dimensional (3D) Model

Scattering formalism gives a simple way to examine the 3D model

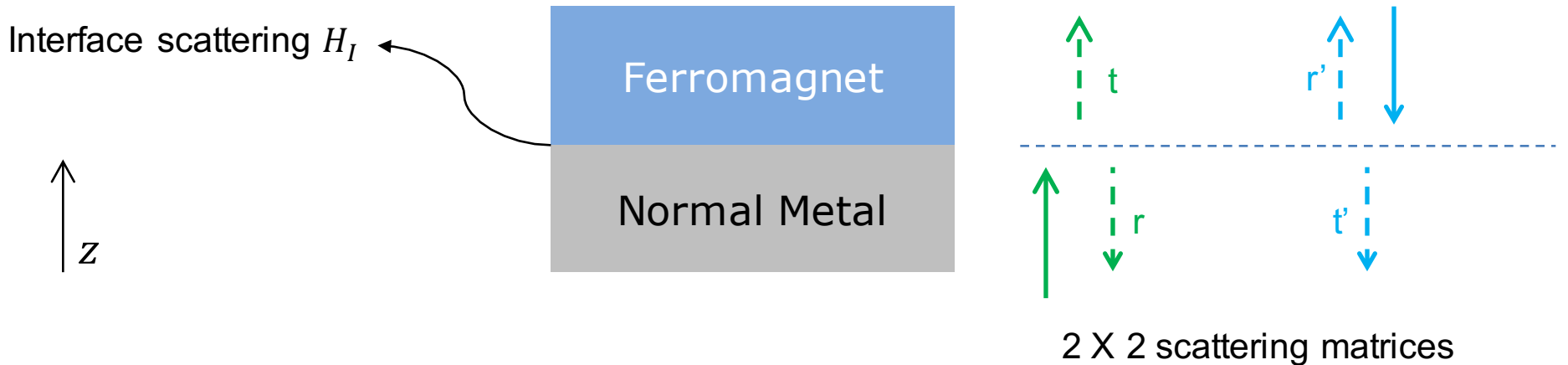


- Difficulty of the model
  - The solution depends on the **details** of  $H_I$
  - Cannot be an analytic theory in general
- How can we deal with  $H_I$  in a general way?
  - Express  $H_I$  by **scattering matrices**!



# Scattering Matrix Formalism

Provides a background for a general analytic theory



- $H_I \leftrightarrow (r, t, r', t')$
- $H_I + \Delta H_I \leftrightarrow (r + \Delta r, t + \Delta t, r' + \Delta r', t' + \Delta t')$

$r$  : reflection matrix  
 $t$  : transmission matrix

- Perturbation theory applicable
  - Our case :  $\Delta H_I = \frac{\hbar^2 h_R}{2m_e} \boldsymbol{\sigma} \cdot (\mathbf{k} \times \hat{\mathbf{z}}) \delta(z)$

# Scattering Matrix Formalism

## Advantages

- General formula **independent of** the scattering potential
- Easy to connect to first-principles calculations
  - Expressions in terms of the reflection and transmission matrices
- Well-studied: analogous to the conventional circuit theory
  - Conductance matrices
    - [Brataas, PRL (2000)]
    - Spin-transfer torque
    - Spin pumping
    - Boundary conditions for the spin drift-diffusion equation
  - Gilbert damping [Brataas, PRL (2008)]



Non-collinear spin injection

Modified scattering matrices will give all of the above quantities, ***and even more!***

## Results and Implications

Effects of the interface spin-orbit coupling go beyond quantitative corrections

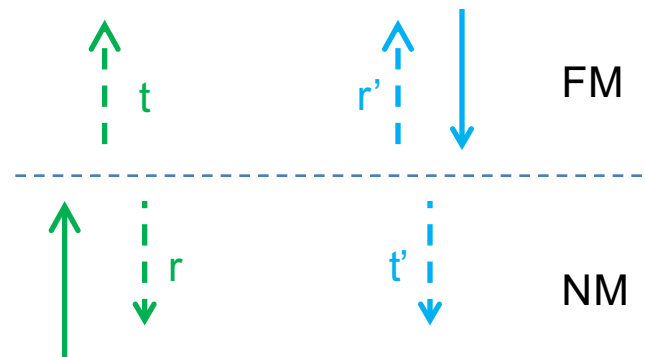
# Results

## Perturbation of the scattering matrices

- Correction due to spin-orbit coupling

- $t_{\mathbf{k}} = t_{\mathbf{k}}^0 - i \frac{\hbar_R}{2k_z} t_{\mathbf{k}}^0 \boldsymbol{\sigma} \cdot (\mathbf{k} \times \hat{\mathbf{z}}) (1 + r_{\mathbf{k}}^0)$
- $t'_{\mathbf{k}} = t'_{\mathbf{k}}{}^{0} - i \frac{\hbar_R}{2k_z} (1 + r_{\mathbf{k}}^0) \boldsymbol{\sigma} \cdot (\mathbf{k} \times \hat{\mathbf{z}}) t'_{\mathbf{k}}{}^{0}$
- $r_{\mathbf{k}} = r_{\mathbf{k}}^0 - i \frac{\hbar_R}{2k_z} (1 + r_{\mathbf{k}}^0) \boldsymbol{\sigma} \cdot (\mathbf{k} \times \hat{\mathbf{z}}) (1 + r_{\mathbf{k}}^0)$
- $r'_{\mathbf{k}} = r'_{\mathbf{k}}{}^{0} - i \frac{\hbar_R}{2k_z} t_{\mathbf{k}}^0 \boldsymbol{\sigma} \cdot (\mathbf{k} \times \hat{\mathbf{z}}) t'_{\mathbf{k}}{}^{0}$

Spin-orbit coupling contributions



- Matrices no longer rotationally symmetric

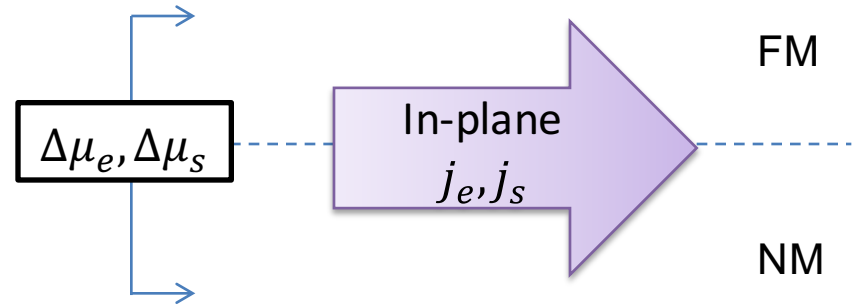
- In-plane current from a perpendicular (spin) potential difference

# Implications – Transverse current generation

The modified scattering matrices implies in-plane current from perpendicular voltage

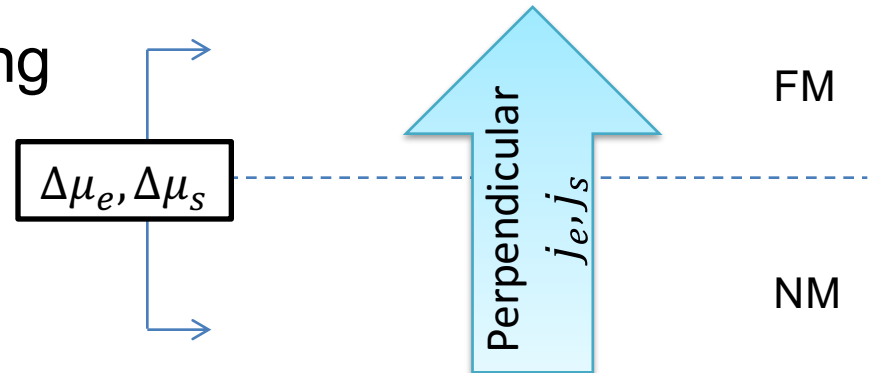
- *In-plane* current generation

- Phenomenology similar to the (inverse) spin Hall effect
- Crucial for interpretations of experiments



