DE LA RECHERCHE À L'INDUSTRIE

THERMOELECTRIC CONVERSION IN DISORDERED NANOWIRES

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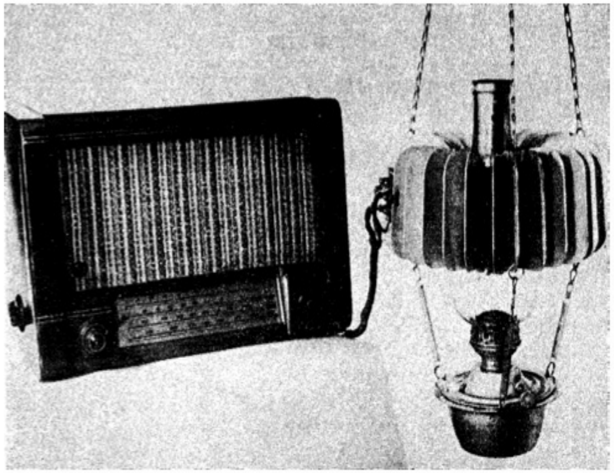
1-slide hystory of thermoelectricity

Thermoelectric effects: simultaneous charge & energy/heat flows

- ❑ The pioneers: Volta (1794), Seebeck (1821), Peltier (1834), Joule (1843), Thomson (1851; later Lord Kelvin).
- ❑ Theoretical formalisation: Onsager (1931).
- ❑ The golden age: ca. 1930 – 1960 (Ioffe).
- ❑ The rebirth: 1990's (Hicks & Dresselhaus).

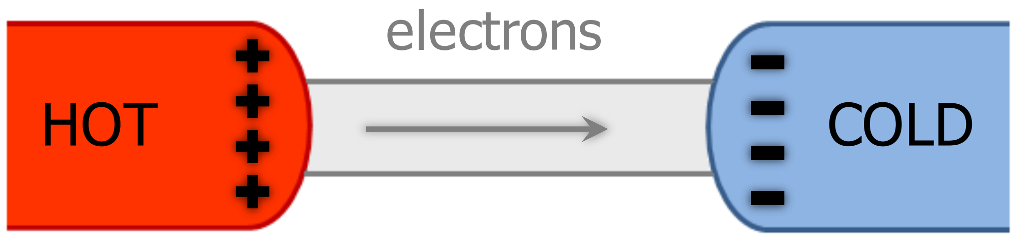
Nanostructuration

- ❑ Today: energy efficiency concerns.



Siberia can listen to Stalin's radio (USSR, 1948)

Thermoelectricity : Rules of the game



$$J_1 = L_{11}X_1 + L_{12}X_2,$$
$$J_2 = L_{21}X_1 + L_{22}X_2.$$

J_1 = charge current

J_2 = heat current

L_{ij} = linear transport coefficients

Seebeck (thermopower) $$S = -\left(\frac{\Delta V}{\Delta T}\right)_{J^e=0} = \frac{1}{eT}\frac{L_{12}}{L_{11}},$$

Figure of merit $$ZT = \frac{L_{12}^2}{\det \mathbf{L}}$$

$$\eta_{max} = \eta_C \frac{\sqrt{ZT+1}-1}{\sqrt{ZT+1}+1},$$

$$\eta(\dot{W}_{\max}) = \eta_{CA}\frac{ZT}{ZT+2},$$

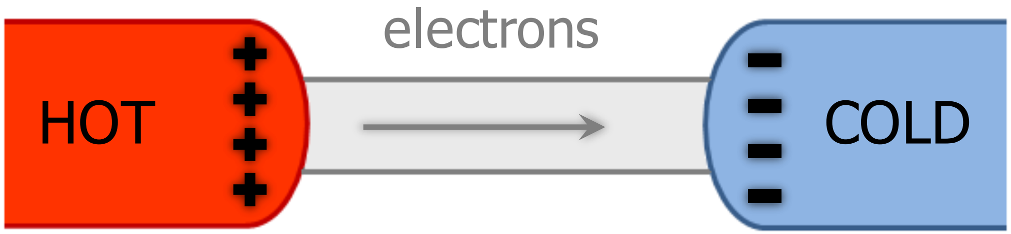
Max efficiency

Carnot efficiency

Efficiency at max power

Curzon-Ahlborn limit

Thermoelectricity : Rules of the game



Thermopower (or Seebeck coeff.):

$$S = -\left(\frac{\Delta V}{\Delta T}\right)_{J^e=0}$$

Maximize the efficiency i.e. the figure of merit :

$$ZT = \frac{GS^2}{K^e + K^{ph}} T$$

… keeping a reasonable electrical output power (power factor) :

$$Q = GS^2$$

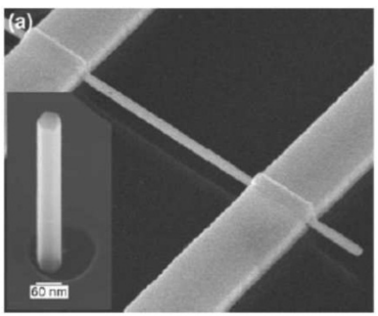
Why semiconductor nanowires?

"… a newly emerging field of low-dimensional thermoelectricity, enabled by materials nanoscience and nanotechnology"

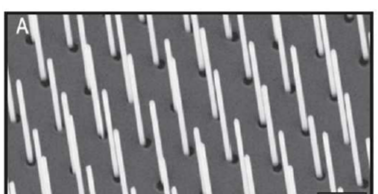
Dresselhaus et al: Adv. Mater. 2007

"… fundamental scientific challenges could be overcome by deeper understanding of charge and heat transport"

Majumdar: Science 2004



SC nanowires



Reduced thermal conductance

Phonon vs electrons mean free path, geometrical designs *(Hochbaum 2008, Heron 2010)*

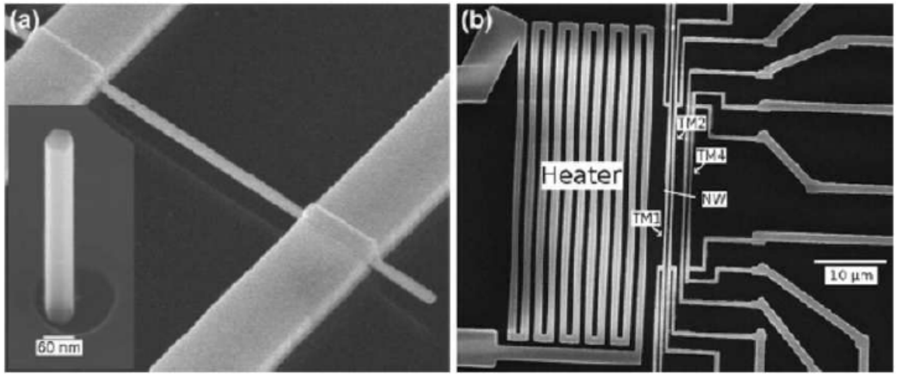
Enhanced thermopower

Field effect transistors *(Brovman 2013, Roddaro 2013 & many others)*

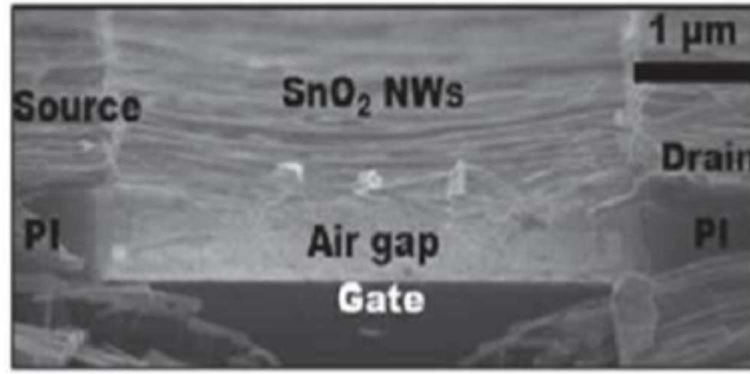
Scalable output power

Arrays of parallel NWs *(Pregl 2013, Stranz 2013)*

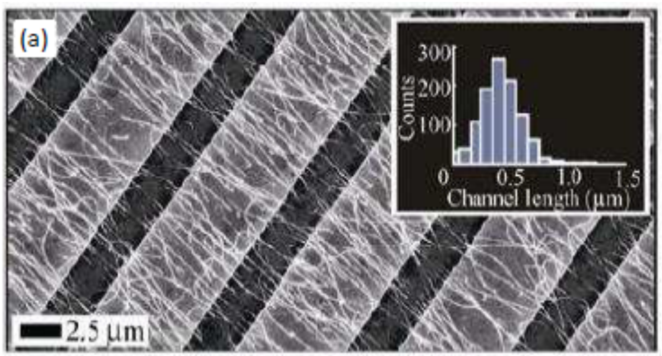
Some experimental realizations



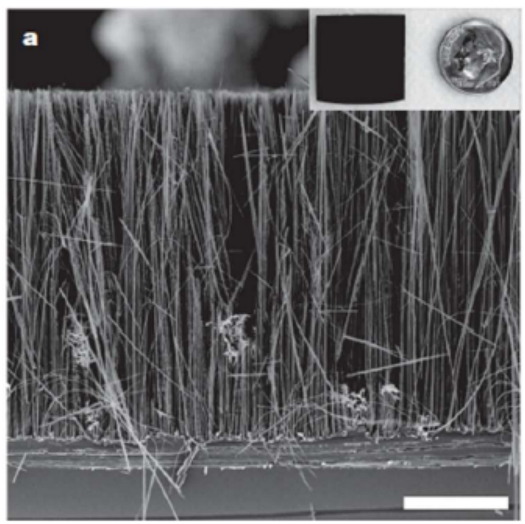
Karg *et al.* (IBM Zurich), 2013



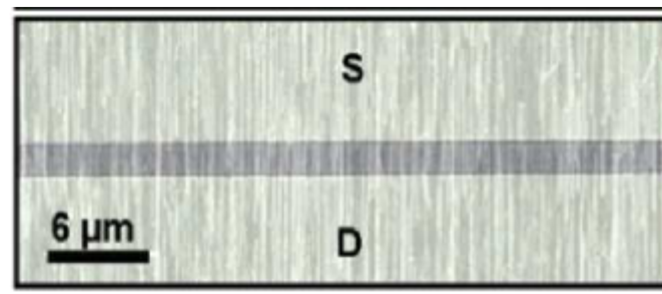
Shin *et al.* (Seoul), 2011



Pregl *et al.* (TU Dresden), 2013



Hochbaum *et al.* (Berkeley CA), 2008



Fan *et al.* (Berkeley CA), 2008

However → Imbalance between experimental / theoretical works !

Outline

Aller à l'idéal et comprendre le réel.

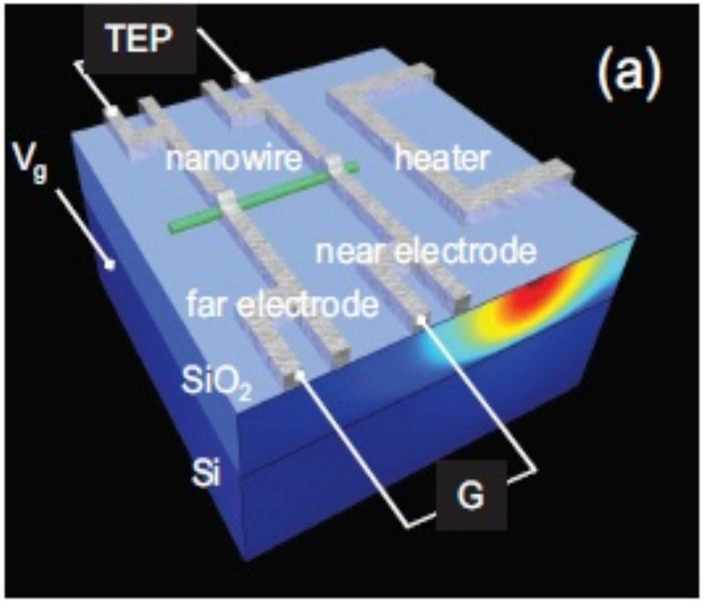
Jean Jaurès

- ❑ Nanowires in the Field Effect Transistor configuration
  The 1D Anderson Model

- ❑ Thermopower of single NW: low T elastic regime
- ❑ Thermopower of single NW: inelastic phonon-assisted regime
- ❑ Parallel nanowires: scalable power factor and hot spot cooling
- ❑ 3-Terminal thermoelectric ratchet (``all-in-one thermocouple'')

- ❑ Conclusions and prospects

Field-Effect Transistors (FET)



Setup used by P. Kim (Columbia) (2013)

- Single (or array of) doped **nanowire(s)** in the FET configuration
- **Substrate**: Electrically and thermally insulating
- **Gate**: «back» or «top»
- **Heater**: for thermoelectric measurements

"Seebeck" configuration: thermal bias

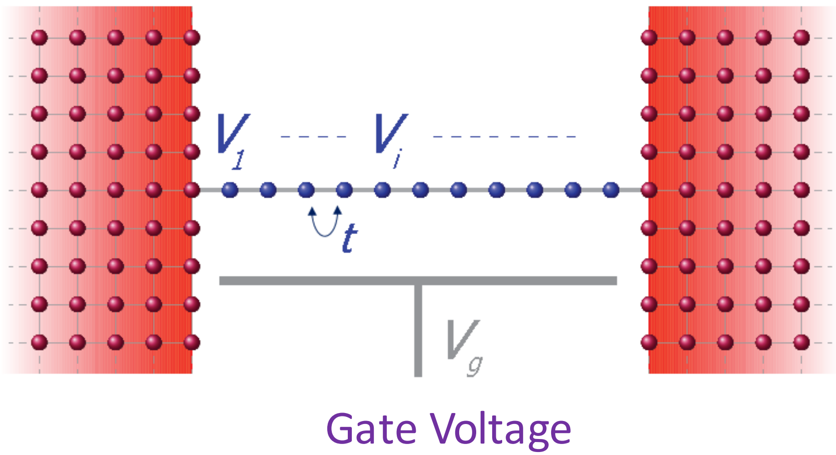
"Peltier" configuration: voltage bias

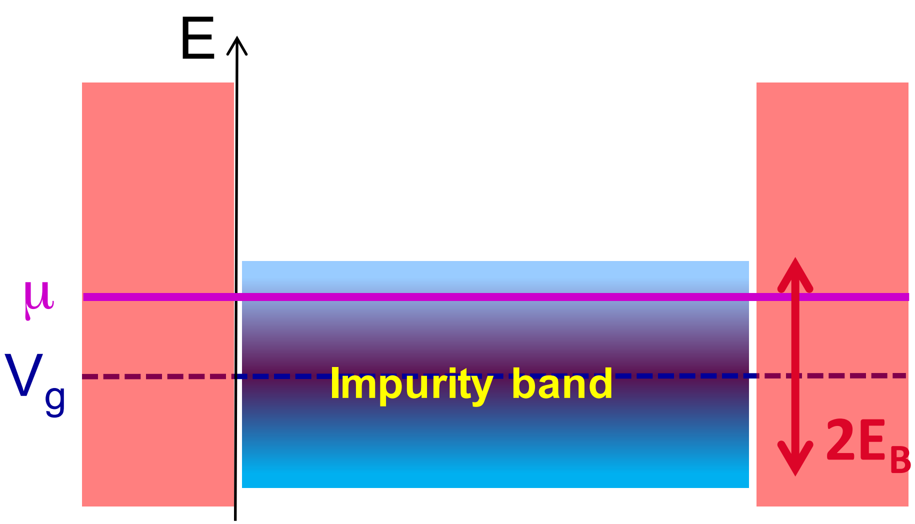
$\Longrightarrow$ $\Pi = ST$: equivalent within linear response if time-reversal symmetry preserved (Kelvin relation)

Goal: effect of gate modulation on the thermopower

1D Anderson Model

Prototypical model of localized system





- 1D electronic lattice with on-site (uniform) disorder $V_i \in$ (-W/2, W/2)

- Tight-binding Hamiltonian

$$\mathcal{H} = -t\sum_{i=1}^{N-1}\left(c_i^\dagger c_{i+1} + \mathrm{h.c.}\right) + \sum_{i=1}^{N}(V_i + V_g)c_i^\dagger c_i$$

- All electrons are localized with localization length ξ (E)

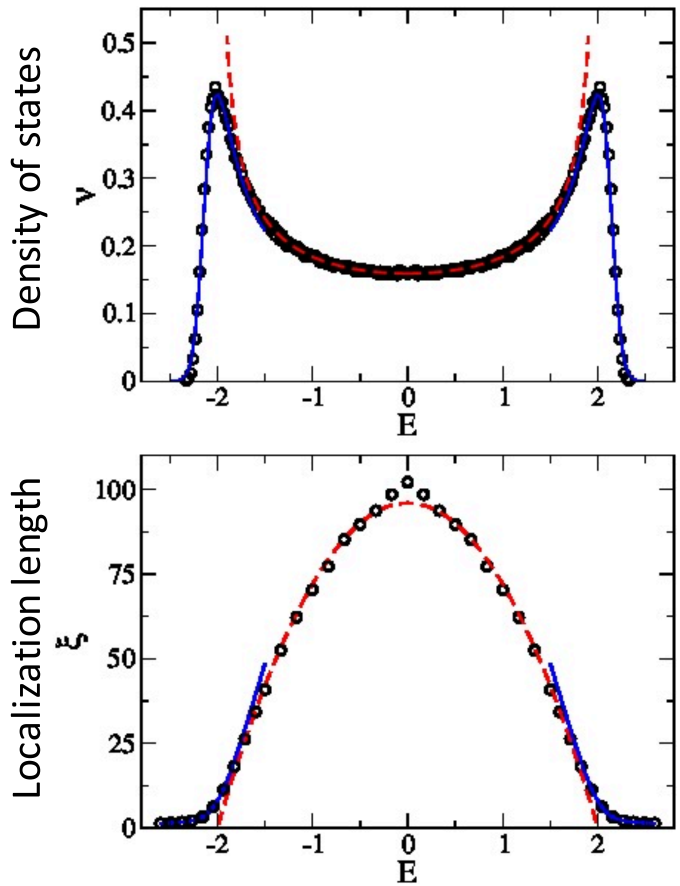
- States distributed within an impurity band $2E_B \approx 4t+W$

- Behavior of the typical thermopower when the gate voltage V_g is varied

1D density of states ν and localization length ξ

Analytical expressions derived in weak disorder limit

W=t : band edge at
~ 2t+W/2 ~ 2.5t



"Bulk" formulas:

$$\nu_b(E) = \frac{1}{2\pi t\sqrt{1-(E/2t)^2}}$$

$$\xi_b(E) = \frac{24}{W^2}\left(4t^2 - E^2\right)$$

"Edge" formulas:

$$\nu_e(E) = \sqrt{\frac{2}{\pi}}\left(\frac{12}{tW^2}\right)^{1/3}\frac{\mathcal{I}_1(X)}{[\mathcal{I}_{-1}(X)]^2}$$

$$\xi_e(E) = 2\left(\frac{12t^2}{W^2}\right)^{1/3}\frac{\mathcal{I}_{-1}(X)}{\mathcal{I}_1(X)}$$

$$\mathcal{I}_n(X) = \int_0^\infty y^{n/2}\, e^{-\frac{1}{6}y^3+2Xy}\, dy$$

$$X = (|E|-2t)t^{1/3}(12/W^2)^{2/3}$$

B. Derrida & E. Gardner, *J. Physique* 45, 1283 (1984)

Outline

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- ❑ Thermopower of single NW: low T elastic regime
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- ❑ Conclusions and prospects

Elastic regime: Thermopower

Theory: Transport mechanism: elastic (coherent) tunnelling

Localized regime: τ decays exponentially with length

$$[\ln \mathcal{T}]_0(E) = -\frac{2N}{\xi(E)}$$

Typical τ depends on the energy via ξ(E) (localization length)

Low Temperatures + Linear Response → Mott formula:

$$\boxed{S \approx \frac{\pi^2}{3}\frac{k_B}{e} k_B T \left.\frac{\mathrm{d}\ln \mathcal{T}}{\mathrm{d}E}\right|_{\mu}}$$

Numerics: Recursive Green Function calculation of S

R. Bosisio, G. Fleury, & J.-L. Pichard, *New J. Phys.* 16:035004 (2014)

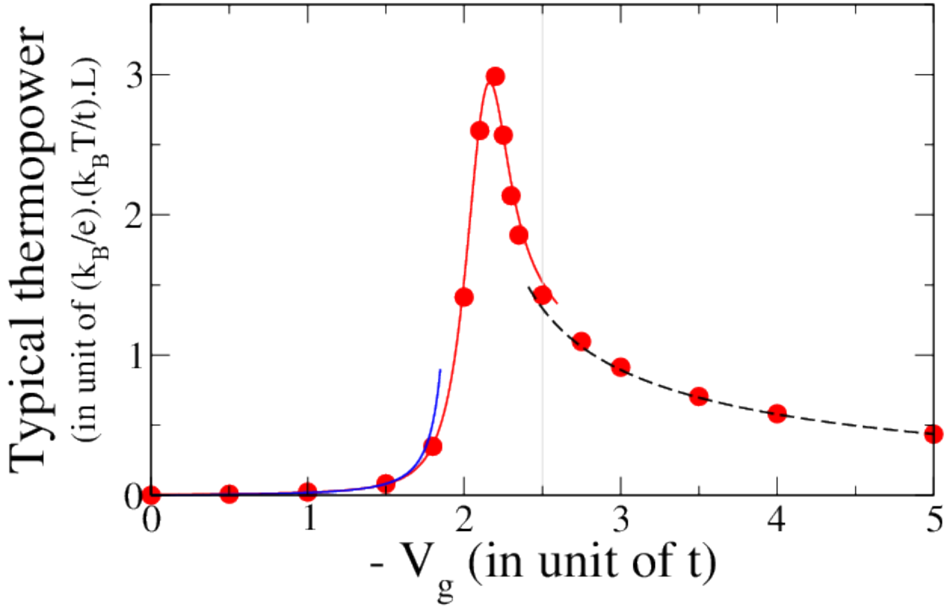
Elastic regime: Summary

Enhancement of the thermopower at the band edges (role of $\xi(E)$)

Analytical description of the results

Sommerfeld Expansion (low T)
Wiedemann-Franz law → Low S

Very low power factor $Q = GS^2$ because of the exponential reduction of G at the band edges



Interest: Ultra-low T Peltier cooling?

OR → toward higher temperatures!

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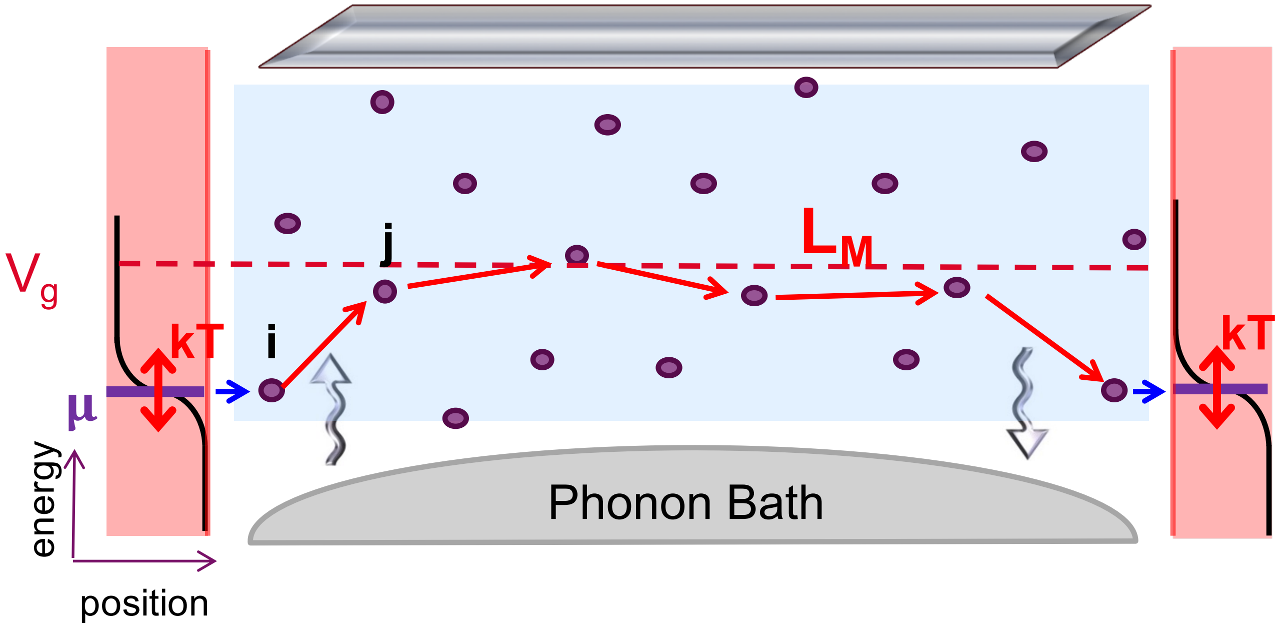
Variable Range Hopping: Transport Mechanism



Mott → competition between tunneling and activated processes

$$G_{ij} \sim e^{-2|x_i - x_j|/\xi} e^{-(|E_i-\mu|+|E_j-\mu|+|E_i-E_j|)/2k_BT}$$

Variable Range Hopping: phonon-assisted transport → *sequence of hops of variable size*

Optimal hop size: *Mott hopping length* $L_M \simeq \left(\frac{\xi}{2\nu T}\right)^{1/2}$

or *Mott hopping energy* $\Delta = \left(\frac{2T}{\xi\nu}\right)^{1/2}$

$$\rightarrow G \sim e^{-\frac{2L_M}{\xi}} = e^{-\frac{\Delta}{k_BT}}$$

Transport Mechanisms

$$L_M \simeq \left(\frac{\xi}{2\nu T}\right)^{1/2}$$

T

Low T: $L_M >> L$ → elastic transport

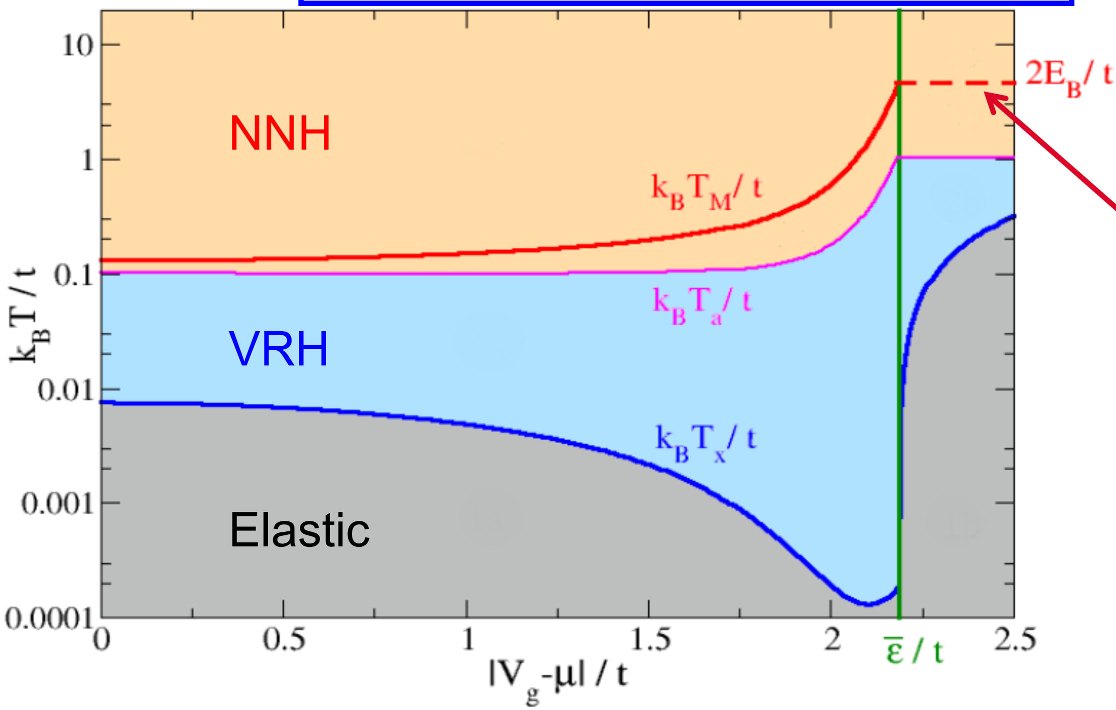
Increasing T: $L_M \sim L$ → onset of inelastic processes (VRH) → $T_x \simeq \frac{\xi}{2\nu L^2}$