- *c.f.)* Without spin-orbit coupling

- No in-plane current



$\Delta\mu_e, \Delta\mu_s$  : chemical potential difference  
e.g.) voltage across the interface

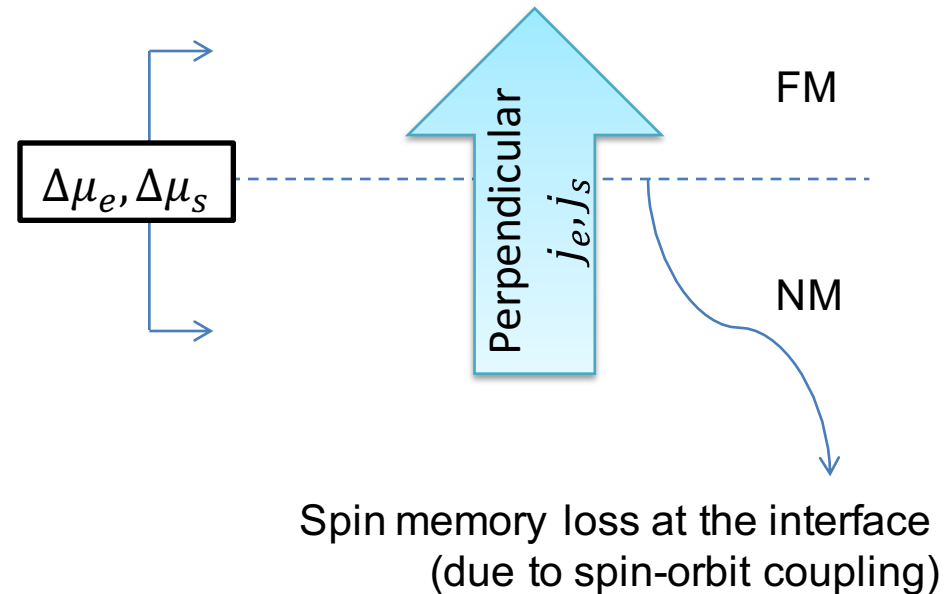
# Implications – Longitudinal transport

Second order calculation shows spin memory loss for perpendicular transport

- First order calculation
  - No correction to the perpendicular transport
    - The same interface conductance :  $G^\uparrow, G^\downarrow, G^{\uparrow\downarrow}$

- Second order calculation
  - Collinear transport for simplicity

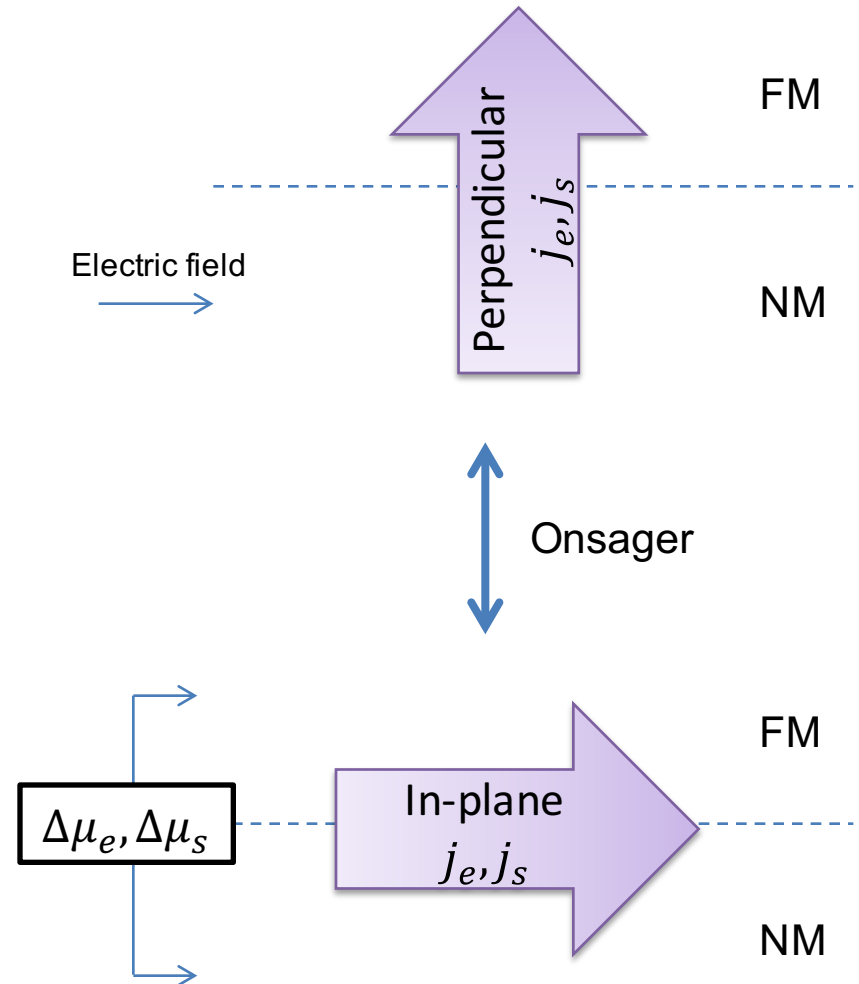
$$\begin{pmatrix} j_\uparrow \\ j_\downarrow \end{pmatrix} = \begin{pmatrix} G_{\text{col}}^\uparrow & G_{\text{sf}} \\ G_{\text{sf}} & G_{\text{col}}^\downarrow \end{pmatrix} \begin{pmatrix} \Delta\mu_\uparrow \\ \Delta\mu_\downarrow \end{pmatrix}$$



# Implications – In-plane bias effects

Onsager reciprocity of the in-plane current gives spin-transfer torque

- Onsager reciprocity
  - Perpendicular spin current induced by in-plane electric field  
→ Spin-transfer torque
- In-plane shift of distribution function  $\Delta k_x = eE\tau/\hbar$ 
  - $STT = \text{Im}[T] \mathbf{m} \times (\hat{\mathbf{y}} \times \mathbf{m}) + \text{Re}[T] \mathbf{m} \times \hat{\mathbf{y}}$
  - $T = h_R \frac{e^2 L E \tau}{8\pi m_e V} \sum \frac{k_{\perp}^2}{k_z} (1 - r_{\mathbf{k}}^{\uparrow} r_{\mathbf{k}}^{\downarrow*}) (r_{\mathbf{k}}^{\downarrow} - r_{\mathbf{k}}^{\uparrow*})$
- Theory easily applicable for **ferromagnetic insulators** and **topological insulators**