Increasing T: $L_M \sim \xi$ → simple activated transport (NNH) → $T_M \simeq \frac{2}{\xi\nu}$

Mott's Hopping Energy: finite range of states contributing to transport

$$\boxed{\Delta = k_B\sqrt{TT_M} \propto \sqrt{\frac{T}{\xi\nu}} \gg k_B T}$$

Relevant energy scale for activated transport



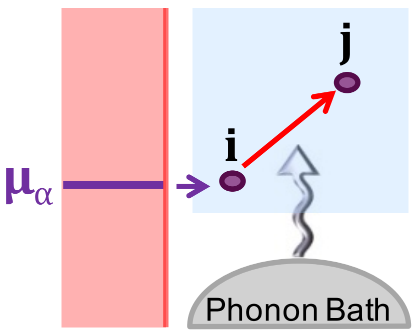
Cut-off required by $\Delta(T_M) = k_B T_M \leq 2E_B$

(T_a : onset of simple activation in 1D (Kurkijärvi 1973, Raikh & Ruzin 1989))

Inelastic (Phonon-Assisted) Regime: method

Essential ingredients to build & solve the Random Resistor Network

1. Transition rates (Fermi golden rule)



Between lead and localized states **[Elastic tunneling rates]**

$$\Gamma_{i\alpha} = \gamma_{i\alpha} f_i \left[1 - f_\alpha(E_i)\right] \quad \alpha = \text{L,R}$$

$$\gamma_{i\alpha} \simeq \gamma_e \exp(-2x_{i\alpha}/\xi_i)$$

Between localized states **[Inelastic hopping rates]**

$$\Gamma_{ij} = \gamma_{ij} f_i (1 - f_j) \left[N_{ij} + \theta(E_i - E_j)\right]$$

$$\gamma_{ij} \simeq \gamma_{ep} \exp(-2x_{ij}/\xi) \longleftarrow$$

ξ energy dependence usually neglected!

$$\longrightarrow \quad \gamma_{ij} \propto |\langle\psi_i|\psi_j\rangle|^2 \simeq \gamma_{ep} \left| \frac{(1/\xi_j)\exp(-x_{ij}/\xi_i) - (1/\xi_i)\exp(-x_{ij}/\xi_j)}{(1/\xi_i - 1/\xi_j)} \right|^2$$

Miller & Abrahams, *PR* (1960), Ambegaokar, Halperin & Langer *PRB* (1971), Jiang, Entin-Wohlman & Imry., *PRB* (2014)

Inelastic (Phonon-Assisted) Regime: method

2. Fermi distributions at equilibrium (no bias)

$$f_i^0 = \left(e^{(E_i-\mu)/kT} + 1\right)^{-1} \qquad f_\alpha = f_\alpha^0(E_i) = \left(e^{(E_i-\mu)/kT} + 1\right)^{-1}$$

3. Occupation probabilities out of equilibrium

$$f_i = f_i^0 + \delta f_i \qquad f_\alpha = f_\alpha^0 + \delta f_\alpha$$

4. Currents

$$I_{ij} = e\left(\Gamma_{ij} - \Gamma_{ji}\right) \qquad I_{i\alpha} = e\left(\Gamma_{i\alpha} - \Gamma_{\alpha i}\right)$$

5. Current conservation at every node i (Kirchoff)
N coupled equations in N variables

$$\left(\sum_{j\neq i} I_{ij}\right) + I_{iL} + I_{iR} = 0 \quad \rightarrow \quad \delta f_i$$

Jiang, Entin-Wohlman & Imry., *PRB* (2014)

Miller – Abrahams Resistor Network

6. Total particle/heat currents

Summing all terms flowing out from L(R) terminal

$$J_L^e = -\sum_i I_{iL} = \sum_i I_{iR},$$

$$J_{L(R)}^Q = \sum_i \left(\frac{E_i - \mu_{L(R)}}{e}\right) I_{L(R)i}.$$

7. Transport coefficients

$$G = \frac{J_L^e}{\delta\mu/e}$$ **Conductance**

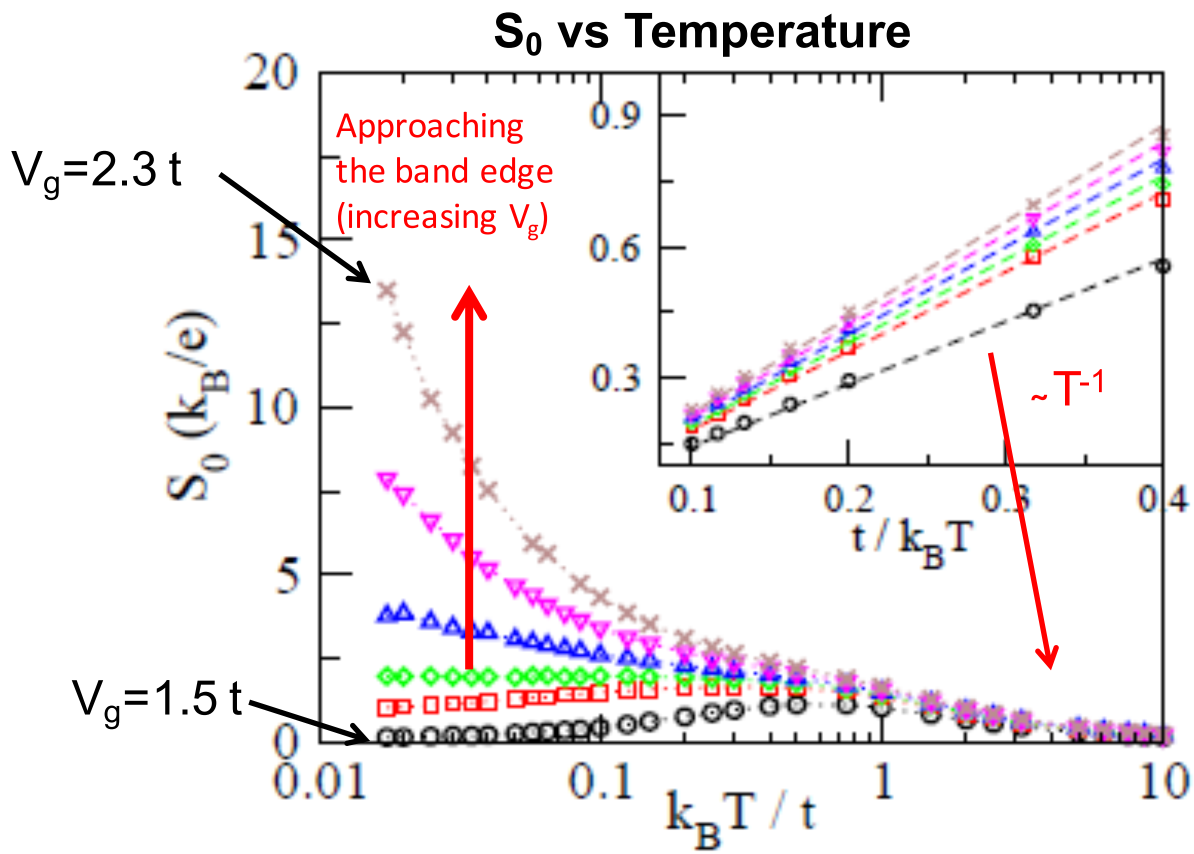
$$\Pi = \frac{J_L^Q}{J_L^e}$$ **Peltier coefficient**

$$S = \frac{\Pi}{T}$$ **Thermopower**

(Having assumed Peltier configuration: T constant everywhere)

Jiang, Entin-Wohlman & Imry., *PRB* (2014)

Inelastic Regime: Typical thermopower



- Thermopower enhancement when the band edges are approached
- Rich behaviour of the T-dependence of the thermopower, "reflecting" the shape of the density of states and localization length

R. Bosisio, C. Gorini, G. Fleury, & J.-L. Pichard, *New J. Phys*. 16:095005 (2014)

Inelastic regime: theory

Theory:

Percolation approach to solve the RRN:
Ambegaokar, Halperin, Langer → conductance (1971)
Zvyagin → thermopower (1973)
thermopower = energy averaged over the percolating path

Integrate between (μ-Δ; μ+Δ)

$$S = \frac{\langle E-\mu\rangle}{eT} = \frac{1}{eT}\,\frac{\int dE\,(E-\mu)\,\nu(E)\,p(E)}{\int dE\,\nu(E)\,p(E)}$$

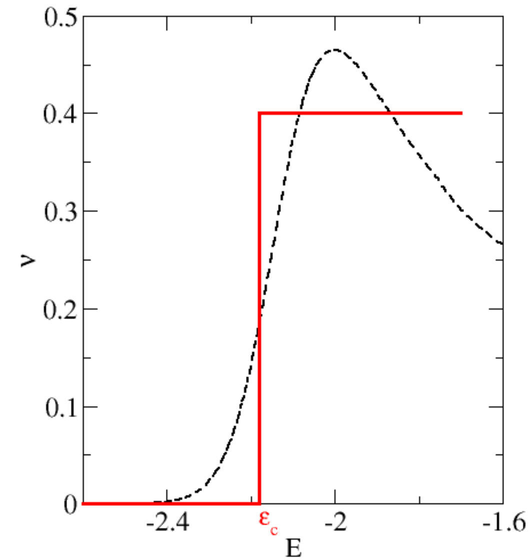
$\sim T^{-1}$: simple activation: energy to «jump» toward ϵ_C

$$S = \frac{k_B}{e}\left(\boxed{\frac{\epsilon_c-\mu}{2k_BT}} + \boxed{\frac{\Delta(T)}{2k_BT}}\right) \quad \text{if} \quad \epsilon_c < \mu \qquad \text{(inside)}$$

$$S = \frac{k_B}{e}\left(\boxed{\frac{\epsilon_c-\mu}{k_BT}} + \boxed{\frac{\Delta(T)}{2k_BT}}\right) \quad \text{if} \quad \epsilon_c > \mu \qquad \text{(outside)}$$

$\sim T^{-1/2}$: Variable Range Hopping (Mott)

$$\ln G(T) \sim \boxed{-\frac{E_A}{k_BT}} \boxed{-\bar{\alpha}\sqrt{\frac{T_M}{T}}}$$



Inelastic Regime: Typical thermopower

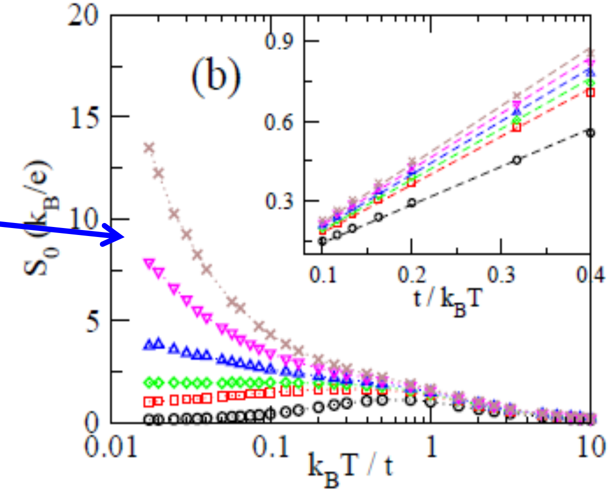
qualitative behavior of the slopes

$$S = \frac{k_B}{e}\left(\frac{\epsilon_c-\mu}{2k_BT}+\frac{\Delta(T)}{2k_BT}\right) \quad \text{if} \quad \epsilon_c<\mu \qquad \text{(inside)}$$

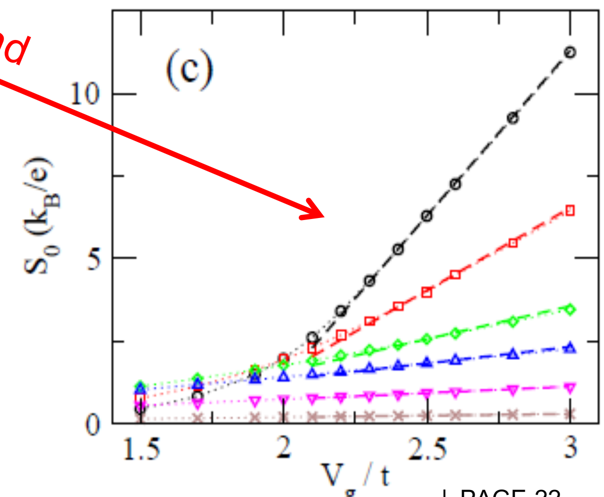
$$S = \frac{k_B}{e}\left(\frac{\epsilon_c-\mu}{k_BT}+\frac{\Delta(T)}{2k_BT}\right) \quad \text{if} \quad \epsilon_c>\mu \qquad \text{(outside)}$$

linear behavior outside the band

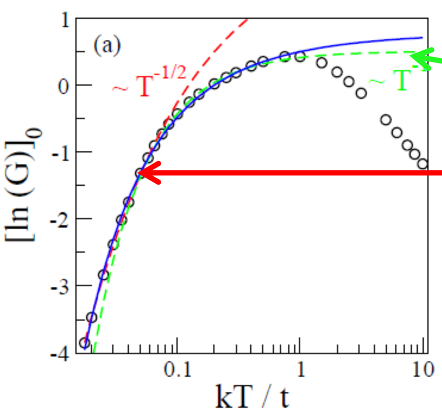
S_0 vs Temperature



S_0 vs Gate Voltage



Electrical Conductance



$$\ln G(T) \sim -\frac{E_A}{k_BT} - \alpha\sqrt{\frac{T_M}{T}}$$

R. Bosisio, C. Gorini, G. Fleury, & J.-L. Pichard, *New J. Phys*. 16:095005 (2014)

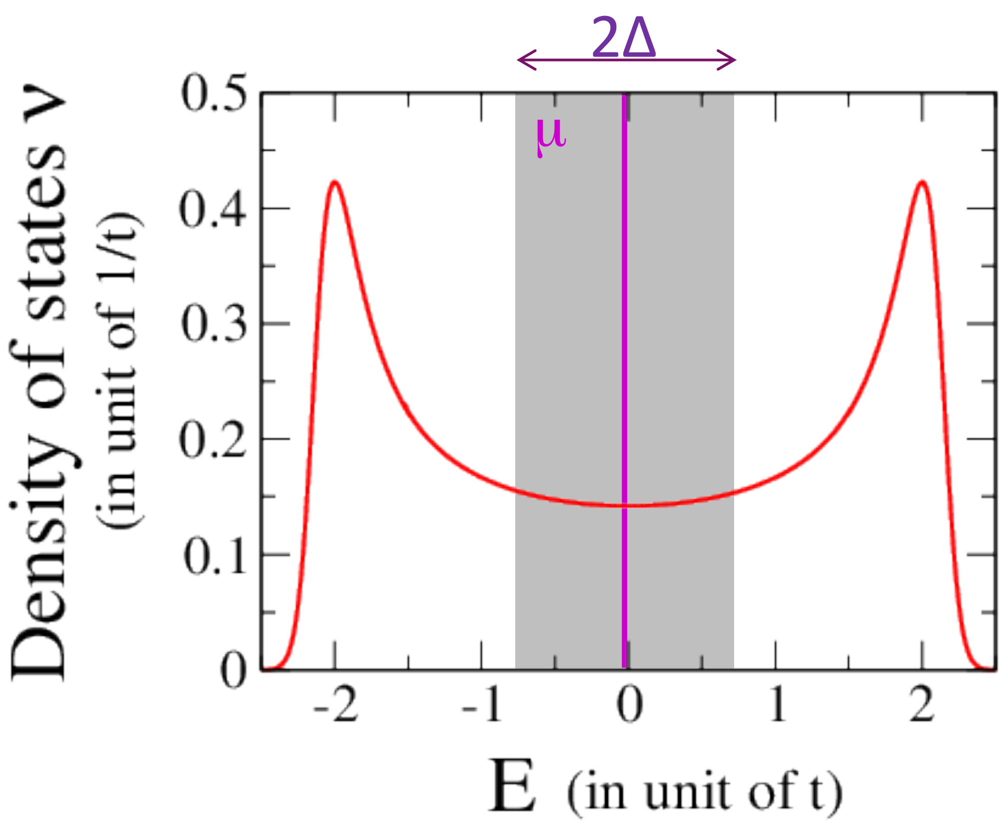
Inelastic Regime: Mott energy

$$S = \frac{\langle E-\mu \rangle}{eT} = \frac{1}{eT}\frac{\int dE\,(E-\mu)\,\nu(E)\,p(E)}{\int dE\,\nu(E)\,p(E)}$$

Integration inside [μ -Δ, μ +Δ]

Mott's Hopping Energy:
finite range of states contributing to transport

$$\Delta = k_B\sqrt{TT_M} \propto \sqrt{\frac{T}{\xi\nu}} \gg k_B T$$



S depends on the asymmetry of the states around μ within [μ -Δ, μ +Δ]

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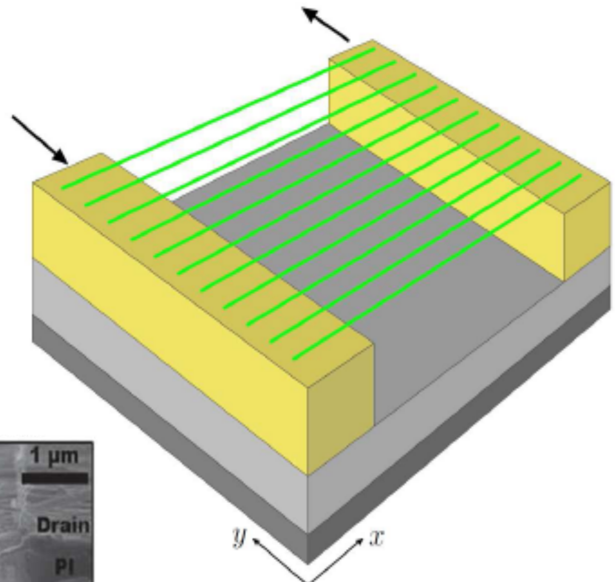
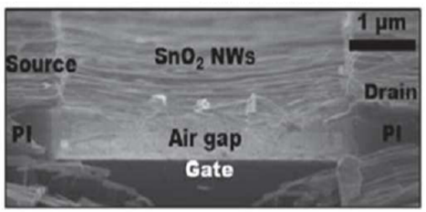
Arrays of parallel nanowires

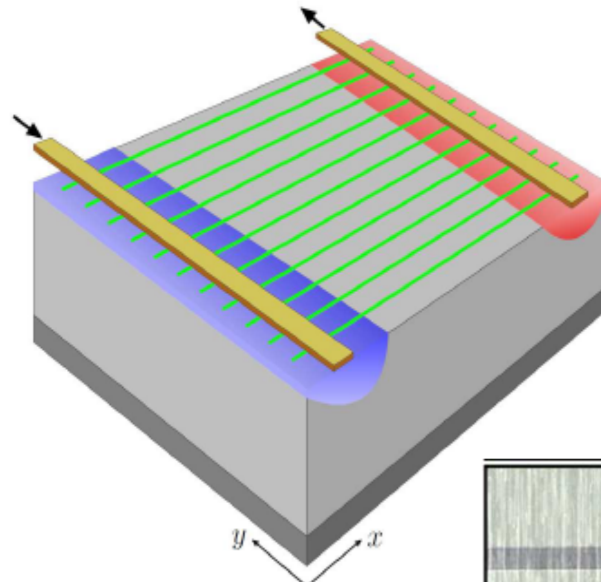
Suspended

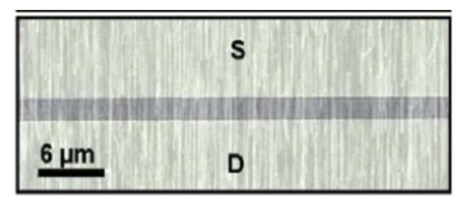




Shin *et al.*, 2011

Deposited





Fan *et al.*, 2008

- Neglect *inter*-wire hopping → independent nanowires
- Transport through each NW: VRH / NNH regime (<u>same</u> treatment as before)

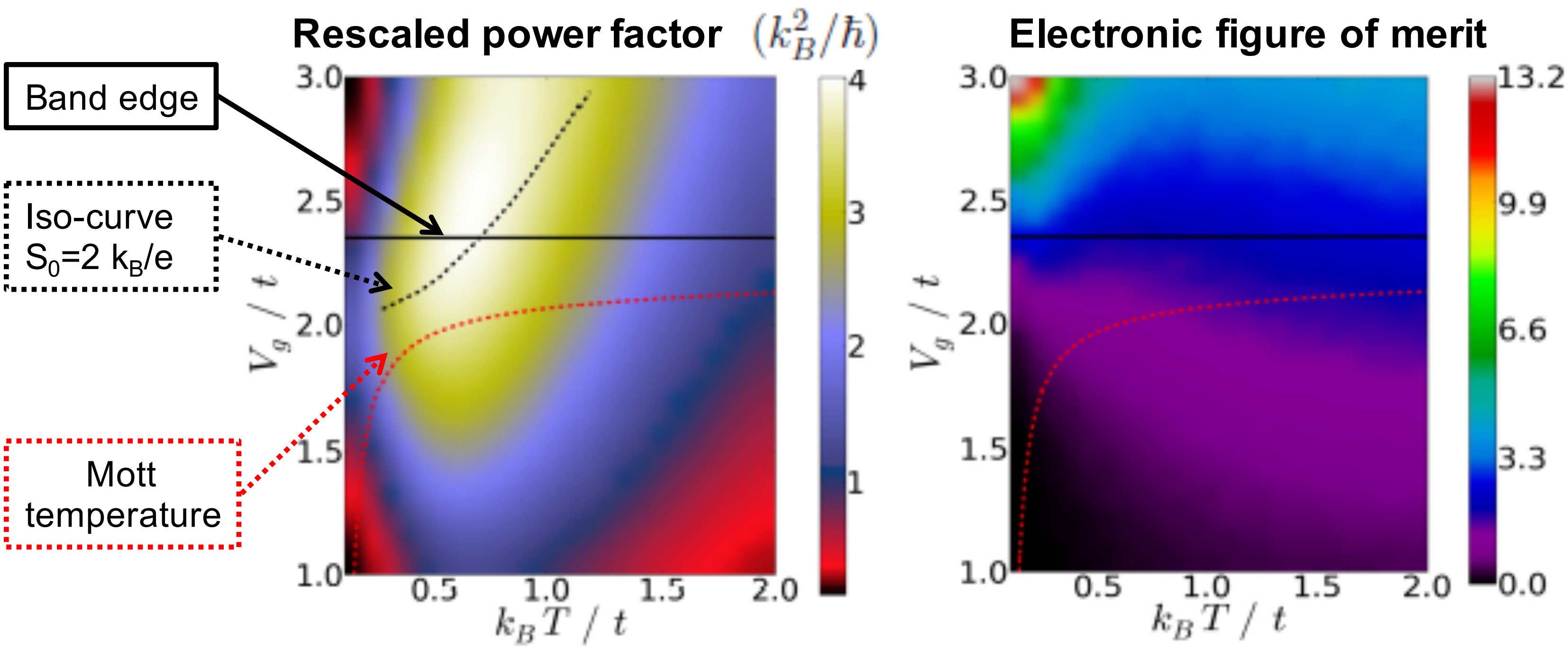
Parallel nanowires: power factor and figure of merit

$$G \approx M\, G_0$$
$$K^e \approx M\, K^e_0$$
$$S \approx S_0$$

Scalable Power Factor

(without affecting the electronic figure of merit)

$$Q = GS^2 \approx M\, G_0 S_0^2$$



Parameters: $M = 150,\ W = t,\ \gamma_e = \gamma_{ep} = t,\ L = 450$

P~μW for 10^5 NWs (1 cm) and δT~10 K

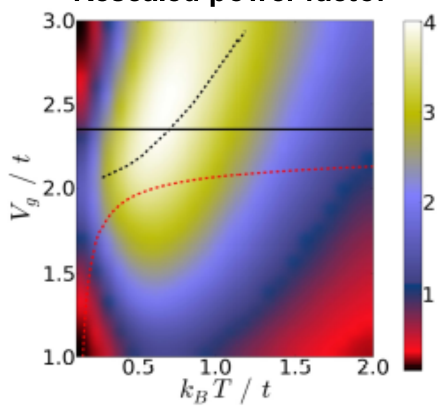
R. Bosisio, C. Gorini, G. Fleury, & J.-L. Pichard, *Phys. Rev. Applied* ***3****, 054002* (2015)

Parallel nanowires: power factor and figure of merit

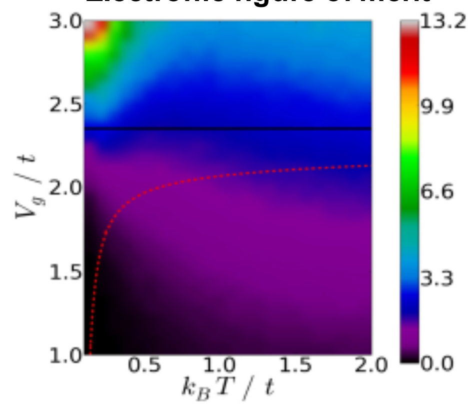
Estimation of the parasitic **phononic contribution** to ZT