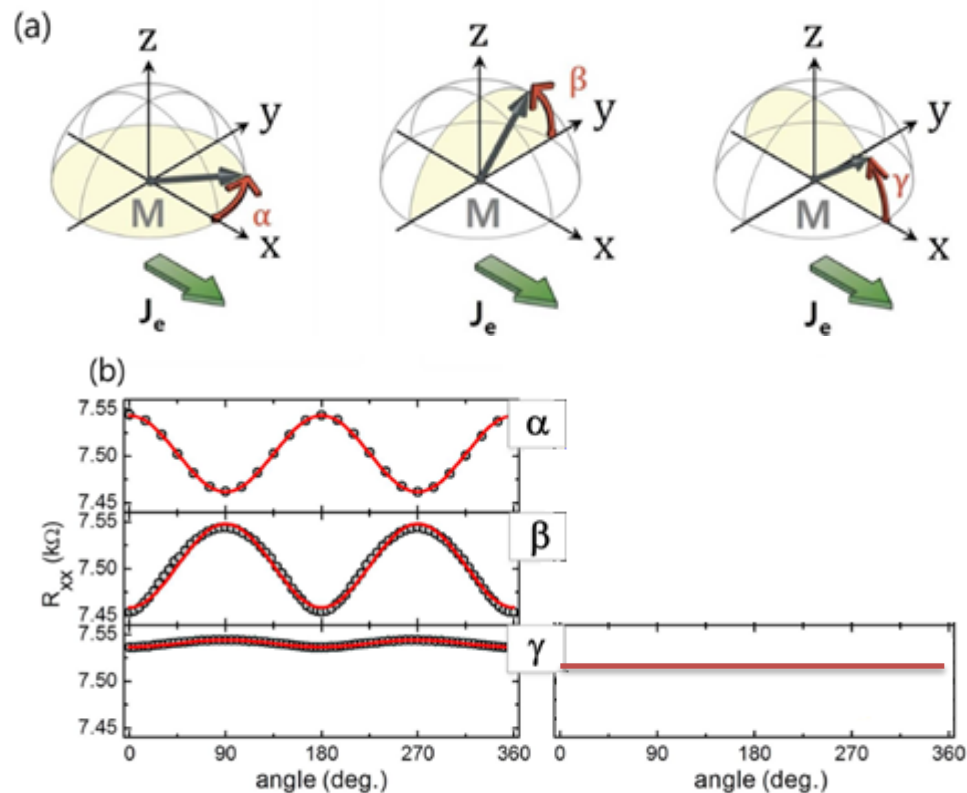




# Implications – In-plane bias effects

: Second order calculation gives anisotropic magnetoresistance

- Second order study for an in-plane bias
  - Anisotropic magnetoresistance (like spin Hall magnetoresistance)
- Anisotropic magnetoresistance  $\propto -(m_x^2 + 3m_y^2)$ 
  - Consistent with the previous report [Zhang, PRB (2015)], but obtained in a different context
- Spin Hall magnetoresistance  $\propto -m_y^2$ 
  - Distinction possible from the different behaviors?



Only spin Hall

Figures from [Cho Sci. Rep. (2015)]

# Future directions

- Reexamination of the existing experimental reports
  - Careful treatment for extracting the spin Hall angle
- Berry phase contribution?
  - Calculation possible from the eigenstates
- Scattering theory of damping
  - Gives chiral damping? [Jue Nat. Mater. (2015)]
- Spin pumping
  - Scattering formalism of spin pumping, developed by Green's function only [Chen PRL (2015)]
- Application for other contexts such as topological insulators, lanthanum aluminate-strontium titanate interface (LAO/STO), ...

# Summary of Part 1

- **2D** Rashba model for magnetic bilayers are well-studied
- Interpretations on experimental results require a **3D** theory
- We adopt the **scattering matrix** formalism and find modified expressions of scattering matrices
  - Modified conductance matrices
  - Spin memory loss at the interface
  - Spin-orbit torque
  - Anisotropic magnetoresistance
- We expect our theory can be applied to other contexts such as **topological insulators**.

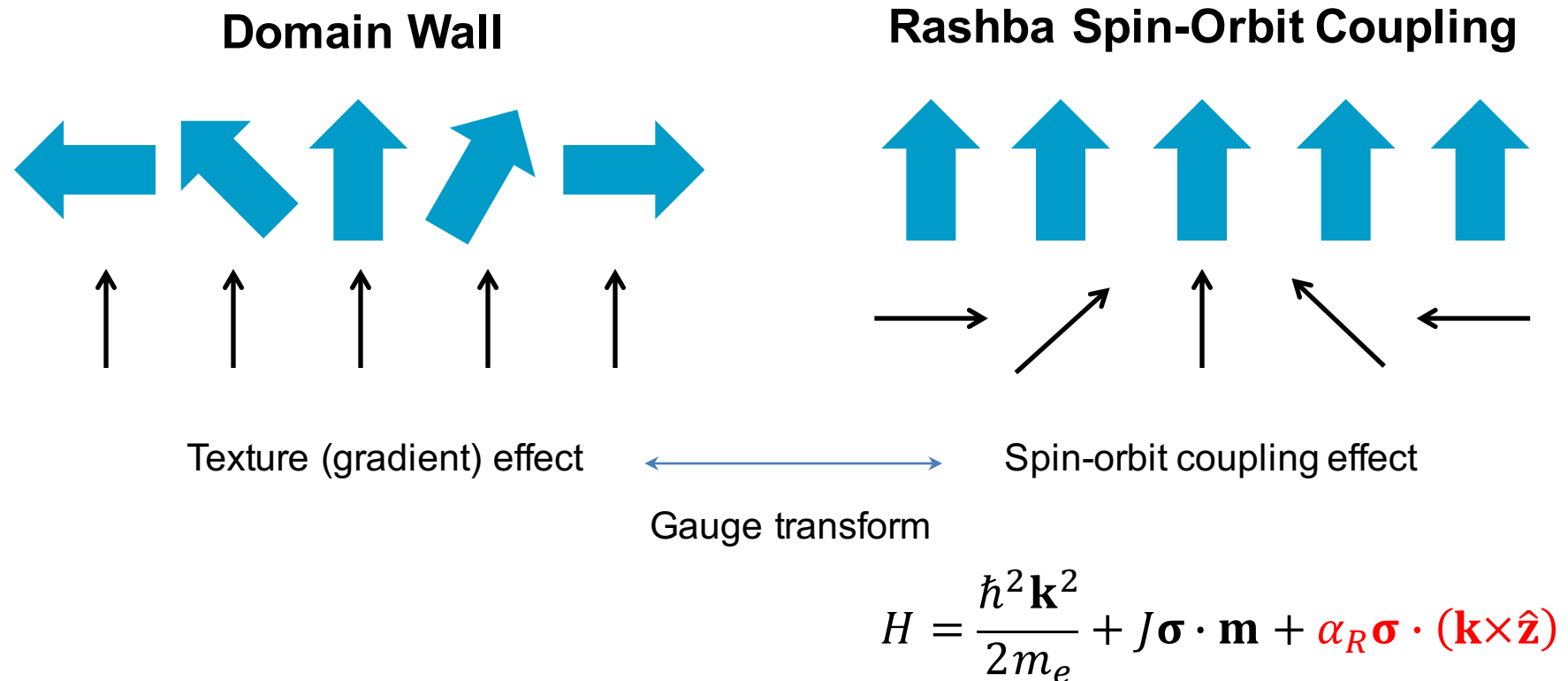
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  - Current-induced domain wall motion
  - *Intrinsic* spin-orbit torque
  - Chiral derivatives
  - *Intrinsic* non-adiabatic spin-transfer torque