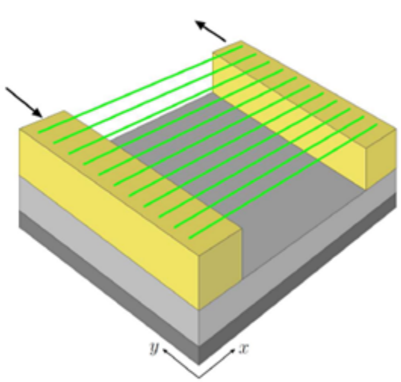
$$ZT = \frac{Z_e T}{1 + K^{ph}/K^e}$$

Rescaled power factor

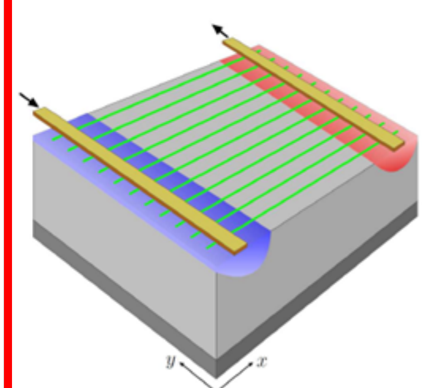


Electronic figure of merit





$$ZT \approx \frac{Z_e T}{3}$$



$$ZT \approx \frac{Z_e T}{21}$$

(For doped Si-NWs and SiO_2 substrate)

Parallel nanowires: Hot Spot cooling

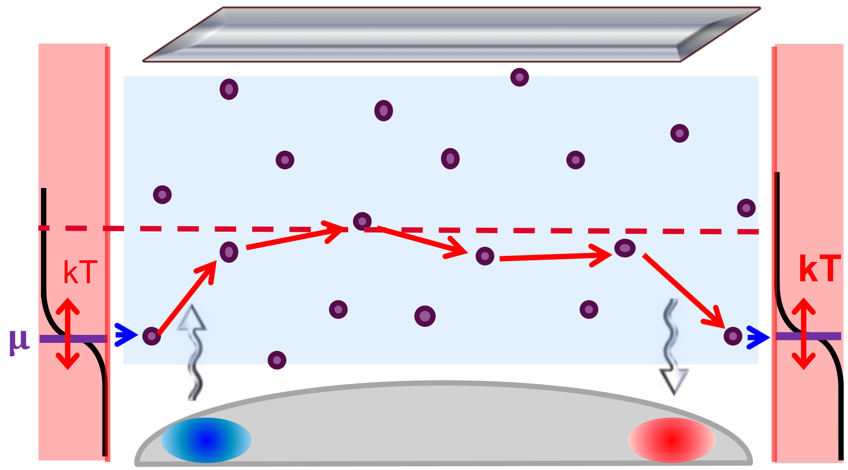
Hopping heat current through each localized state *i*

$$I_i^Q = \sum_j I_{ij}^Q = \sum_j (E_j - E_i) I_{ij}^N$$

E_i randomly distributed → Local fluctuations

$$\boxed{\mathcal{I}_{x,y}^Q = \sum_{i \in \Lambda_{ph} \times \Lambda_{ph}(x,y)} I_i^Q}$$

Λ_{ph}: inelastic phonon mean free path = *thermalization length* in the substrate

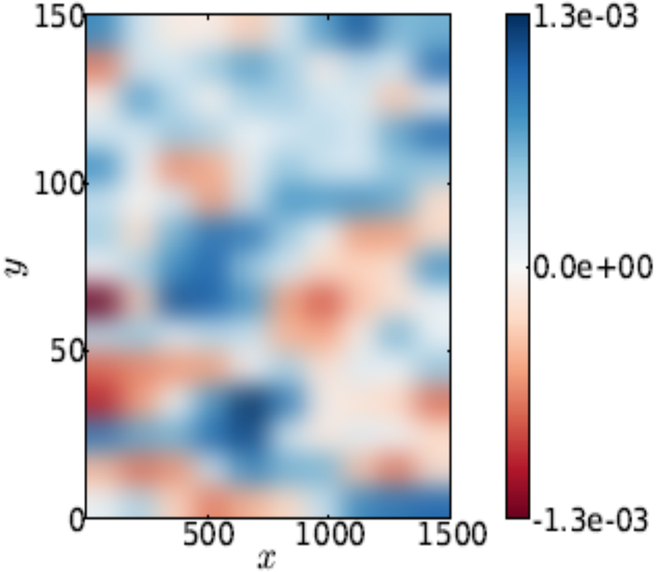


R. Bosisio, C. Gorini, G. Fleury, & J.-L. Pichard, *Phys. Rev. Applied* ***3****, 054002* (2015)

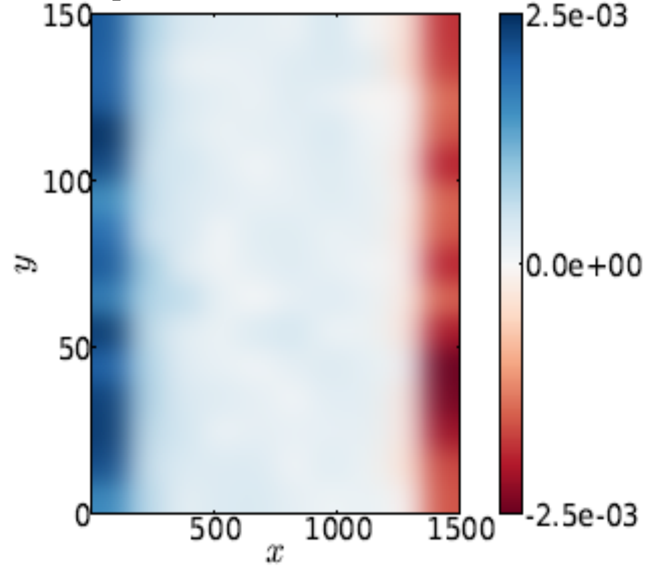
Parallel nanowires: Hot Spot cooling

Top view of the substrate

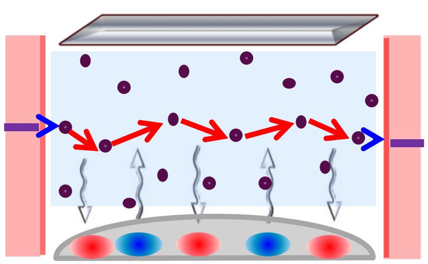
V_g=0: band *center*

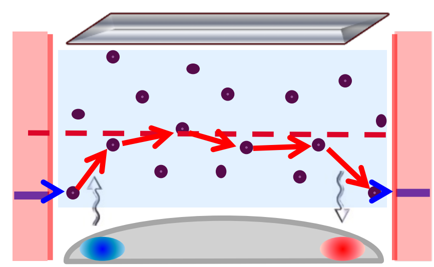


V_g=2.25 t: band *edge*



Formation of hot and cold spots near the boundaries of the substrate





Parameters: $M = 150,\ W = t,\ \gamma_e = \gamma_{ep} = t,\ k_B T = 0.25t,\ \Lambda_{ph} = 150a$

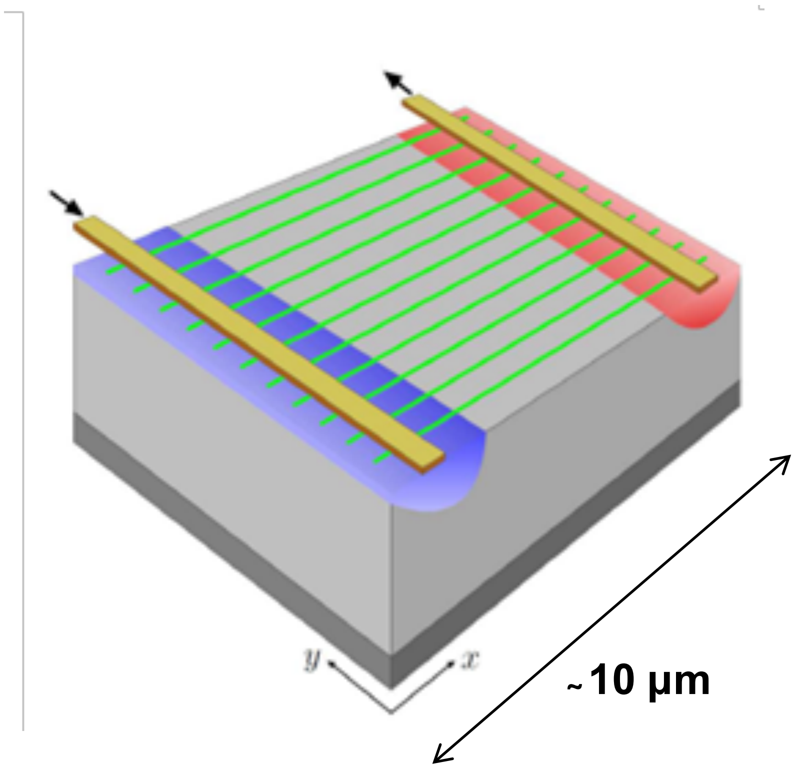
(Heat currents in unit of $t^2/\hbar$)

R. Bosisio, C. Gorini, G. Fleury, & J.-L. Pichard, *Phys. Rev. Applied* ***3****, 054002* (2015)

Parallel nanowires: Hot Spot cooling

Opportunities for heat management in microstructures

E.g.: **hot spot cooling** in microelectronics: transferring heat some microns away



Estimation of the cooling power density:

~10 W/cm^2

at $\delta\mu$=1mV ~ 7.7 x 10^{-2}t
and T = 75K ~ 0.5t
(still linear response)

Microprocessors ~ 100 W/cm^2

R. Bosisio, C. Gorini, G. Fleury, & J.-L. Pichard, *Phys. Rev. Applied* ***3****, 054002* (2015)

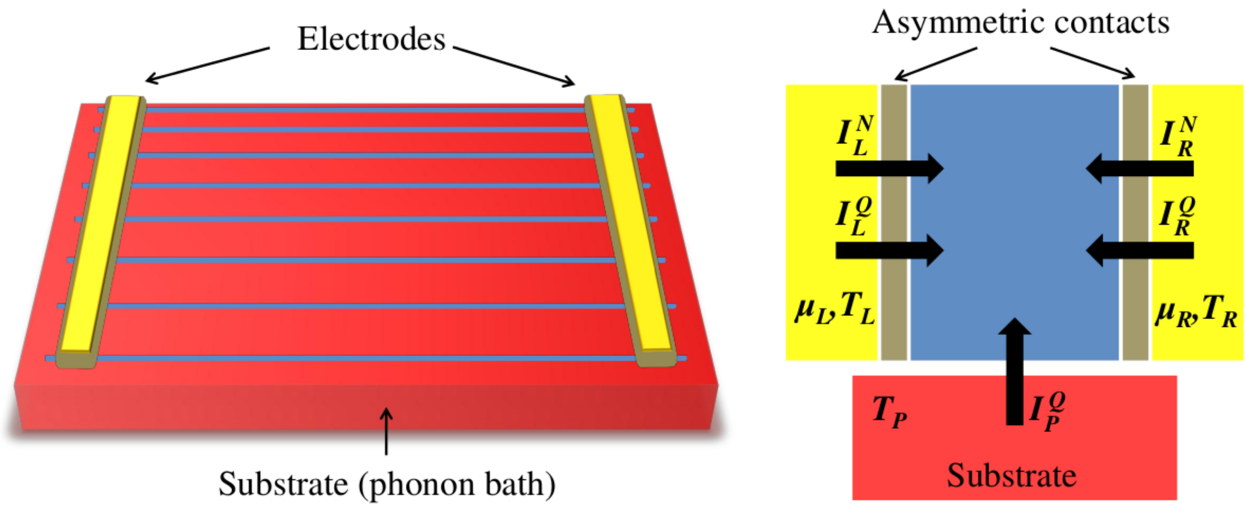
Outline

- ✓ Nanowires in the Field Effect Transistor configuration
  The 1D Anderson Model

- ✓ Thermopower of single NW: low T elastic regime
- ✓ Thermopower of single NW: inelastic phonon-assisted regime
- ✓ Parallel Nanowires: scalable power factor and hot spots cooling
- ❑ 3-Terminal thermoelectric ratchet

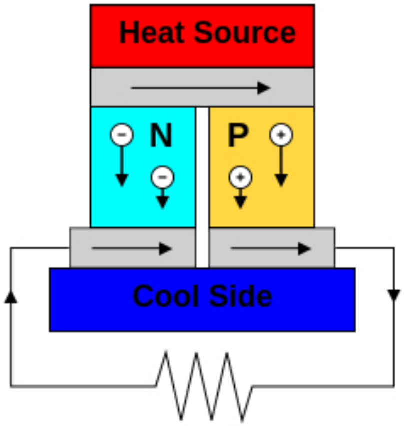
- ❑ Conclusions and prospects

3-Terminal thermoelectric ratchet



(a) (b)

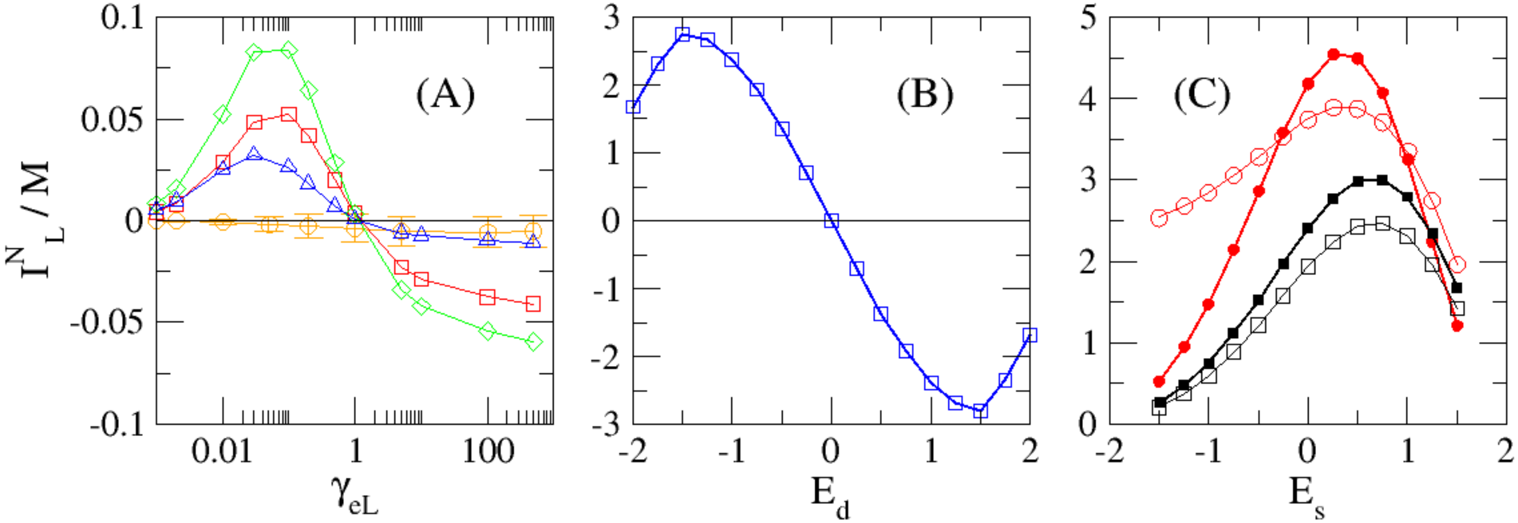
- ❑ Microscale heat harvester or cooler
- ❑ Non – local thermal machine (``all-in-one thermocouple'')
- ❑ Simple (ideally Si – based)

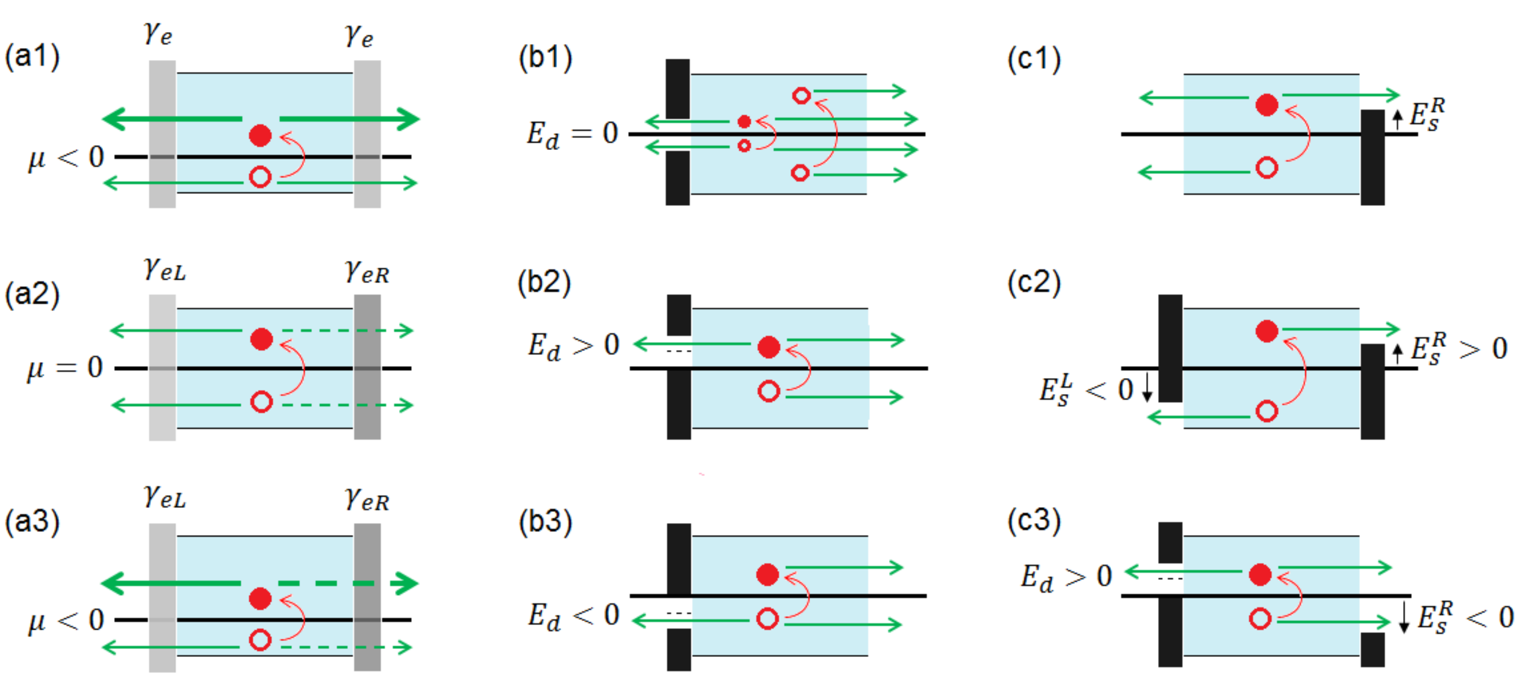


Bosisio, Fleury, Pichard & Gorini, arXiv (2015)

Sànchez & Büttiker, *PRB* (2011) → Low T, coherent, feasible but extremely low power

3-Terminal thermoelectric ratchet





The end

Aller à l'idéal et comprendre le réel.

Jean Jaurès

- **Variable Range Hopping → inelastic, large window for e-h asymmetry**

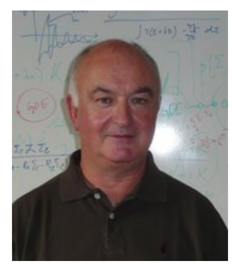
- Parallel nanowires → high Seebeck AND good power factor Q
- Gate-controlled local heat exchanges (hot spots cooling)
- 3-Terminal thermoelectric ratchet:
    1. Simple setup, energy harvester/cooler
    2. Good power factor Q and electronic ZT (0.1 - 100)

Universität Regensburg

SFB 689

Riccardo Bosisio

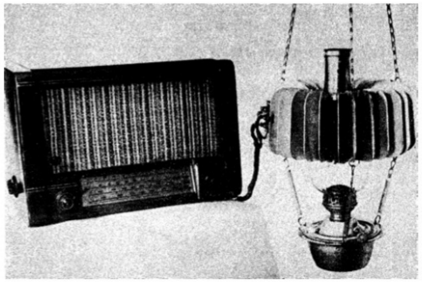
Geneviève Fleury

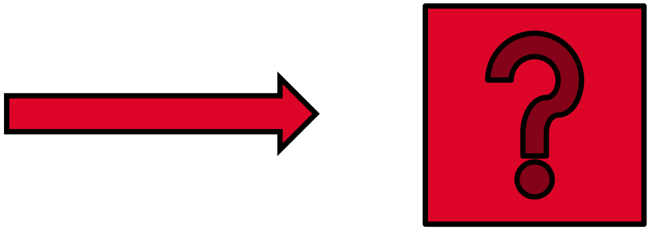


Jean-Louis Pichard

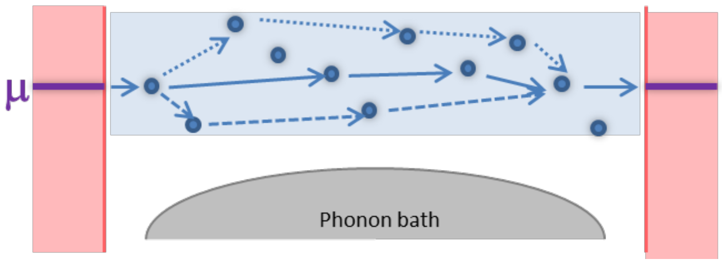
Wishlist

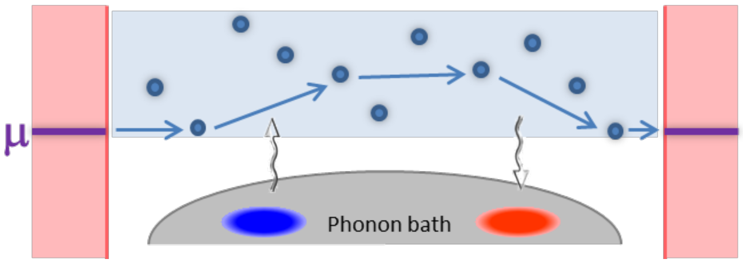
- Rigorous treatment of e-ph coupling
- Combining DFT calculations of γ_e and γ_{ep}
- Non-linear regime (higher output powers), multi-terminal setups
- Heat-to-spin conversion I: magnetism and/or spin-orbit in wires
- Heat-to-spin conversion II: hot magnon bath
- Time-dependent configurations.

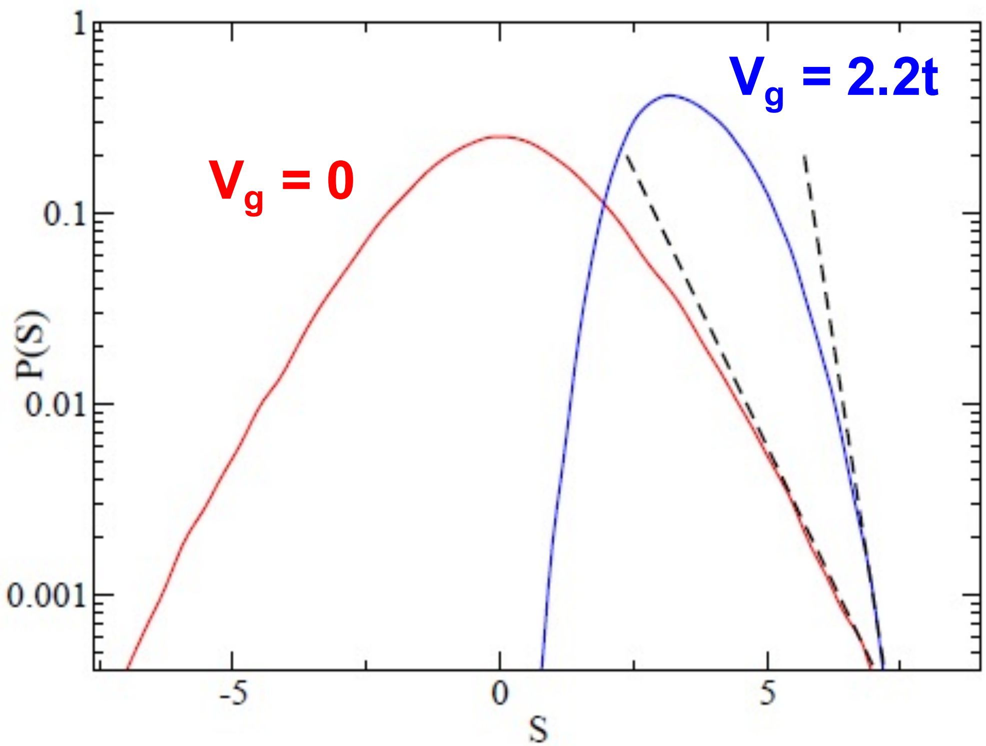




Inelastic Regime: Thermopower fluctuations





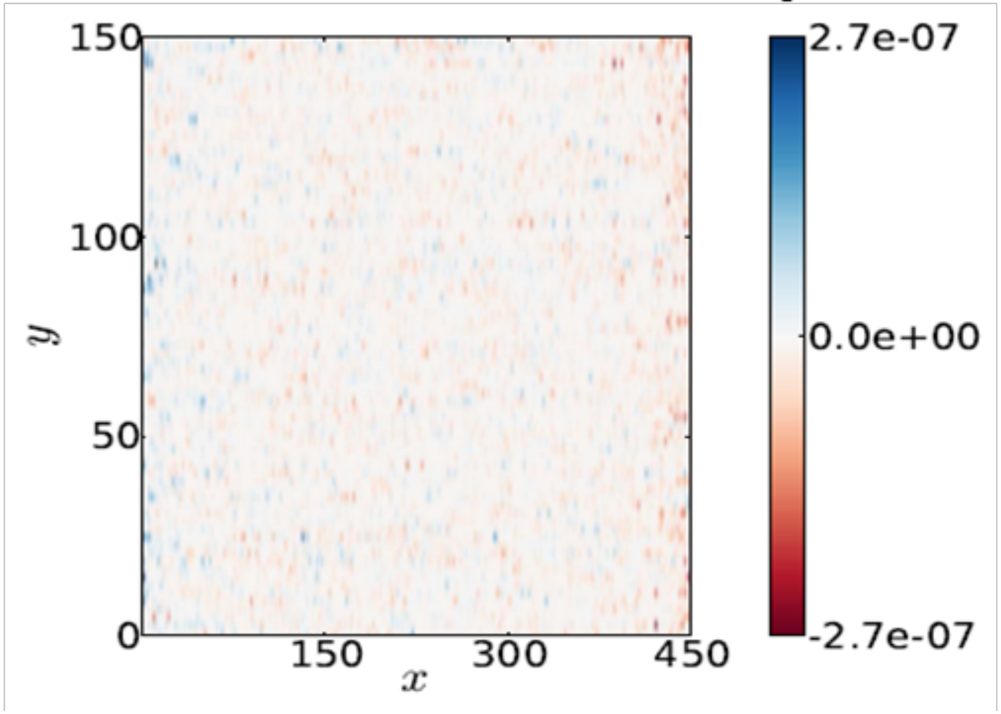


Numerical simulations
for kT=t and W=t

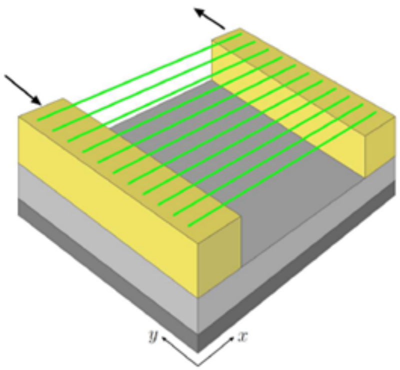
R. Bosisio, C. Gorini, G. Fleury, & J.-L. Pichard, *New J. Phys*. 16:095005 (2014)