# Motivation

A new physics in one side implies a new physics in the other side

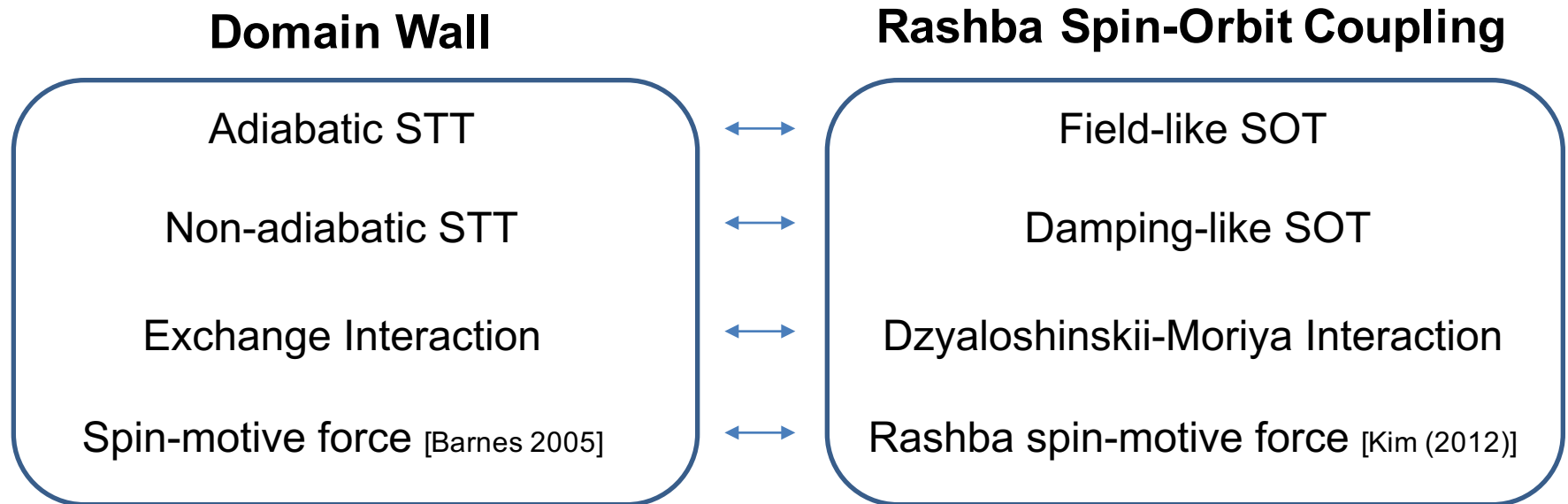
- One-to-One Correspondence [Kim, PRL (2013)]
  - *There is one-to-one correspondence between magnetic texture effects and Rashba spin-orbit coupling effects*



# Motivation

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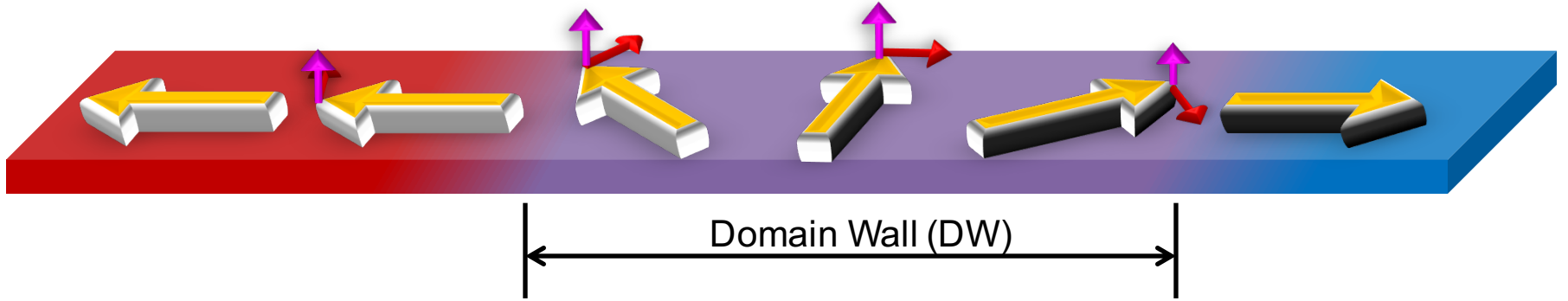
## **Current-Induced Domain Wall Motion**

Introduction to non-adiabatic spin-transfer torque



# Spin-Transfer Torques (STTs) in Domain Walls

Adiabatic and non-adiabatic STTs



## Adiabatic STT

- $\mathbf{T}_{\text{adia}} = b_J \mathbf{m} \times (\partial_x \mathbf{m} \times \mathbf{m})$
- Drives the DW motion
- Angular momentum conservation

Current  $j_e$

$$b_J = \frac{P j_e \mu_B}{e M_s}$$

## Non-adiabatic STT ( $\beta$ ) [Zhang, PRL (2004)]

- $\mathbf{T}_{\text{non}} = -\beta b_J \mathbf{m} \times \partial_x \mathbf{m}$
- Determines the DW velocity
- Something beyond : main mechanism still unclear
  - Spin relaxation

$$v_{\text{DW}} = \frac{\beta}{\alpha} b_J$$

$\alpha$  : Gilbert damping

# *Intrinsic vs Extrinsic* in Spin-Orbit Coupling Systems

Can be applied to a magnetic textured system

## ▪ *Extrinsic*

- **Dependent** on scattering
- Examples
  - *Extrinsic* spin Hall effect
  - *Extrinsic* spin-orbit torque

## ▪ *Intrinsic*

- **Independent** of scattering
- Examples
  - *Intrinsic* spin Hall effect
  - *Intrinsic* spin-orbit torque

## ▪ Textured system

- Magnetic texture → Effective spin-orbit coupling
- **Non-adiabatic STT** from spin relaxation [Zhang, PRL (2004)]: *Extrinsic*
  - Hard to enhance  $\beta/\alpha$  significantly
- **Non-adiabatic STT** from an *intrinsic* mechanism (?)
  - Efficient DW motion : mechanism not enhancing  $\alpha$
  - Better physical understanding

# ***Intrinsic non-adiabatic spin-transfer torque***

## ***Intrinsic Spin-Orbit Torque***

Provides a template for studying the intrinsic spin-transfer torque in DW motion

# Spin-Orbit Torques (SOTs) in Rashba systems

Extrinsic Field-like and Damping-like SOTs

- Rashba spin-orbit coupling
  - Structural inversion asymmetry

$$H = \frac{\hbar^2 \mathbf{k}^2}{2m_e} + J \boldsymbol{\sigma} \cdot \mathbf{m} + \frac{\hbar^2 k_R}{2m_e} \boldsymbol{\sigma} \cdot (\mathbf{k} \times \hat{\mathbf{z}})$$

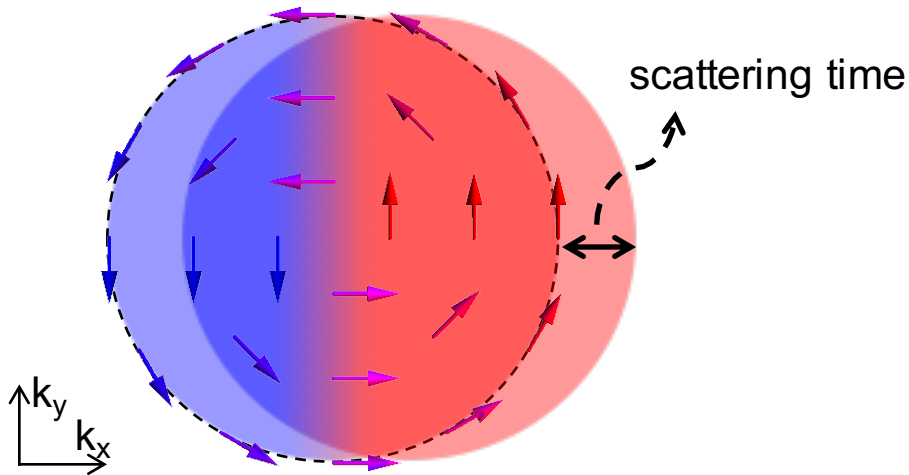
Ferromagnet (Co, CoFeB...)

Heavy Metal (Pt, Ta...)