Parallel nanowires: Hot Spot cooling

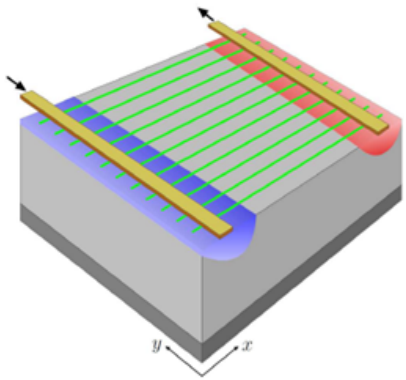
I^Q_i (top view) for T=0.05t and V_g=2.25t



Parasitic Phonon Contribution to ZT



$$ZT \approx \frac{Z_e T}{1 + 2/\tilde{\gamma}_e}$$



$$ZT \approx \frac{Z_e T}{1 + 20/\tilde{\gamma}_e}$$

$$ZT = \frac{Z_e T}{1 + K^{ph}/K^e}$$

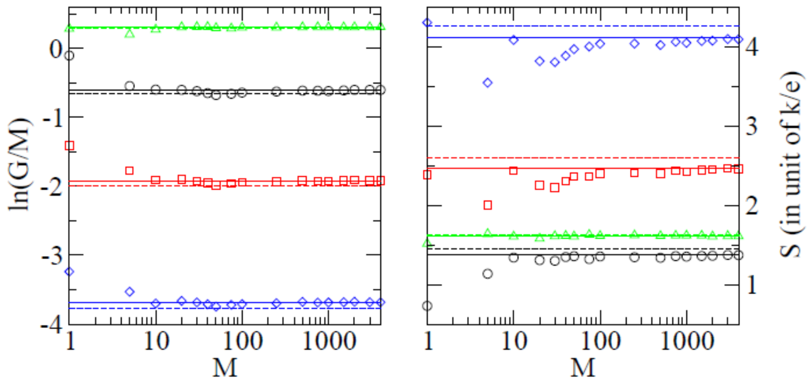
$$\frac{Z}{Z_e} = \left[1 + \frac{\kappa^{sub}\Sigma^{sub} + M\kappa_0^{nw}\Sigma^{nw}}{M\kappa_0^{e}\Sigma^{nw}} \right]^{-1}$$

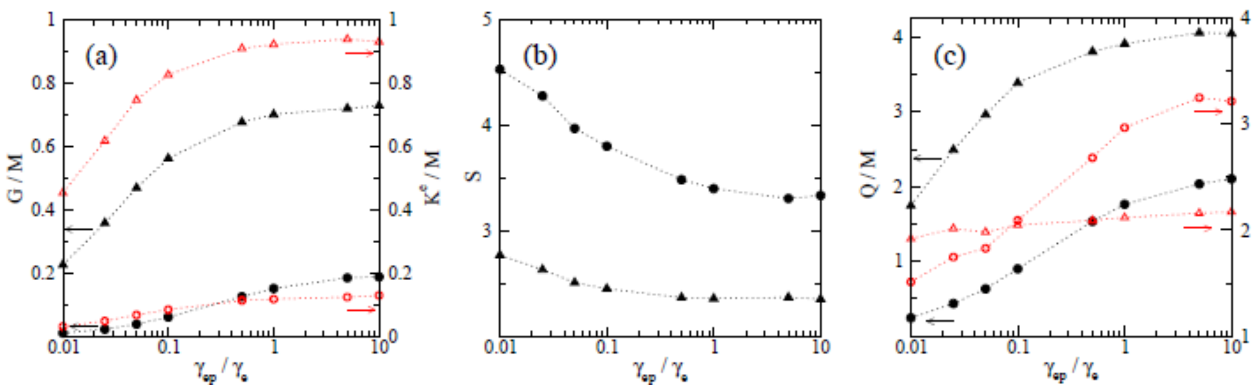
$$\kappa_0^{nw} \approx 2\,\mathrm{W}/(\mathrm{K.m}) \text{ at } T \approx 100\,\mathrm{K}$$

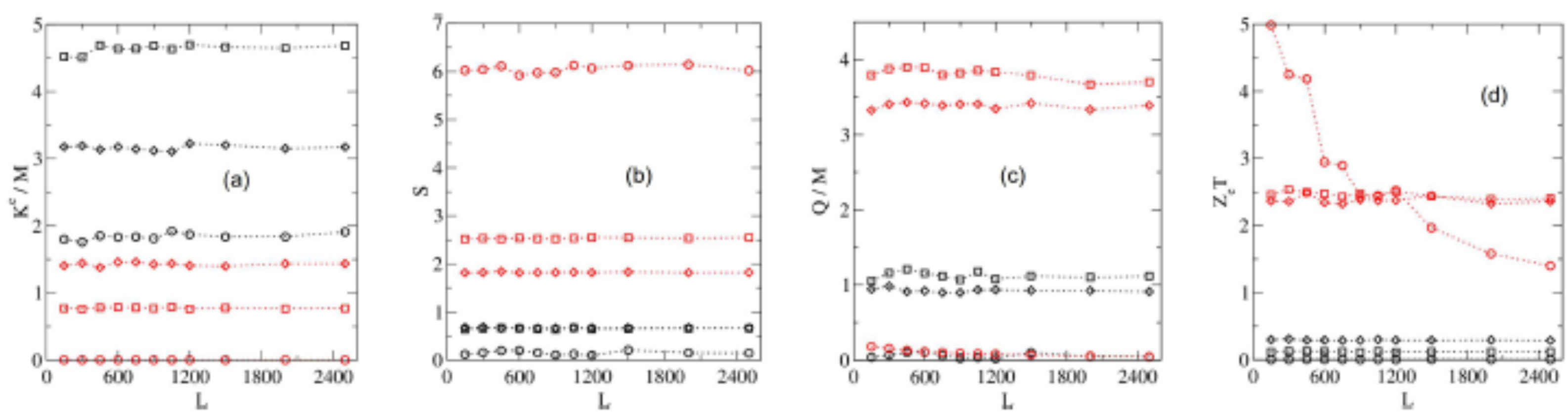
$$\kappa^{sub} \approx 0.7\,\mathrm{W}/(\mathrm{K.m})$$

(For doped Si-NWs and SiO_2 substrate)

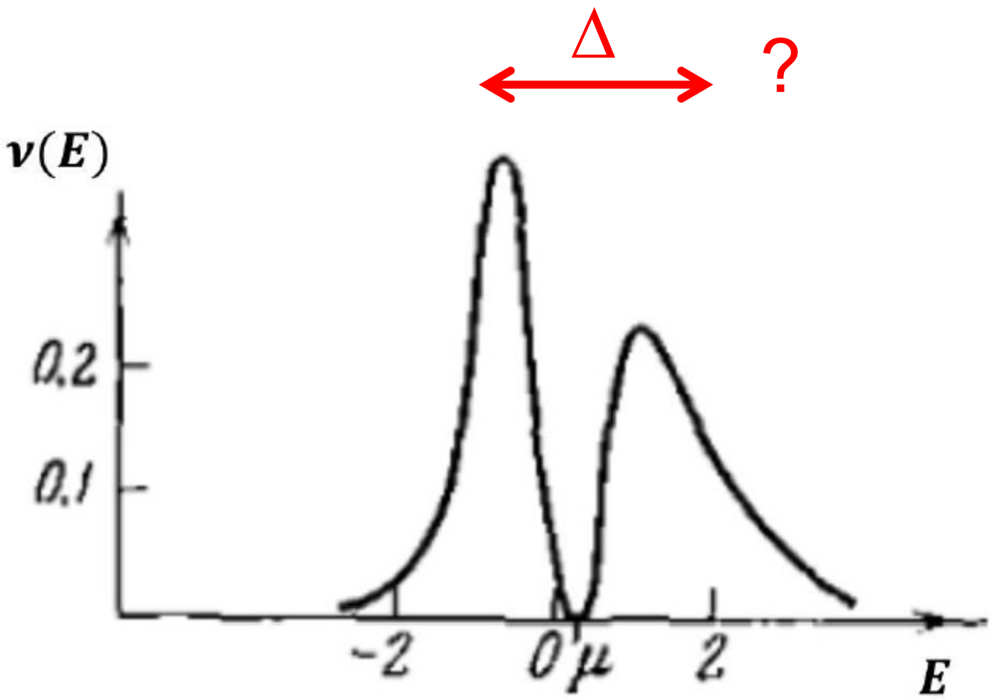
Parallel nanowires: addenda



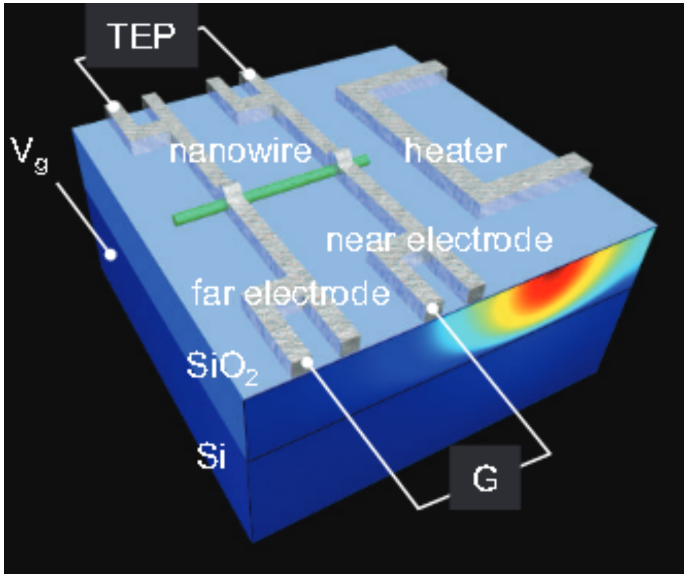


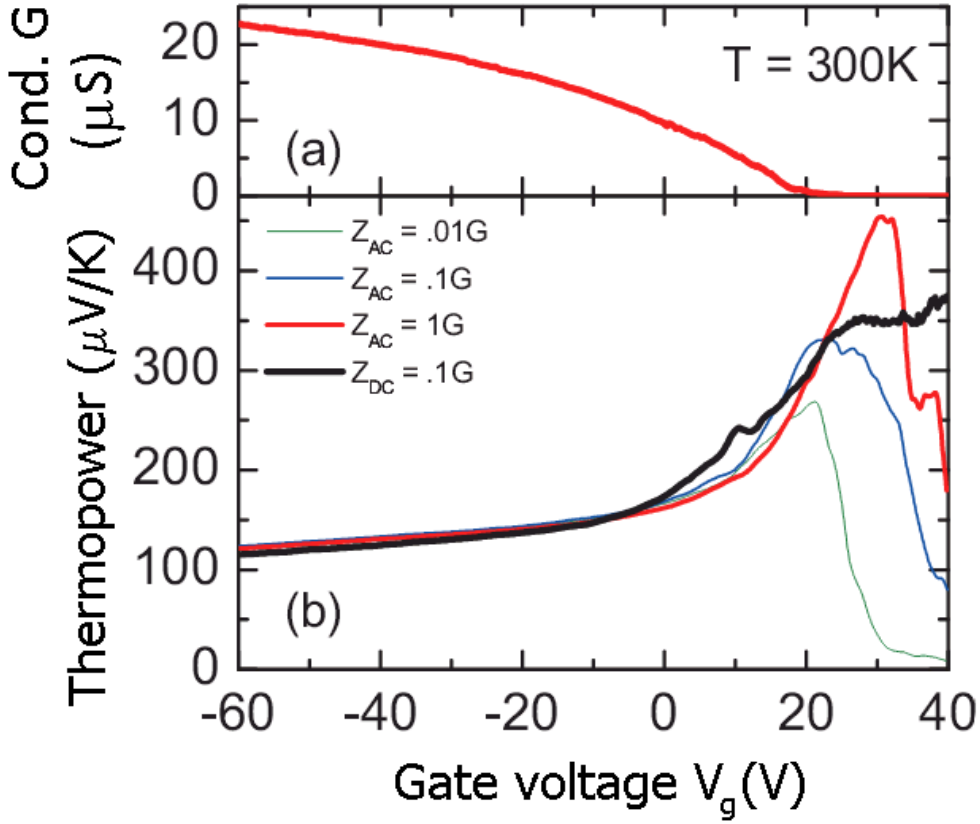


Density of states with Coulomb gap



Thermopower Enhancement: experimental results





Group of Philip Kim (preprint 2013)

Similar data in the group of F. Giazotto or H. Riel

(Field effect transistor device configuration)

Largely enhanced thermopower near the band edge of the nanowire

(Experiments mainly carried out at Room Temperature)

Linear response: Onsager formalism

(Two-terminal) Coupled 1D charge and heat transport

Fluxes

$$\begin{cases} J^e = L_{11} X_1 + L_{12} X_2 \\ J^Q = L_{21} X_1 + L_{22} X_2 \end{cases}$$

Affinities (forces)

$$X_1 = \frac{\Delta\mu}{T} = \frac{e\Delta V}{T} \qquad X_2 = \frac{\Delta T}{T^2}$$