- Field-like SOT [Manchon PRB (2008)]

- Damping-like SOT

[Wang PRL, Kim PRB, Pesin PRB (2012)]



- *Similar to the non-adiabatic STT*
- Mechanism still unclear
  - Spin relaxation
- $\mathbf{T}_{\text{damp}} = -\beta k_R b_J \mathbf{m} \times (\hat{\mathbf{y}} \times \mathbf{m})$

- Current : More y spins, less -y spins
- $\mathbf{T}_{\text{field}} = k_R b_J \hat{\mathbf{y}} \times \mathbf{m}$

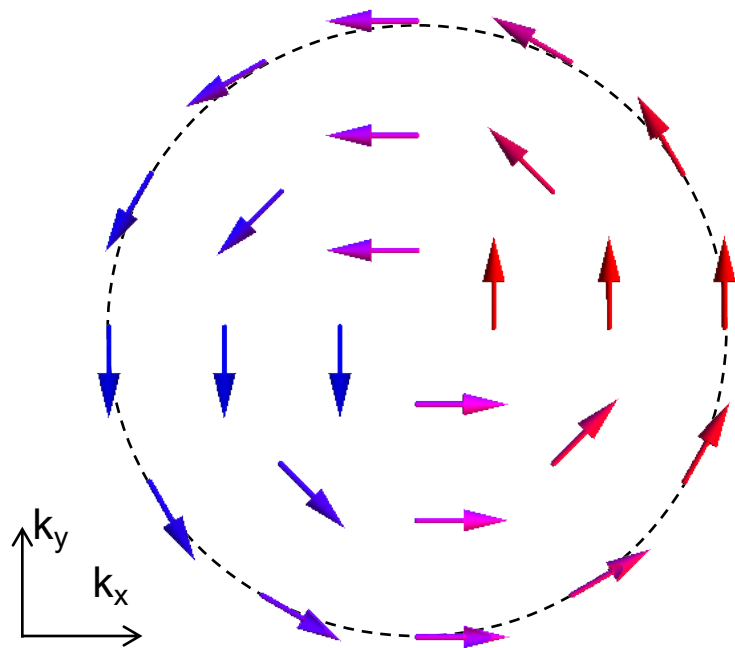
Figure without  $\mathbf{m}$  for simplicity,

# Intrinsic Damping-like SOT

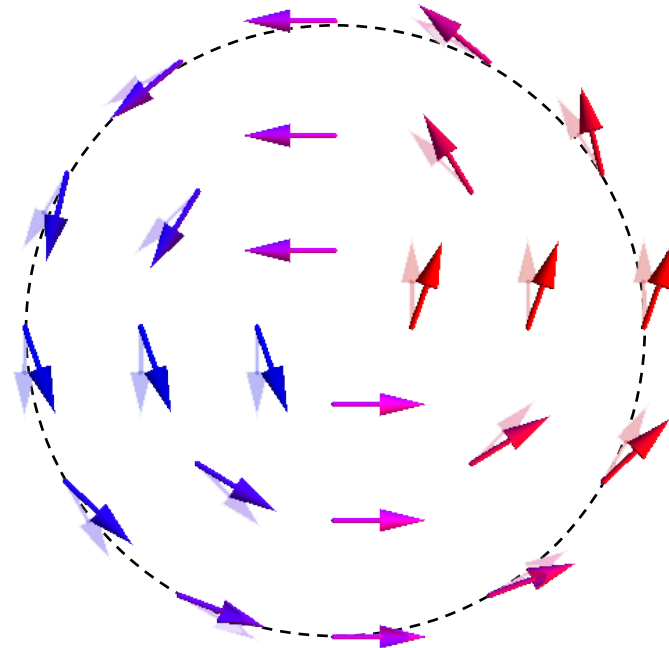
Damping-like SOT independent of scattering

- *Intrinsic* Damping-like SOT [Kurebayashi. Nat. Nano. (2014)]
  - Wave function change due to an electric field

Without an electric field



With an electric field



Additional source of damping-like SOT

$$\mathbf{T}_{\text{damp}}^{\text{int}} \propto \mathbf{m} \times (\hat{\mathbf{y}} \times \mathbf{m})$$

Independent of scattering  $\rightarrow$  *intrinsic*

# *Intrinsic non-adiabatic spin-transfer torque*

## **Chiral Derivatives**

One-to-one correspondence between spin-orbit coupling systems and textured systems

# One-to-One Correspondence

Between texture effects and spin-orbit coupling effects

## ■ One-to-One Correspondence [Kim, PRL (2013)]

- Chiral derivatives  $\partial_x \mathbf{m} \rightarrow k_R \hat{\mathbf{y}} \times \mathbf{m}$
- Example) *Adiabatic STT*  $\rightarrow$  *Field-like SOT*

$$b_J \mathbf{m} \times (\partial_x \mathbf{m} \times \mathbf{m}) \rightarrow k_R b_J \hat{\mathbf{y}} \times \mathbf{m}$$

### Domain Wall

Adiabatic STT

Non-adiabatic STT

Exchange Interaction

Spin-motive force



### Rashba Spin-Orbit Coupling

Field-like SOT

Damping-like SOT

Dzyaloshinskii-Moriya Interaction

Rashba spin-motive force



# One-to-One Correspondence

Between texture effects and spin-orbit coupling effects

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### Domain Wall

Adiabatic STT

*Extrinsic* Non-adiabatic STT

*Intrinsic* Non-adiabatic STT (?)

Exchange Interaction

Spin-motive force



### Rashba Spin-Orbit Coupling

Field-like SOT

*Extrinsic* Damping-like SOT

*Intrinsic* Damping-like SOT

Dzyaloshinskii-Moriya Interaction

Rashba spin-motive force

# ***Intrinsic non-adiabatic spin-transfer torque***

## ***Intrinsic Non-adiabatic Spin-Transfer Torque***

Non-adiabatic spin-transfer torque independent of scattering rate

# Intrinsic Non-adiabatic STT

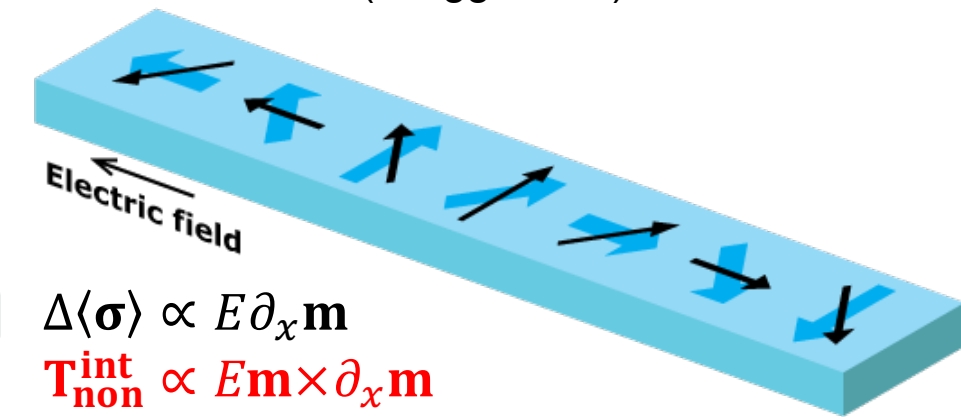
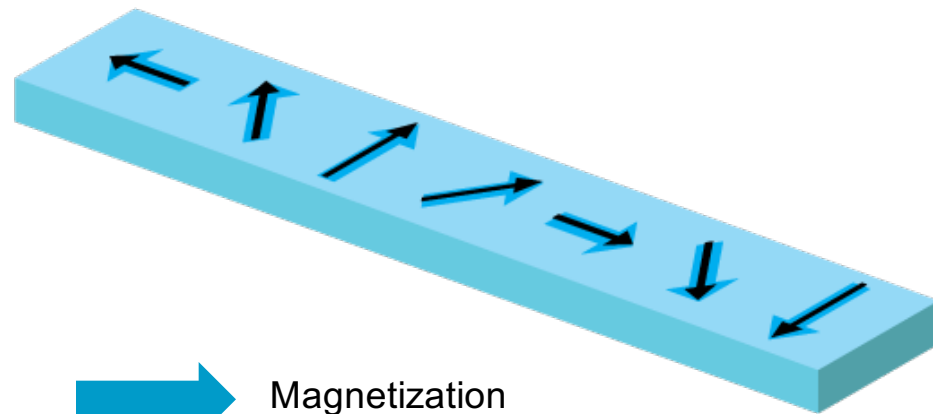
Non-adiabatic STT independent of scattering