Positivity of entropy production

$$L_{11} \geq 0, \quad L_{22} \geq 0, \quad \det \mathbf{L} \geq 0$$

Onsager relation (TRS)

$$L_{12} = L_{21}$$

Transport coefficients & Onsager matrix elements

$$G = \left(\frac{J^e}{\Delta V}\right)_{\Delta T = 0} = \frac{e^2}{T} L_{11}$$

$$K = \left(\frac{J^Q}{\Delta T}\right)_{J^e = 0} = \frac{1}{T^2} \frac{\det \mathbf{L}}{L_{11}}$$

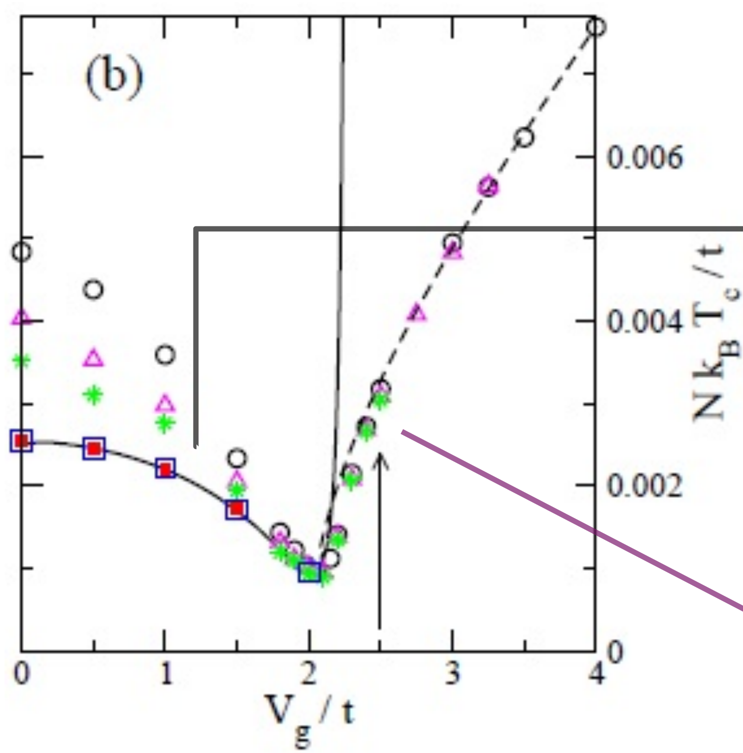
$$S = -\left(\frac{\Delta V}{\Delta T}\right)_{J^e = 0} = \frac{1}{eT} \frac{L_{12}}{L_{11}}$$

$$\Pi = \left(\frac{J^Q}{J^e}\right)_{\Delta T = 0} = \frac{1}{e} \frac{L_{21}}{L_{11}}$$

Elastic Regime: validity of Sommerfeld expansion

Q: Validity of the Mott's formula for S

Validity of Sommerfeld Expansion → Wiedemann-Franz (WF) law, Mott's formula



Sommerfeld temperature is proportional to the mean energy level spacing in the system:

$$k_B T_c \propto \Delta_F$$

(Proportionality constant depends on required precision)

Result for a tunnel barrier:

$$N k_B T_c \propto t\sqrt{[(E_F - V_g)/(2t)]^2 - 1}$$

Estimation for Si nanowire: ~100 mK

Elastic Regime: Typical thermopower of a tunnel barrier

In the basis $\{1, N, 2, \ldots, N-2\}$ $\quad G = [E - \mathcal{H} - \Sigma_L - \Sigma_R]^{-1}$

$$G = \begin{pmatrix} A & B \\ \tilde{B} & C \end{pmatrix}^{-1} \qquad A = (E - V_g - \sigma)\mathbf{1_2}$$

σ = nonvanishing part of Σ

$$\mathcal{T}(E) = \mathrm{Tr}\left[\begin{pmatrix} \gamma & 0 \\ 0 & 0 \end{pmatrix} G_A \begin{pmatrix} 0 & 0 \\ 0 & \gamma \end{pmatrix} G_A^\dagger\right] = \gamma^2 |G_A^{(1N)}|^2 \qquad \gamma = -2\mathrm{Im}(\sigma)$$

$$S = t\,\frac{\mathrm{d}\ln\mathcal{T}}{\mathrm{d}E}\bigg|_{E_F}$$

$$\boxed{\frac{S_0^{TB}}{N} \underset{N\to\infty}{\approx} \frac{1}{N}\,\frac{2t}{\Gamma(E_F)}\,\frac{\mathrm{d}\Gamma}{\mathrm{d}E}\bigg|_{E_F} \pm \frac{1}{\sqrt{\left(\frac{E_F - V_g}{2t}\right)^2 - 1}}}$$

Three-terminal quantum thermal machines

Efficiency: depends on the sign of the heat currents $$\eta = \frac{\dot{W}}{\sum_{i_+} J_i^Q}$$

$$\eta_1 \equiv \frac{\dot{W}}{J_1^Q} \qquad \eta_2 \equiv \frac{\dot{W}}{J_2^Q} \qquad \eta_{12} \equiv \frac{\dot{W}}{J_1^Q + J_2^Q}$$

Carnot limit

$$\eta_{C,1} = 1 - \frac{T_3}{T_1} + \frac{J_2^Q}{J_1^Q}(1-\zeta_{32}) = \eta_C^{II} + \frac{J_2^Q}{J_1^Q}(1-\zeta_{32})$$

$$\eta_{C,2} = \eta_C^{II} - \frac{T_3}{T_1}\left[\frac{J_1^Q}{J_2^Q}(1-\zeta_{13}) - (1-\zeta_{12})\right]$$

$$\eta_{C,12} = 1 - \frac{T_3}{T_1}\left(1 + \frac{\zeta_{12}-1}{1+\frac{J_1^Q}{J_2^Q}}\right) = \eta_C^{II} - \frac{T_3}{T_1}\frac{\zeta_{12}-1}{1+\frac{J_1^Q}{J_2^Q}}$$

Not a function of the temperatures only !!

Efficiencies at maximum power

$$\eta_1(\dot{W}_{\max}) = \frac{\eta_{C,1}}{2}\frac{\mathcal{Z}_{11}T}{C_1 + \mathcal{Z}_{11}T}$$

$$\eta_2(\dot{W}_{\max}) = \frac{\eta_{C,2}}{2}\frac{\mathcal{Z}_{22}T}{C_2 + \mathcal{Z}_{22}T}$$

$$\eta_{12}(\dot{W}_{\max}) \simeq \frac{\eta_{C,12}}{2}\frac{\mathcal{Z}_{12}T}{C_{12} + \mathcal{Z}_{12}T}$$

Generalized figures of merit

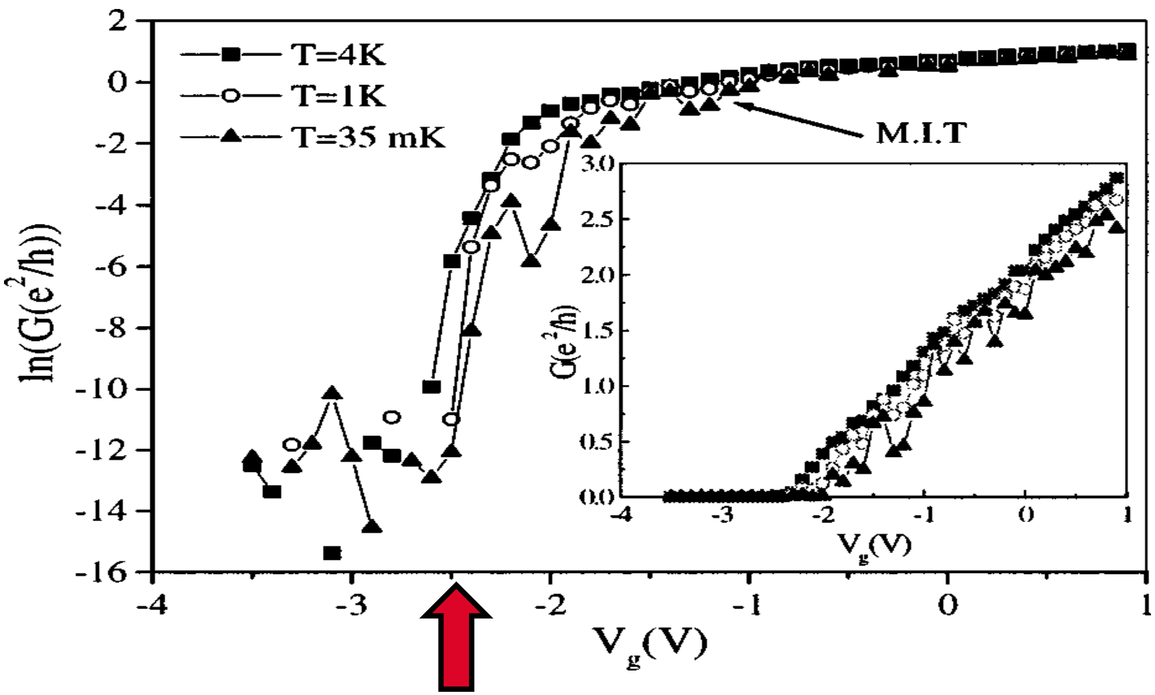
$$\boxed{\mathcal{Z}_{ij} = f(G_{kl}S_{mn}S_{pq}/K_{uv})} \quad k,l,m,n,u,v \in \{1,2\}$$

$$C_{ij}, C_i = f(K_{ij}, X_1^T, X_2^T)$$

F. Mazza, R. Bosisio, G. Benenti, V. Giovannetti., R. Fazio & F. Taddei, *New J. Phys.* (2014)

GATE MODULATED CARRIER DENSITY

Anderson Insulator → ← **Conductor**

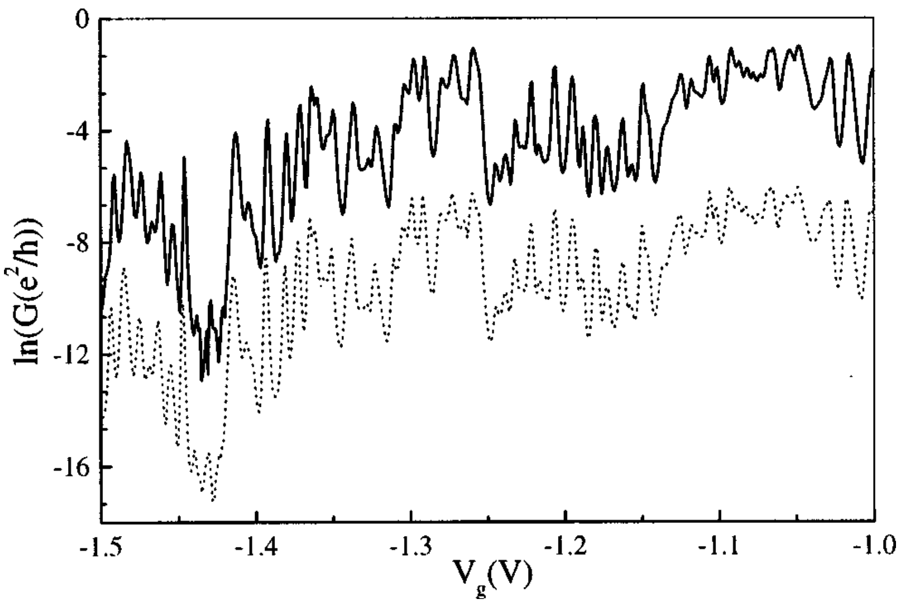


ln *G*(*V* gate) at three temperatures in a large gate voltage range
(details of the conductance pattern are not seen for this gate voltage sampling)
Inset: the same curve in a linear scale. Note the linearity at voltages above the transition.

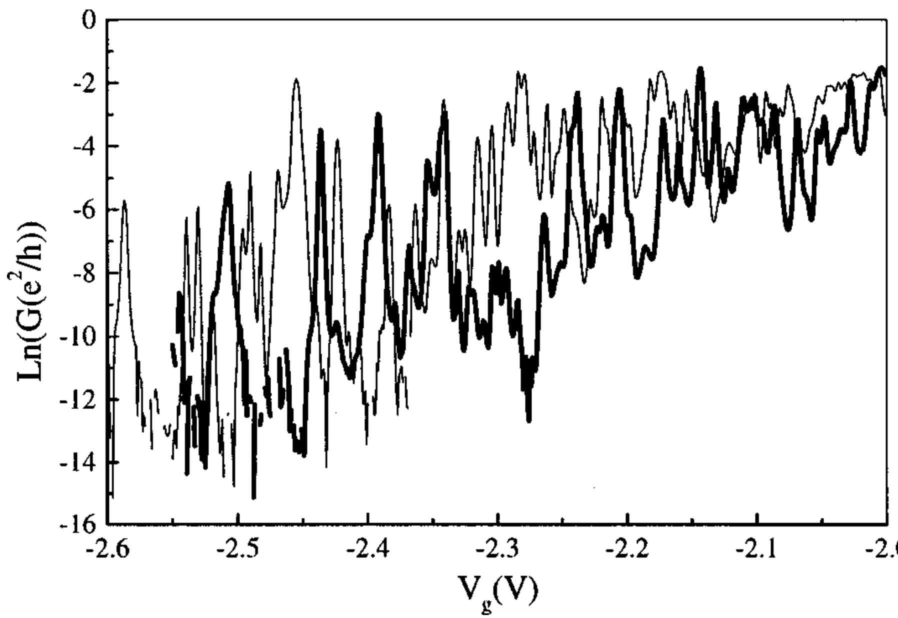
Edge of the impurity band = -2,5 V
(Complete depletion of the disordered nanowire)

CONDUCTANCE FLUCTUATION S INDUCED BY VARYING THE GATE VOLTAGE

Bulk of the impurity band



Edge of the impurity band



Mott Formula :

$$S \approx \frac{\partial \ln(G)}{\partial V_g}$$

Larger thermopower at the band edges