## ▪ Intrinsic Non-adiabatic STT

- Wave function change due to an electric field

Without an electric field

With an electric field  
(exaggerated)



Magnetization

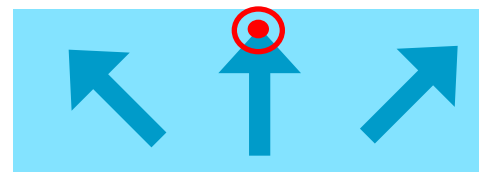


Spin expectation value

$$\Delta\langle\sigma\rangle \propto E\partial_x\mathbf{m}$$

$$\mathbf{T}_{\text{non}}^{\text{int}} \propto E\mathbf{m}\times\partial_x\mathbf{m}$$

Non-adiabatic STT form



$$\mathbf{T} = -\frac{n_s\mu_B\hbar eE_x}{2m_eJM_s}\mathbf{m}\times\partial_x\mathbf{m}$$

# ***Intrinsic Non-adiabatic STT* vs *Intrinsic Damping-like SOT***

The features are the same

- *Intrinsic Non-adiabatic STT*
- *Intrinsic Damping-like SOT*

## Mathematical forms

$$\mathbf{T}_{\text{non}}^{\text{int}} \propto \mathbf{m} \times \partial_x \mathbf{m} \quad \xleftrightarrow[\partial_x \mathbf{m} \leftrightarrow k_R \hat{\mathbf{y}} \times \mathbf{m}]{\text{Chiral derivatives}} \quad \mathbf{T}_{\text{damp}}^{\text{int}} \propto \mathbf{m} \times (\hat{\mathbf{y}} \times \mathbf{m})$$

## Features

- Additional component for each state
- From wave function change
- Electric-field-induced
- Fermi **sea** contribution
- *Intrinsic* (**independent of scattering**)

# Magnitude of *Intrinsic* Non-adiabatic STT

*Intrinsic* Non-adiabatic STT can be the dominant contribution

- DW velocity  $\mathbf{T}_{\text{non}} = \mathbf{T}_{\text{non}}^{\text{ext}} + \mathbf{T}_{\text{non}}^{\text{int}} = -(\beta_{\text{ext}} + \beta_{\text{int}})b_J \mathbf{m} \times \partial_x \mathbf{m}$

$$v_{\text{DW}} = \frac{\beta_{\text{ext}} + \beta_{\text{int}}}{\alpha} b_J$$

## ▪ Magnitudes

- $\beta_{\text{int}} / \beta_{\text{ext}} \sim T_{\text{sr}} / T_{\text{mr}}$
- $1/T_{\text{mr}} \sim 10^{14}$  to  $10^{15} \text{ s}^{-1}$  (momentum relaxation)
- $1/T_{\text{sr}} \sim 10^{12} \text{ s}^{-1}$  (spin relaxation)
- $\beta_{\text{int}}$  can be much larger than  $\beta_{\text{ext}}$ !

## ▪ Impurity effects

- Impurity effect :  $\beta_{\text{int}}$  and  $\beta_{\text{ext}}$  similar magnitudes
- Future challenge : Determining the dominant contribution

# Summary of Part 2

- STT-SOT Correspondence
  - Recent discovery on *intrinsic* damping-like SOT
  - Corresponding *intrinsic* non-adiabatic STT
  
- Features of *intrinsic* non-adiabatic STT
  - Originates from wave function change
  - **Electric-field**-induced
  - Scattering-time-**independent**
  - Fermi **sea** contribution
  - Mechanism **not** enhancing damping
  - Can be the dominant contribution of the non-adiabatic STT

More information – [Kim *et al.*, PRB **92**, 224426 (2015)]

# Summary

- Magnetism: Promising candidate for next generation device
- Spin-orbit coupling
  - Arises in symmetry broken nanostructures
  - Raises efficiency of possible spintronic devices
  - Effects on spin dynamics not clearly understood
- Theoretical study of spin-orbit coupling effects
  - Advances spintronic device application
  - Predicts qualitatively different physics
  - Deepen the understanding of magnetic systems

## Collaborators

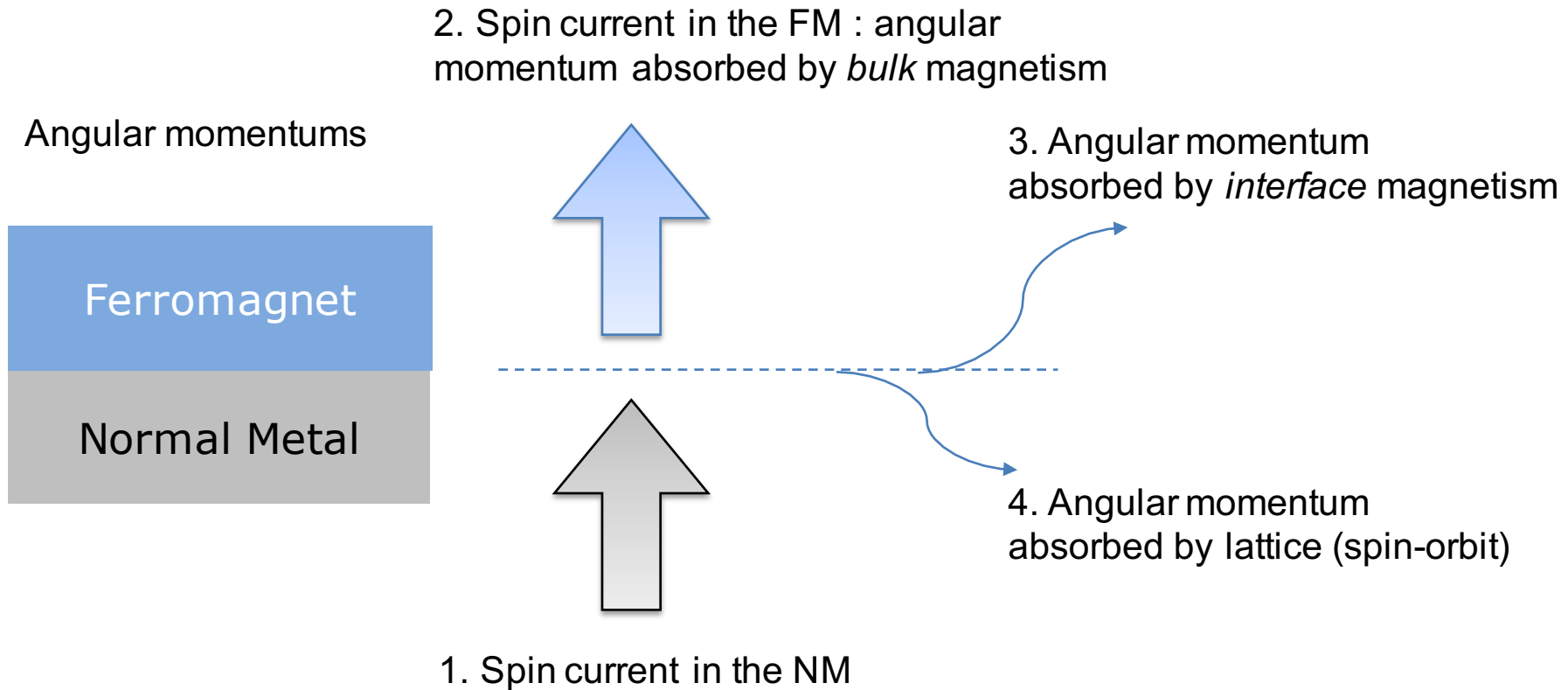
- Mark Stiles (NIST)
- Hyun-Woo Lee (POSTECH)
- Kyung-Jin Lee (Korea Univ.)

## Acknowledgement

- Vivek Amin (NIST)
- Dongwook Go (POSTECH)
- Guru Khalsa (Cornell Univ.)



# Backup slides



Angular momentum conservation :  $1 = 2+3+4$

Spin-transfer torque =  $2+3 = 1-4$

1 : calculate from the modified reflection matrix

4 : given by the quantum boundary condition (current absorption by the delta function potential)

$$\frac{V}{L} \frac{1}{2} \text{Tr}[\hat{j}_u^N] = \frac{(\hat{\mathbf{u}} \times \hat{\mathbf{z}})}{4i} \cdot \left[ (G_{ht}^{\uparrow\uparrow\uparrow} - G_{ht}^{\downarrow\downarrow\downarrow})(\Delta\mu_0^F - \Delta\mu_0^N)\mathbf{m} + (G_{ht}^{\uparrow\uparrow\uparrow} + G_{ht}^{\downarrow\downarrow\downarrow})(\Delta\mu_s^F - \mathbf{m} \cdot \mathbf{s}\Delta\mu_s^N)\mathbf{m} \right. \\ \left. + (G_{hr}^{\uparrow\downarrow\downarrow} - G_{hr}^{\downarrow\uparrow\uparrow*})\Delta\mu_s^N[\mathbf{m} \times (\mathbf{s} \times \mathbf{m}) - i\mathbf{s} \times \mathbf{m}] \right] + \text{c.c.}, \quad (104)$$

$$\frac{V}{L} \frac{1}{2} \text{Tr}[\boldsymbol{\sigma} \cdot \mathbf{m}\hat{j}_u^N] = \frac{(\hat{\mathbf{u}} \times \hat{\mathbf{z}})}{4i} \cdot \left[ (G_{ht}^{\uparrow\uparrow\uparrow} + G_{ht}^{\downarrow\downarrow\downarrow})(\Delta\mu_0^F - \Delta\mu_0^N)\mathbf{m} + (G_{ht}^{\uparrow\uparrow\uparrow} - G_{ht}^{\downarrow\downarrow\downarrow})(\Delta\mu_s^F - \mathbf{m} \cdot \mathbf{s}\Delta\mu_s^N)\mathbf{m} \right. \\ \left. - (G_{hr}^{\uparrow\downarrow\downarrow} + G_{hr}^{\downarrow\uparrow\uparrow*})\Delta\mu_s^N[\mathbf{m} \times (\mathbf{s} \times \mathbf{m}) - i\mathbf{s} \times \mathbf{m}] \right] + \text{c.c.}, \quad (105)$$

$$\frac{V}{L} \frac{1}{2} \text{Tr}[\boldsymbol{\sigma}_\perp \hat{j}_u^N] = \frac{1}{4i} \left[ (G_{ht}^{\uparrow\downarrow\downarrow} - G_{ht}^{\downarrow\uparrow\uparrow*})\Delta\mu_0^F - (G_{ht}^{\uparrow\downarrow\downarrow} + G_{ht}^{\downarrow\uparrow\uparrow*})\Delta\mu_s^F + (G_{hr}^{\uparrow\downarrow\downarrow} - G_{hr}^{\downarrow\uparrow\uparrow*})\Delta\mu_0^N \right. \\ \left. - \mathbf{m} \cdot \mathbf{s}(G_{hr}^{\uparrow\downarrow\downarrow} + G_{hr}^{\downarrow\uparrow\uparrow*})\Delta\mu_s^N \right] \{ \mathbf{m} \times [(\hat{\mathbf{u}} \times \hat{\mathbf{z}}) \times \mathbf{m}] + i\mathbf{m} \times (\hat{\mathbf{u}} \times \hat{\mathbf{z}}) \} \\ + \frac{(\hat{\mathbf{u}} \times \hat{\mathbf{z}}) \cdot \mathbf{m}}{4i} (G_{hr}^{\uparrow\downarrow\downarrow} + G_{hr}^{\downarrow\uparrow\uparrow*})\Delta\mu_s^N[\mathbf{m} \times (\mathbf{s} \times \mathbf{m}) - i\mathbf{s} \times \mathbf{m}] + \text{c.c.}, \quad (106)$$

where c.c. means the complex conjugate and the interface Hall conductances are defined by

$$G_{hr}^{ss's''} = -h_R \frac{e^2}{2h} \sum_{\mathbf{k}_\perp} ' \frac{E_\perp}{E_F - E_\perp} (1 + r_{\mathbf{k}}^s)(1 + r_{\mathbf{k}}^{s'})r_{\mathbf{k}}^{s''*}, \quad (107)$$

$$G_{ht}^{ss's''} = -h_R \frac{e^2}{2h} \sum_{\mathbf{k}_\perp} ' \frac{E_\perp}{E_F - E_\perp} (1 + r_{\mathbf{k}}^s)t_{\mathbf{k}}^{s's''*}. \quad (108)$$

The interface Hall conductances are the central result of this note. Unlike the conventional interface conductances, the conductances includes three spin indices. This is because there are two origins that can flip the injected spin  $s$ , spin-orbit coupling and magnetism. A few remarks are in order. First, what we have calculated are current densities, not currents. For  $\hat{j}_z$ , the total current is given by  $(V\hat{j}_z/L)$  which is calculated above. However, for in-plane current, the cross sectional area is not  $V/L$  so that the aspect ratio should be taken into account. The in-current densities do not go to zero in the thermodynamic limit since the summation over transverse modes gives additional  $V/L$  factor. Second,  $G_{hr}^{ss's''}$  is symmetric under exchange between  $s$  and  $s'$ . Third, the unitarity implies that  $\text{Im}[G_{hr}^{ssss}] = -\text{Im}[G_{ht}^{ssss}]$ . This is shown by using  $|r_{\mathbf{k}}^s|^2 + |t_{\mathbf{k}}^{s'}|^2 = 1$  to see

$$G_{hr}^{ssss} + G_{ht}^{ssss} = -h_R \frac{e^2}{2h} \sum_{\mathbf{k}_\perp} ' \frac{E_\perp}{E_F - E_\perp} |1 + r_{\mathbf{k}}^s|^2 \quad (109)$$

is real. The meaning of this constraint is two-fold. First, it is a necessary condition for the unitarity. Second, it is a necessary condition for the absence of charge current at equilibrium, which turns out to be proportional to  $\text{Im}[G_{hr}^{\uparrow\uparrow\uparrow} + G_{ht}^{\uparrow\uparrow\uparrow} - G_{hr}^{\downarrow\downarrow\downarrow} - G_{ht}^{\downarrow\downarrow\downarrow}]$ .

# Backup slides

$$\frac{V}{L} \frac{1}{2} \text{Tr}[\Delta \hat{j}_u] = -\frac{(\hat{\mathbf{u}} \times \hat{\mathbf{z}})}{4i} \cdot \left[ (\Delta G_h^{\uparrow\uparrow} - \Delta G_h^{\downarrow\downarrow})(\Delta\mu_0^F - \Delta\mu_0^N) \mathbf{m} + (\Delta G_h^{\uparrow\uparrow} + \Delta G_h^{\downarrow\downarrow})(\Delta\mu_0^F - \mathbf{m} \cdot \mathbf{s} \Delta\mu_0^N) \mathbf{m} \right. \\ \left. - 2\Delta G_h^{\uparrow\downarrow} \mathbf{m} \times (\mathbf{s} \times \mathbf{m}) \Delta\mu_s^N \right] + \text{c.c.}, \quad (117)$$

$$\frac{V}{L} \frac{1}{2} \text{Tr}[\boldsymbol{\sigma} \cdot \mathbf{m} \Delta \hat{j}_u] = -\frac{(\hat{\mathbf{u}} \times \hat{\mathbf{z}})}{4i} \cdot \left[ (\Delta G_h^{\uparrow\uparrow} + \Delta G_h^{\downarrow\downarrow})(\Delta\mu_0^F - \Delta\mu_0^N) \mathbf{m} + (\Delta G_h^{\uparrow\uparrow} - \Delta G_h^{\downarrow\downarrow})(\Delta\mu_0^F - \mathbf{m} \cdot \mathbf{s} \Delta\mu_0^N) \mathbf{m} \right. \\ \left. - 2i\Delta G_h^{\uparrow\downarrow} \mathbf{s} \times \mathbf{m} \right] + \text{c.c.}, \quad (118)$$

where the conductance for the interface discontinuity is given by

$$\Delta G_h^{ss'} = -h_R \frac{e^2}{2h} \sum_{\mathbf{k}_\perp} ' \frac{E_\perp}{E_F - E_\perp} (1 + r_{\mathbf{k}}^s)(1 + r_{\mathbf{k}}^{s'}). \quad (119)$$

# Backup slides

$$\hat{t}_{\mathbf{k},\text{ex}} = 2i\sqrt{\frac{|\hat{K}_z|}{|k_z|}}(i\hat{K}_z + ik_z - \kappa)^{-1}k_z, \quad (35)$$

$$\hat{t}'_{\mathbf{k},\text{ex}} = 2i(i\hat{K}_z + ik_z - \kappa)^{-1}\hat{K}_z\sqrt{\frac{|k_z|}{|\hat{K}_z|}}, \quad (36)$$

$$\hat{r}_{\mathbf{k},\text{ex}} = (i\hat{K}_z + ik_z - \kappa)^{-1}(ik_z - i\hat{K}_z + \kappa), \quad (37)$$

$$\hat{r}'_{\mathbf{k},\text{ex}} = \sqrt{|\hat{K}_z|}(i\hat{K}_z + ik_z - \kappa)^{-1}(i\hat{K}_z - ik_z + \kappa)\frac{1}{\sqrt{|\hat{K}_z|}}. \quad (38)$$

Lastly, from Eq. (39),

$$\hat{r}'_{\mathbf{k},\text{ex}} = \hat{r}'_{\mathbf{k},\text{ex}}{}^0 - i\frac{\hbar R}{2k_z}t_{\mathbf{k},\text{ex}}^0\boldsymbol{\sigma} \cdot (\mathbf{k} \times \hat{\mathbf{z}})\hat{t}'_{\mathbf{k},\text{ex}}{}^0. \quad (48)$$

Taking projection,

$$\hat{t}_{\mathbf{k}} = \hat{t}_{\mathbf{k}}^0 - i\frac{\hbar R}{2k_z}t_{\mathbf{k},\text{ex}}^0\boldsymbol{\sigma} \cdot (\mathbf{k} \times \hat{\mathbf{z}})(\hat{1} + \hat{r}_{\mathbf{k}}^0), \quad (49)$$

$$\hat{t}'_{\mathbf{k}} = \hat{t}'_{\mathbf{k}}{}^0 - i\frac{\hbar R}{2k_z}(1 + \hat{r}'_{\mathbf{k},\text{ex}}{}^0)\boldsymbol{\sigma} \cdot (\mathbf{k} \times \hat{\mathbf{z}})\hat{t}'_{\mathbf{k}}{}^0, \quad (50)$$

$$\hat{r}_{\mathbf{k}} = \hat{r}_{\mathbf{k}}^0 - i\frac{\hbar R}{2k_z}(1 + \hat{r}'_{\mathbf{k},\text{ex}}{}^0)\boldsymbol{\sigma} \cdot (\mathbf{k} \times \hat{\mathbf{z}})(\hat{1} + \hat{r}_{\mathbf{k}}^0), \quad (51)$$

$$\hat{r}'_{\mathbf{k}} = \hat{r}'_{\mathbf{k}}{}^0 - i\frac{\hbar R}{2k_z}t_{\mathbf{k},\text{ex}}^0\boldsymbol{\sigma} \cdot (\mathbf{k} \times \hat{\mathbf{z}})\hat{t}'_{\mathbf{k}}{}^0. \quad (52)$$

# Intrinsic Non-adiabatic STT in Real Situations

Suppression due to impurities

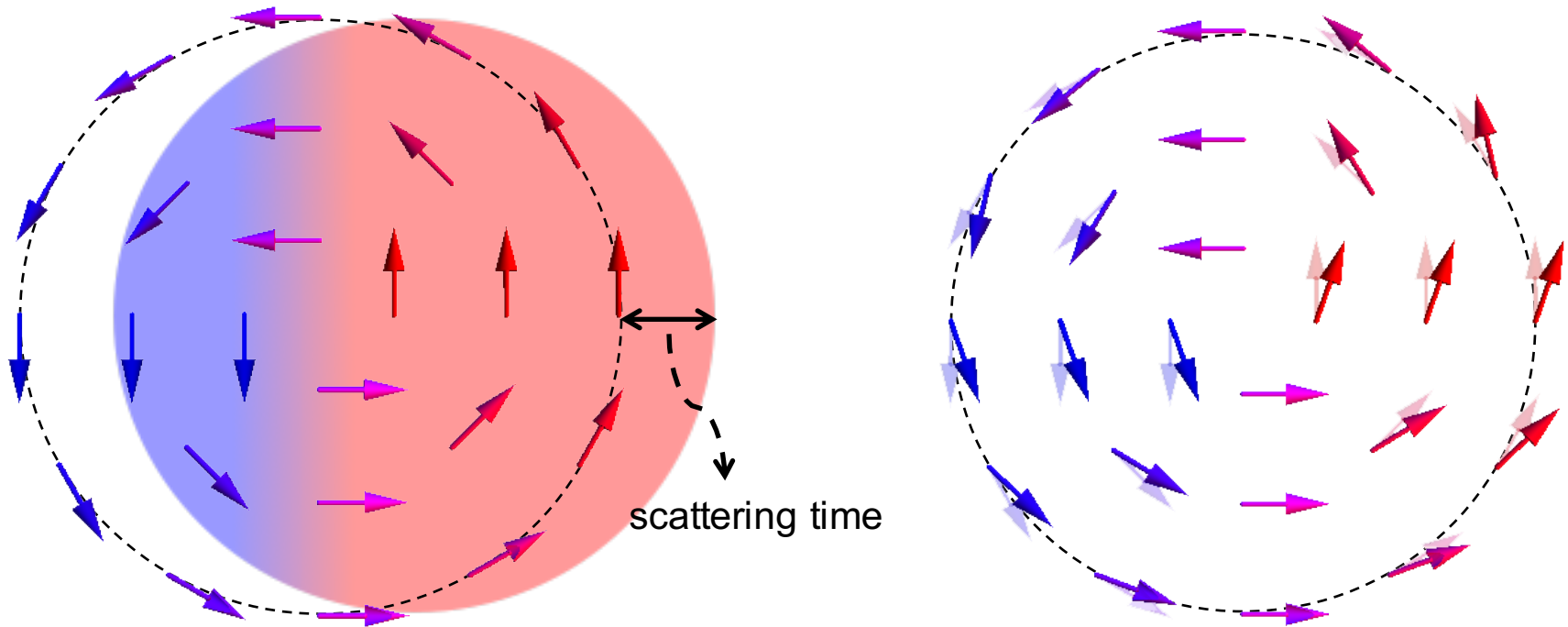
- Large suppression of  $\beta_{\text{int}}$  due to impurity (even tiny amount)

$\beta_{\text{int}}$	Free electron model	General dispersion
No magnetic impurities	0	Non-zero
With magnetic impurities	Non-zero	Non-zero

- Same thing happens in *intrinsic* spin / anomalous Hall effects  
[Inoue (2003, 2004, 2006)]

# Comparison

Between two roles of electric fields – Distribution shift and wave function change



- Change the occupation
- Current-induced
- Fermi **surface** contribution
- *Extrinsic*
- *Studied intensively*

- Additional component for each state
- Electric-field-induced
- Fermi **sea** contribution
- *Intrinsic*
- *Has rarely received attention*